1)

$$M\left(a(p+q),\ \frac{1}{2}a(p^2+q^2)\right)$$

$$T\left(a(p+q),\ apq\right)$$

$$N\left(a(p+q),\ \frac{\frac{a}{2}(p^2+q^2+2pq)}{2},\ \frac{a(p+q)^2}{4}\right)$$

7e)

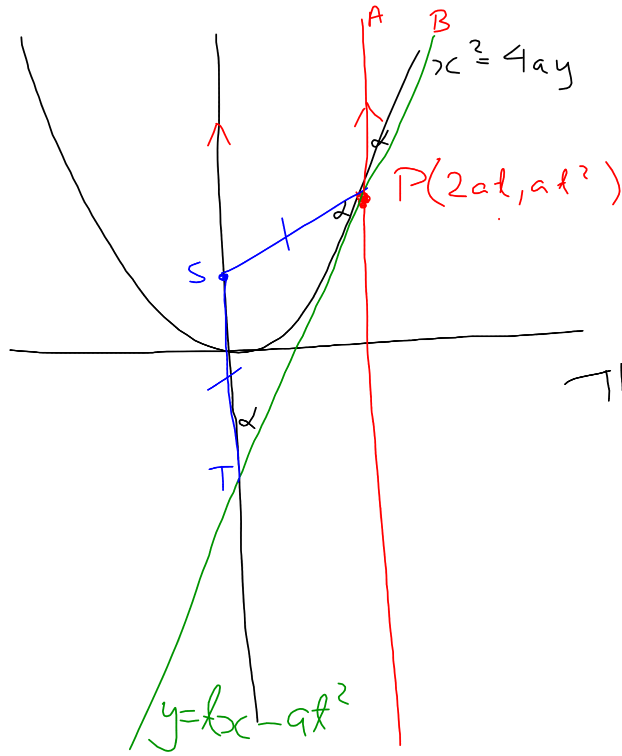
$$y = -\frac{2}{p+q}x + a$$

$\underline{x=0}$, $y = 0 + a$
$= a$

$\therefore$ line passes through focus
but PQ is a focal chord

Thus $(0, a)$ is the pt of intersection

11(c)



Let $\angle SPT = \alpha$

$\angle STP = \alpha$ (base $\angle$'s isosceles $\triangle$)

$\angle STP = \angle APB$ (corresponding $\angle$'s $=$, y axis $\parallel PA$)

$\therefore \angle APB = \alpha$

Thus

$\underline{\angle APB = \angle SPT}$

12 d)

$Q\left(\frac{2ay_1}{x_1}, 0\right)$ $\qquad R(0, -y_1)$

$$m_{SQ} \times m_{PQ} = \frac{a - 0}{0 - \frac{2ay_1}{x_1}} \times \frac{y_1 - 0}{x_1 - \frac{2ay_1}{x_1}}$$

$$= \frac{-x_1}{2\cancel{y_1}} \times \frac{x_1 \cancel{y_1}}{x_1^2 - 2ay_1}$$

$$= \frac{-x_1^2}{2(4ay_1 - 2ay_1)}$$

$$= \frac{-x_1^2}{4ay_1}$$

$$= \underline{-1} \qquad (\because x_1^2 = 4ay_1)$$

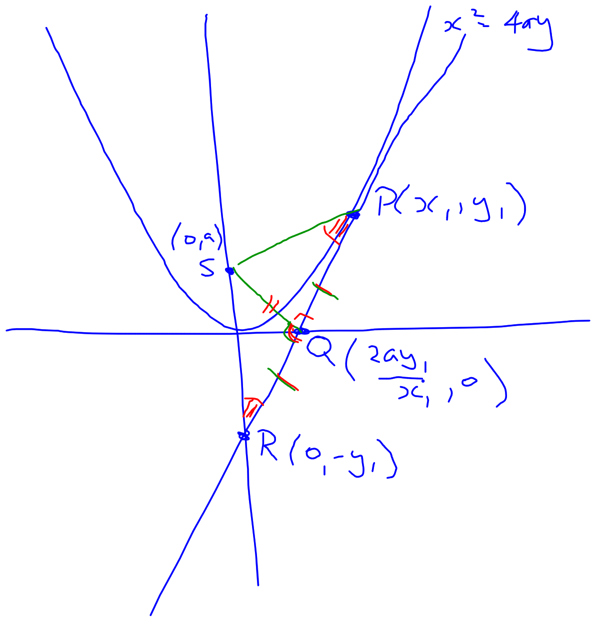
$P(x_1, y_1)$

$R(0, -y_1)$

$M\left(\frac{x_1}{2}, 0\right)$ which lies on x axis

$\therefore$ x axis (tangent at vertex) bisects PR.

