3c) $R(2t, 2t^2+2)$

$x = 2t$

$t = \frac{x}{2}$

$y = 2t^2 + 2$

$y = 2\left(\frac{x^2}{4}\right) + 2$

$\underline{y = \frac{x^2}{2} + 2}$.

$2y = x^2 + 4$

$x^2 = 2y - 4$

$x^2 = 2(y-2)$

$\therefore$ Vertex is $(0,2)$

5d)

$x = a(p+q)$

$p+q = \frac{x}{a}$

$y = \frac{1}{2}a(p^2+q^2)$

$= \frac{1}{2}a\left[(p+q)^2 - 2pq\right]$

$= \frac{1}{2}a\left[\frac{x^2}{a^2} - 2(-1)\right]$ $(pq = -1)$

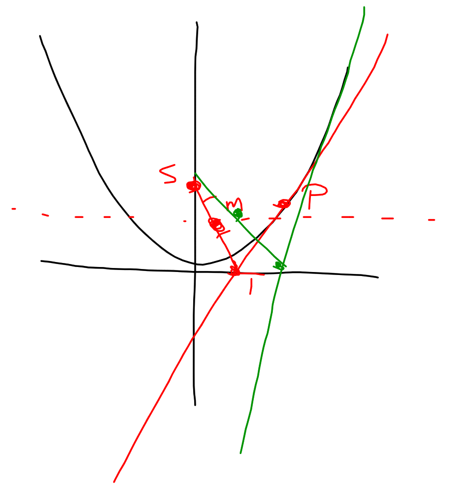
$= \frac{x^2}{2a} + a$

$2ay = x^2 + 2a^2$

$x^2 = 2ay - 2a^2$

$\underline{x^2 = 2a(y-a)}$

7c) $T\left(-\frac{a}{p}, 0\right)$, $S(0, a)$

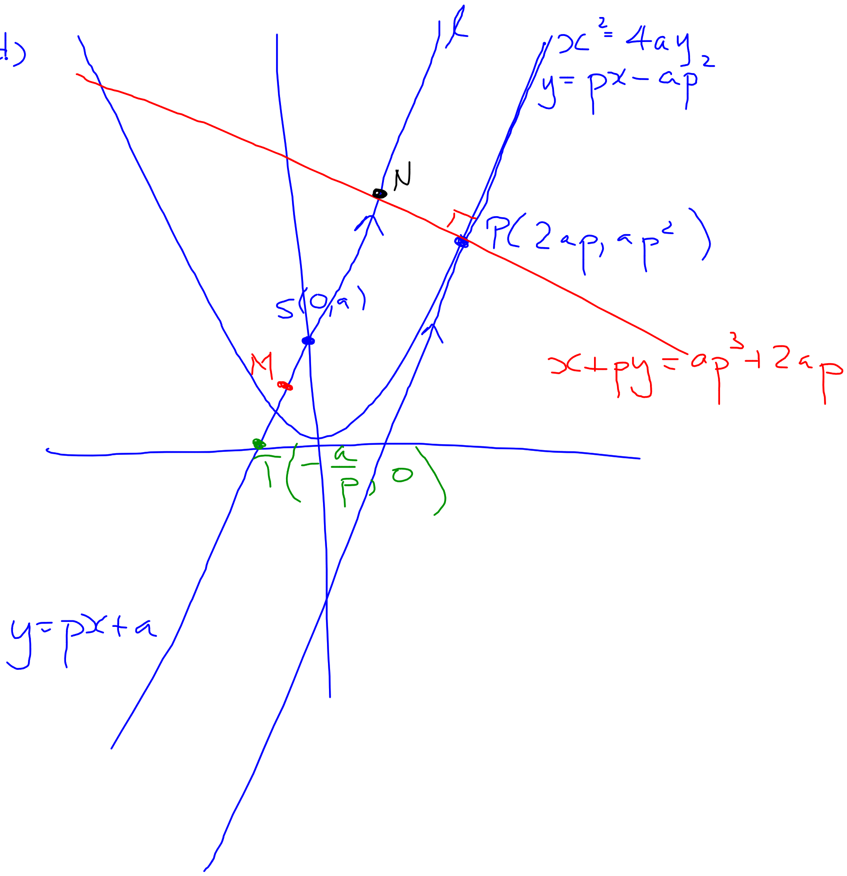


$$M = \left(-\frac{a}{2p}, \frac{a}{2}\right)$$

$\therefore$ locus is $y = \frac{a}{2}$

excluding $\left(0, \frac{a}{2}\right)$

7 d)



