1a) $t = 1$, $x = 2t$ $y = t^2$

$\frac{dx}{dt} = 2$ $\frac{dy}{dt} = 2t$ OR $y = \frac{1}{4}x^2$

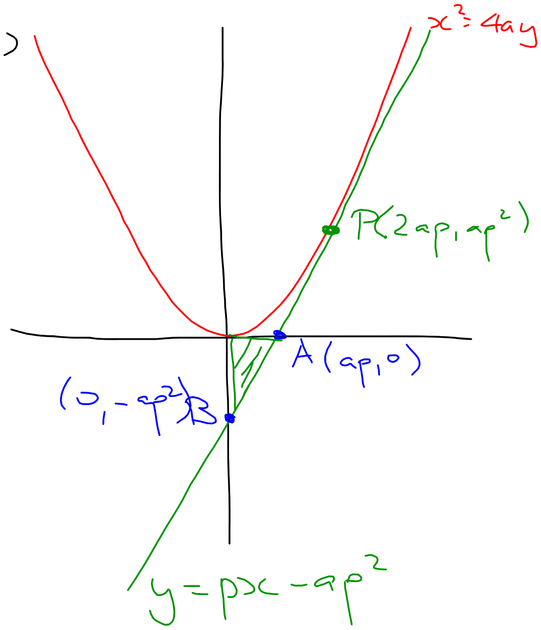
$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$ $\frac{dy}{dx} = \frac{1}{2}x$

$= 2t \times \frac{1}{2}$ when $x = 2$, $\frac{dy}{dx} = 1$

$= t$

when $t = 1$, $\frac{dy}{dx} = 1$

3b)



$\underline{A}$: $y = 0$

$0 = px - ap^2$

$px = ap^2$

$x = ap$

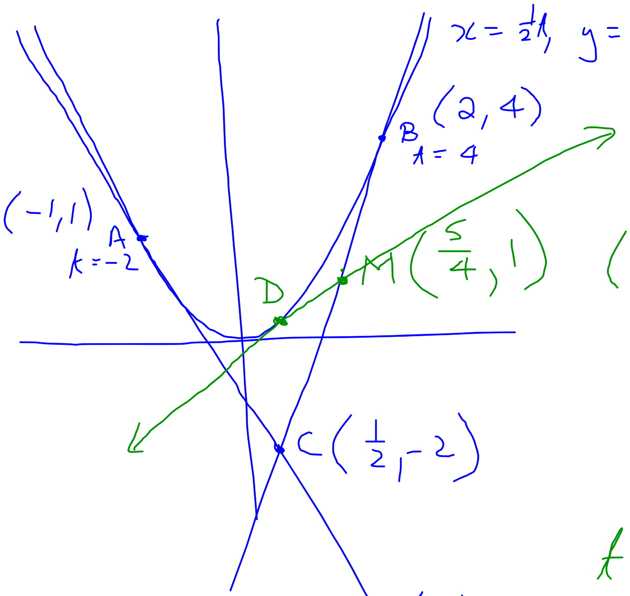
$\underline{\underline{A(ap, 0)}}$

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times ap \times ap^2$

$= \underline{\underline{\frac{1}{2}a^2p^3}}$

8c) $x = \frac{1}{2}t, \; y = \frac{1}{4}t^2$



$y = tx - at^2$

$y = tx - \frac{1}{4}t^2$

$(\frac{5}{4}, 1)$: $1 = \frac{5}{4}t - \frac{1}{4}t^2$

$t^2 - 5t + 4 = 0$

$(t-4)(t-1) = 0$

$t = 4$ or $t = 1$

$t = 1$, $D(\frac{1}{2}, \frac{1}{4})$

$m_{tangent} = 1$ $\quad m_{AB} = \frac{4-1}{2+1} = \frac{3}{3} = 1 = m_{tangent}$

$\therefore AB \parallel$ tangent at D

9a) $x = 6t \quad y = 3t^2$

$$x - y - 3 = 0$$

$$6t - 3t^2 - 3 = 0$$

$$t^2 - 2t + 1 = 0$$

$$(t-1)^2 = 0$$

$$t = 1$$

$\therefore$ pt contact is $\underline{(6, 3)}$

9c)