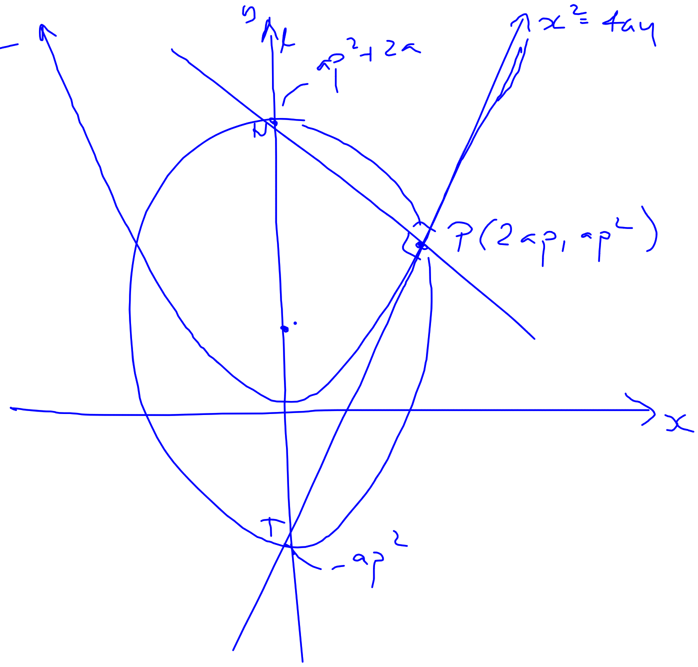
e)



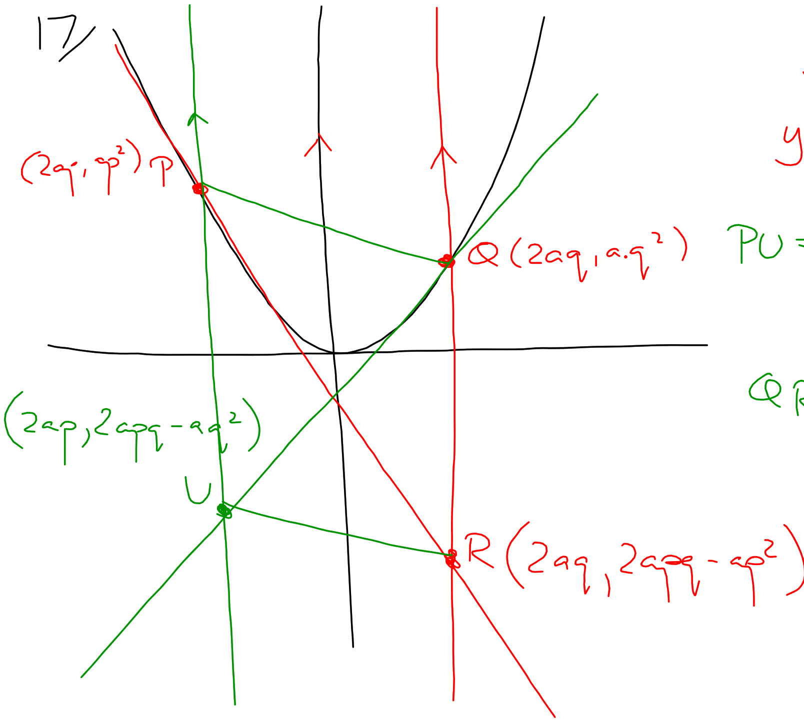
$\triangle PQS \equiv \triangle RQS$ (SAS)

$\therefore \angle SPQ = \angle SRQ$ (matching $\angle$'s in $\equiv \triangle$'s)