$x + py = ap^3 + 2ap$

$px - y = -a$

$x + p^2x + ap = ap^3 + 2ap$

$x(p^2+1) = ap^3 + ap$

$= ap(p^2+1)$

$x = ap$

$y = ap^2 + a$

$= a(p^2+1)$

$N\left(ap, a(p^2+1)\right)$

$x = ap$

$p = \frac{x}{a}$

$y = a(p^2+1)$

$y = a\left(\frac{x^2}{a^2}+1\right)$

$y = \frac{x^2}{a} + a$

9c)

$pq = -4$

$x = a(p+q)$

$p+q = \frac{x}{a}$

$y = \frac{1}{2}a(p^2+q^2)$

$y = \frac{1}{2}a\left[(p+q)^2 - 2pq\right]$

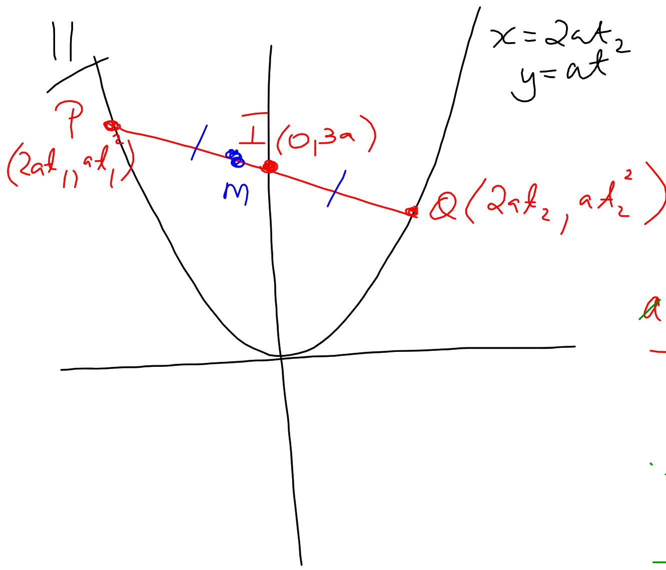
$y = \frac{1}{2}a\left[\frac{x^2}{a^2} + 8\right]$

$y = \frac{x^2}{2a} + 4a$

$\frac{x^2}{2a} = y - 4a$

$\underline{x^2 = 2a(y-4a)}$

11



$$m_{PQ} = m_{PI}$$

$$\frac{at_2^2 - at_1^2}{2at_2 - 2at_1} = \frac{3a - at_1^2}{0 - 2at_1}$$

$$\frac{\cancel{a}\cancel{(t_2-t_1)}(t_2+t_1)}{2\cancel{a}\cancel{(t_2-t_1)}} = \frac{\cancel{a}(3-t_1^2)}{-2\cancel{a}t_1}$$

$$-t_1(t_2+t_1) = 3 - t_1^2$$

$$-t_1t_2 - t_1^2 = 3 - t_1^2$$

$$t_1t_2 = -3$$

11b) $t_1 t_2 = -3$

$$M\left(a(t_1+t_2),\ \frac{a}{2}(t_1^2+t_2^2)\right)$$

$x = a(t_1+t_2)$

$t_1+t_2 = \frac{x}{a}$

$$y = \frac{a}{2}(t_1^2+t_2^2)$$

$$= \frac{a}{2}\left[(t_1+t_2)^2 - 2t_1t_2\right]$$

$$= \frac{a}{2}\left[\frac{x^2}{a^2}+6\right]$$

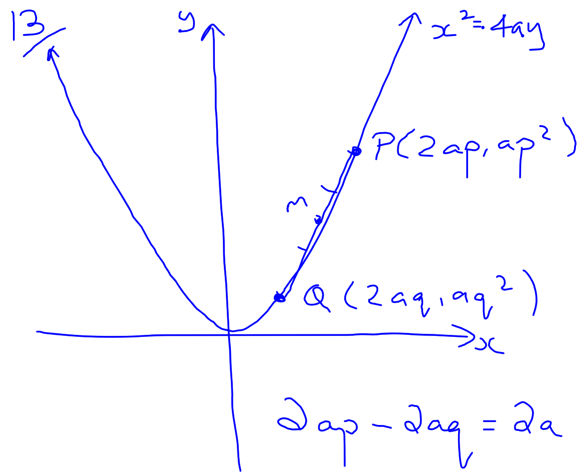
$$= \frac{x^2}{2a} + 3a$$

$$2ay = x^2 + 6a^2$$

$$x^2 = 2ay - 6a^2$$

$$\underline{x^2 = 2a(y-3a)}$$

13)



