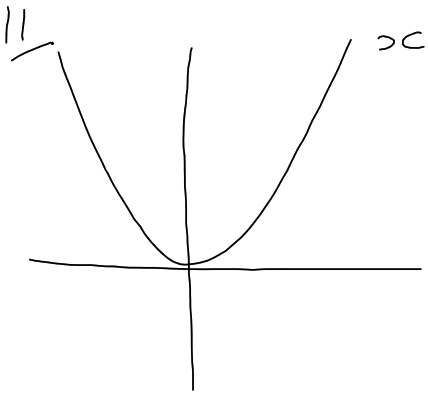

$x^2 = 4a(1-2x)$

$x^2 + 8ax - 4a = 0$

$\Delta = 0$

$64a^2 + 16a = 0$

$16a(4a+1) = 0$

$a = 0$ or $a = -\frac{1}{4}$

12a) $x^2 = 4ay$

$y = mx + b$

$x^2 = 4a(mx + b)$

$x^2 - 4amx - 4ab = 0$

tangent when $\Delta = 0$

ie $16a^2m^2 + 16ab = 0$

$am^2 + b = 0$

$b = -am^2$

$\therefore y = mx - am^2$

b) $x^2 = 12y$

$4a = 12$

$a = 3$

tangent $\parallel y = 7x$

$\therefore m = 7$

$y = 7x - 147$

13a) $mx - y + m^2 = 0 \Rightarrow y = mx + m^2$

$x^2 = -4y$

$x^2 = -4mx - 4m^2$

$x^2 + 4mx + 4m^2 = 0$

tangent if $\Delta = 0$

$\Delta = 16m^2 - 16m^2$

$= 0$

$\therefore$ line is a tangent

13b)

a) $mx - y + m^2 = 0$ touches $x^2 = -4y$

hence find tangents through $(1,2)$

$$m - 2 + m^2 = 0$$

$$m^2 + m - 2 = 0$$

$$(m+2)(m-1) = 0$$

$$m = -2 \text{ or } m = 1$$

tangents are

$$-2x - y + 4 = 0 \text{ and } x - y + 1 = 0$$

15 a) $ax+by=1 \Rightarrow y=\frac{1-ax}{b}$

$x^2=12y$

$x^2=12\left(\frac{1-ax}{b}\right)$

$bx^2=12-12ax$

$bx^2+12ax-12=0$

tangent when $\Delta=0$

$144a^2+48b=0$

$3a^2+b=0$

b) tangent $(0,-27)$

$-27b=1$

$b=\frac{1}{-27}$

$3a^2-\frac{1}{27}=0$

$a^2=\frac{1}{81}$

$a=\pm\frac{1}{9}$

$\frac{1}{9}x-\frac{1}{27}y=1$ and $-\frac{1}{9}x-\frac{1}{27}y=1$

$3x-y=27$ $\quad -3x-y=27$

c) tangent passes through (4,1)

$\therefore 4a+b=1$

$b=1-4a$

$3a^2+b=0$

$3a^2+1-4a=0$

$3a^2-4a+1=0$

$(3a-1)(a-1)=0$

$a=\frac{1}{3}$ or $a=1$

$b=-\frac{1}{3}$ $\quad b=-3$

$\therefore$ tangents are $\frac{1}{3}x-\frac{1}{3}y=1$ and $x-3y=1$

$x-y=3$

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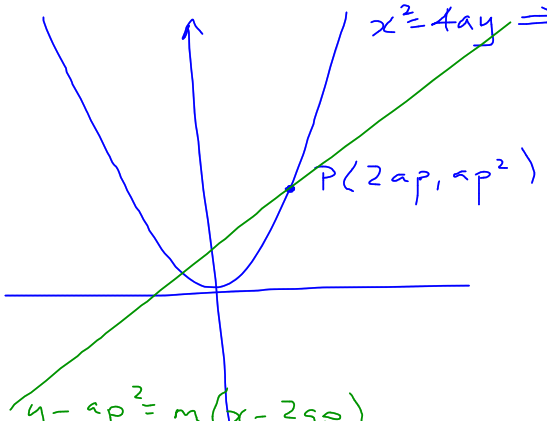

$y - ap^2 = m(x - 2ap)$

$at^2 - ap^2 = m(2at - 2ap)$

$at^2 - 2amt + (2apm - ap^2) = 0$

$t^2 - 2mt + (2pm - p^2) = 0$

b) $\Delta = 4m^2 - 4(2pm - p^2)$

$= 4(m^2 - 2pm + p^2)$

$= 4(m-p)^2$

tangent when $\Delta = 0$

$4(m-p)^2 = 0$

$m = p$

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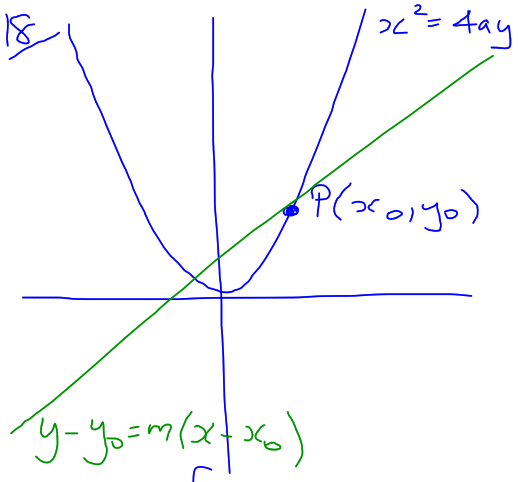

$y - y_0 = m(x - x_0)$

$x^2 = 4a\left[m(x - x_0) + y_0\right]$

$x^2 = 4a\left[m(x - x_0) + \frac{x_0^2}{4a}\right]$

$x^2 - 4amx + (4amx_0 - x_0^2) = 0$

b) $\Delta = 16a^2m^2 - 4(4amx_0 - x_0^2)$

$= 4(x_0^2 - 4amx_0 + 4a^2m^2)$

$= 4(x_0 - 2am)^2$

tangent when $\Delta = 0$

$4(x_0 - 2am)^2 = 0$

$x_0 = 2am$

$m = \frac{x_0}{2a}$

$\therefore$ tangent is

$y - y_0 = \frac{x_0}{2a}(x - x_0)$

$y - y_0 = \frac{x_0x}{2a} - \frac{x_0^2}{2a}$

$= \frac{x_0x}{2a} - \frac{4ay_0}{2a}$

$= \frac{x_0x}{2a} - 2y_0$

$y + y_0 = \frac{x_0x}{2a}$

$x_0x = 2a(y + y_0)$