$x = 2t \quad y = t^2$

$x + y + a = 0$

$2t + t^2 + a = 0.$

$t^2 + 2t + a = 0$

$\Delta = 4 - 4a$

tangent if $\Delta = 0$

$\therefore \underline{a = 1}$

11/ $x^2 = 16y$

$y = \frac{1}{16}x^2$

$\frac{dy}{dx} = \frac{1}{8}x$

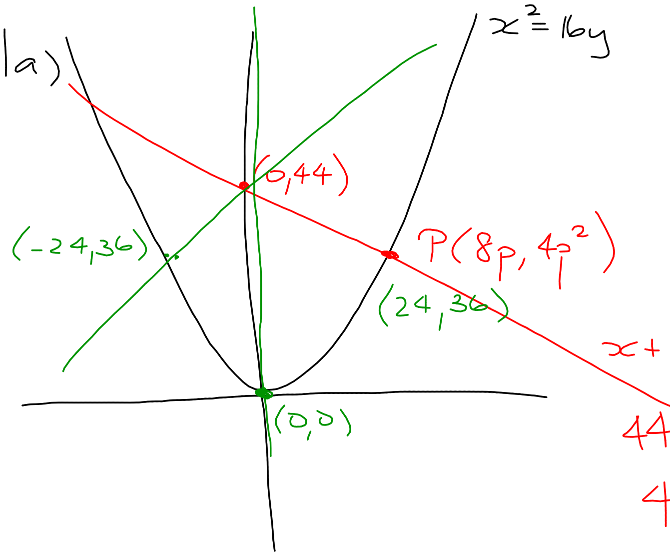
at $x = 8p$, $\frac{dy}{dx} = p$

$y - 4p^2 = -\frac{1}{p}(x - 2ap)$

$py - 4p^3 = -x + 2ap$

$\underline{x + py = 2ap + 4p^3}$

11a)



$x + py = 8p + 4p^3$

$44p = 8p + 4p^3$

$4p^3 - 36p = 0$

$4p(p^2 - 9) = 0$

$p = 0$ or $p = \pm 3$

$\therefore$ the 3 points are $(0,0)$, $(-24,36)$ and $(24,36)$

12 b)
$$(p-q)(p^2+pq+q^2)$$
$$= p^3+p^2q+pq^2-p^2q-pq^2-q^3$$
$$= \underline{p^3-q^3}$$

c) $\left(-apq(p+q),\ a(p^2+q^2+pq+2)\right)$

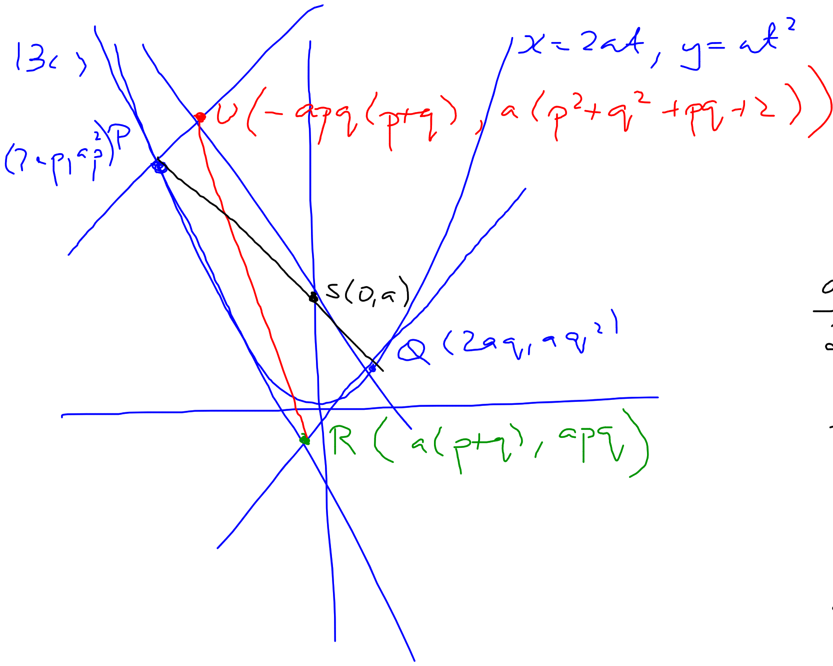
d) $\underline{pq=2}$

$$x^2 = a^2p^2q^2(p+q)^2$$
$$= 4a^2(p+q)^2$$

$$4ay = 4a^2\left(p^2+q^2+pq+2\right)$$
$$= 4a^2(p^2+2pq+q^2)$$
$$= 4a^2(p+q)^2$$

$\therefore x^2 = 4ay$

point lies on parabola



If PQ is focal chord

$$m_{PS} = m_{PQ}$$

$$\frac{ap^2 - a}{2ap - 0} = \frac{ap^2 - aq^2}{2ap - 2aq}$$

$$\frac{p^2 - 1}{2p} = \frac{p^2 - q^2}{2p - 2q}$$

$$= \frac{p+q}{2}$$

$$2p^2 - 1 = 2p^2 + 2pq$$

$$\underline{\underline{pq = -1}}$$

$x_U = -apq(p+q)$ $\qquad$ $x_R = a(p+q)$

$= a(p+q) \qquad (pq=-1)$

$= x_R$

$\therefore \underline{UR \parallel y \text{ axis}}$