1a) 1 and -3 , $x = 2t$ $y = t^2$

Cartesian

$(2,1)$ and $(-6,9)$

$m = \dfrac{9-1}{-6-2}$

$= -1$

$y - 1 = -1(x-2)$

$y - 1 = -x + 2$

$\underline{x + y - 3 = 0}$

parametrics

$y = \frac{1}{2}(p+q)x - apq$

$p = 1$, $q = -3$, $a = 1$

$y = \frac{1}{2}(1-3)x - (1)(1)(-3)$

$y = -x + 3$

$\underline{x + y - 3 = 0}$

1(c) -1 and -2, $x = t$ $y = \frac{1}{2}t^2$

$(-1, \frac{1}{2})$ and $(-2, 2)$

$$m = \frac{2 - \frac{1}{2}}{-2 + 1}$$

$$= -\frac{3}{2}$$

1d) $x = 2at \quad y = at^2$

$x = \frac{1}{2}t, \quad y = \frac{1}{4}t^2$

$t = -2, \ (-1, 1)$

$t = 4, \ (2, 4)$

OR

$(p+q)x - 2y = 2apq$

$a = \frac{1}{4}, \ p = -2, \ q = 4$

$2x - 2y = -4$

$x - y + 2 = 0$

2a) $p = 2,\ q = -\frac{1}{2}$

$x = 6t\ ,\ y = 3t^2$

$\therefore \underline{\underline{a = 3}}$

$(p+q)x - 2y = 2apq$

$\frac{3x}{2} - 2y = -6$

$\underline{3x - 4y + 12 = 0}$

b) $x^2 = 4ay$

$x^2 = 12y$

$\underline{\underline{y = \frac{1}{12}x^2}}$ focus $\underline{(0,3)}$

c) $\underline{\underline{(0,3)}}$:

$3(0) - 4(3) + 12$

$= 0 - 12 + 12$

$\underline{\underline{= 0}}$

$\therefore$ chord is focal chord

3c)

<u>PQ</u>: $y - \frac{1}{2}(p+q)x + apq = 0$

directrix: $y = -a$

$$-a - \frac{1}{2}(p+q)x + apq = 0$$

$$(p+q)x = 2apq - 2a$$

$$x = \frac{2a(pq-1)}{p+q}$$

3/

c) PQ intersects directrix

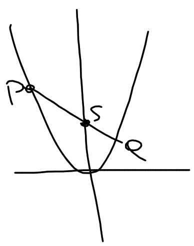
at $\left(\dfrac{2a(pq-1)}{p+q}, -a \right)$

e) $pq = -1$

f) $\left(\dfrac{-4a}{p+q}, -a \right)$

4/ d) $PS + QS = a(p^2 + q^2 + 2)$

e) $pq = -1$ ie it is a focal chord.



If focal chord $PQ = PS + QS$

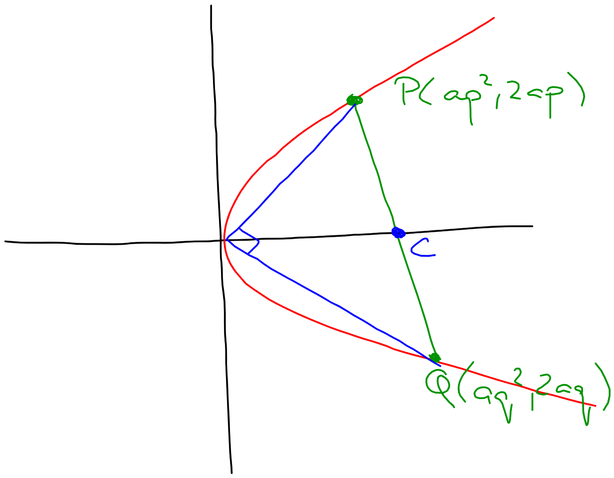
$$= a(p^2 + q^2 + 2)$$

$$= a\left(p^2 + \left(-\frac{1}{p}\right)^2 + 2\right)$$

$$= a\left(p^2 + 2 + \frac{1}{p^2}\right)$$

$$= a\left(p + \frac{1}{p}\right)^2$$

5/ $x = at^2 \quad y = 2at$



$$m_{PQ} = \frac{2ap - 2aq}{ap^2 - aq^2}$$

$$= \frac{2a(p-q)}{a(p+q)(p-q)}$$

$$= \frac{2}{(p+q)}$$

$$y - 2ap = \frac{2}{p+q}(x - ap^2)$$

$$(p+q)y - 2ap^2 - 2apq = 2x - 2ap^2$$

$$\underline{2x - (p+q)y + 2apq = 0}$$

x intercept occurs when $y = 0$

$$2x + 2apq = 0$$

$$\underline{x = -apq}$$

$$m_{OP} \times m_{OQ} = -1$$

$$\frac{2ap}{ap^2} \times \frac{2aq}{aq^2} = -1$$

$$\frac{4}{pq} = -1$$

$$\underline{pq = -4}$$

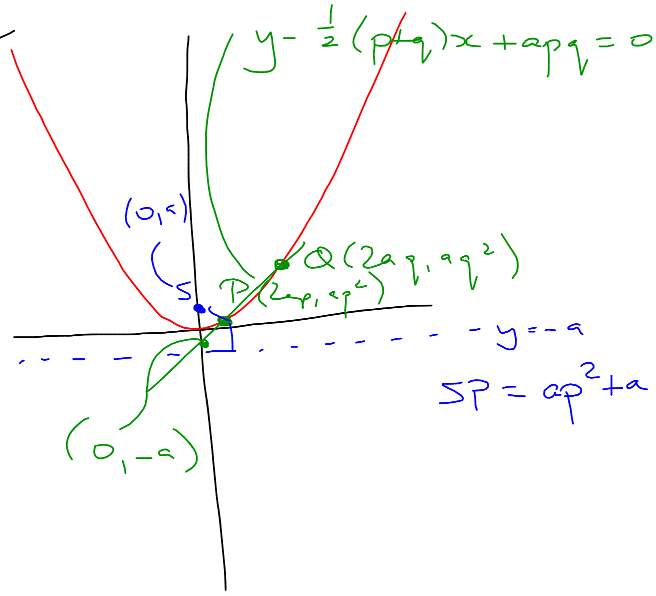
$\therefore$ x intercept is

$$x = -a \times -4$$

$$x = 4a$$

which is independent of p and q.

7)



b) $pq = -1$

c) $\frac{1}{SP} + \frac{1}{SQ}$

$$= \frac{1}{a(p^2+1)} + \frac{1}{a(q^2+1)}$$

$$= \frac{q^2+1+p^2+1}{a(p^2+1)(q^2+1)}$$

$$= \frac{p^2+q^2+2}{a(p^2+q^2+p^2q^2+1)}$$

$$= \frac{p^2+q^2+2}{a(p^2+q^2+1+1)}$$

$$= \frac{1}{a}$$