13



17/



$y = px - ap^2$

$y = 2apq - ap^2$

$$PU = ap^2 - (2apq - aq^2)$$
$$= ap^2 - 2apq + aq^2$$

$$QR = aq^2 - (2apq - ap^2)$$
$$= aq^2 - 2apq + ap^2$$
$$= PU$$

$\therefore$ PQRU is $\parallel$gram

(opposite sides = and $\parallel$)

b) $A = bh$

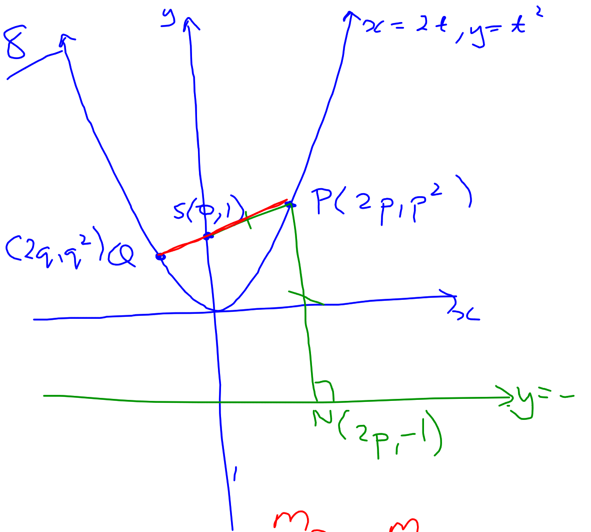
$= (ap^2 + aq^2 - 2apq)(2ap - 2aq)$

$= 2a^2(p^2 - 2pq + q^2)(p-q)$

$= 2a^2(p-q)^2(p-q)$

$= 2a^2(p-q)^3$

18



a) $PS = PN$ (def of parabola)

$= p^2 + 1$

b) $PS + QS$

$= p^2 + 1 + q^2 + 1$

$= p^2 + 2 + q^2$

c) $PQ = PS + QS$

$= p^2 + 2 + q^2$

$= p^2 + 2 + \frac{1}{p^2}$

$= \left(p + \frac{1}{p}\right)^2$

$m_{PQ} = m_{PS}$

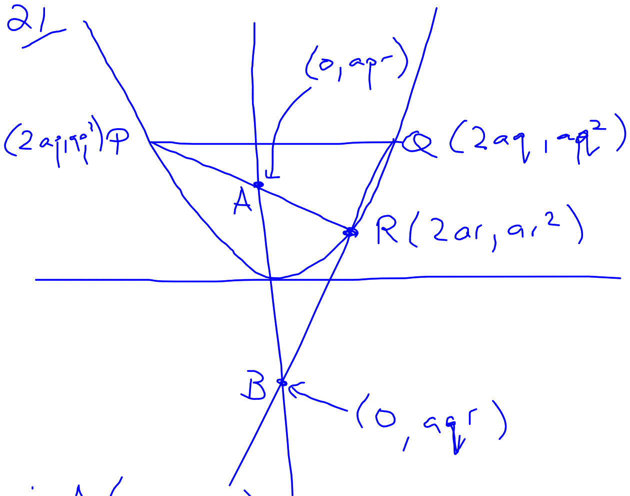
$\frac{p^2 - q^2}{2p - 2q} = \frac{p^2 - 1}{2p}$

$\frac{(p+q)\cancel{(p-q)}}{\cancel{(p-q)}} = \frac{p^2 - 1}{p}$

$p^2 + pq = p^2 - 1$

$\underline{pq = -1} \Rightarrow q = -\frac{1}{p}$

21)



$\therefore A(0, apr)$

$B(0, -apr)$

$M_{AB} = (0, 0)$

$$m_{PR} = \frac{ap^2 - ar^2}{2ap - 2ar}$$

$$= \frac{p+r}{2}$$

$$y - ap^2 = \frac{p+r}{2}(x - 2ap)$$

$$2y - 2ap^2 = (p+r)x - 2ap^2 + 2apr$$

$$(p+r)x - 2y = -2apr$$

$x = 0$ $\quad y = apr$

$m_{PQ} = 0$

$$\frac{ap^2 - aq^2}{2ap - 2aq} = 0$$

$$p^2 - q^2 = 0$$

$$p^2 = q^2$$

$$q = \pm p$$

$\therefore q = -p$ (∵ if $p = q$, P, Q same pt)