$2ap - 2aq = 2a$

$\underline{\underline{p - q = 1}}$

$p^2 - 2pq + q^2 = 1$

$p^2 + 2pq + q^2 = \frac{x^2}{a^2}$

$2p^2 + 2q^2 = \frac{x^2}{a^2} + 1$

$M = \left(\frac{2ap+2aq}{2}, \frac{ap^2+aq^2}{2}\right)$

$= \left\{a(p+q), \frac{a(p^2+q^2)}{2}\right\}$

$x = a(p+q)$

$p+q = \frac{x}{a}$

$y = \frac{a}{2}(p^2+q^2)$

$= \frac{a}{2}\left(\frac{1}{2}\left(\frac{x^2}{a^2}+1\right)\right)$

$= \frac{a}{4}\left(\frac{x^2}{a^2}+1\right)$

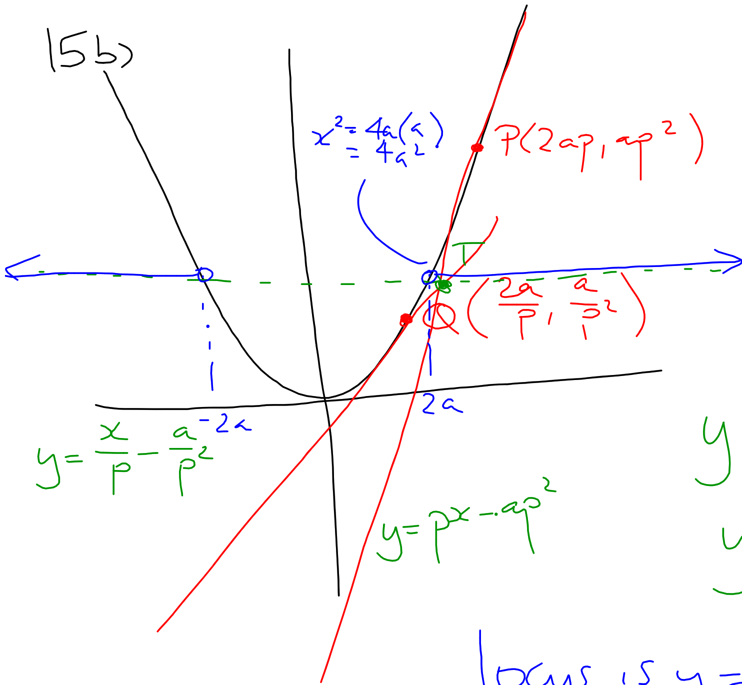
$\underline{y = \frac{x^2}{4a} + \frac{a}{4}}$

b) $4ay = x^2 + a^2$

$x^2 = 4a\left(y - \frac{a}{4}\right)$

parabola, vertex $\left(0, \frac{a}{4}\right)$

15b)



$$px - ap^2 = \frac{x}{p} - \frac{a}{p^2}$$

$$\left(p - \frac{1}{p}\right)x = a\left(p^2 - \frac{1}{p^2}\right)$$

$$x = a\left(p + \frac{1}{p}\right)$$

$$y = ap\left(p + \frac{1}{p}\right) - ap^2$$

$$y = a$$

locus is $y = a$, for $x < -2a$ and $x > 2a$

17 normal$_A$: $x + t_1 y = a t_1^3 + 2a t_1$

normal$_B$: $x + t_2 y = a t_2^3 + 2a t_2$

a) $m_{normal_A} \times m_{normal_B} = -1$

$-\frac{1}{t_1} \times -\frac{1}{t_2} = -1$

$\frac{1}{t_1 t_2} = -1$

$\underline{t_1 t_2 = -1}$

b) $x + t_1\left(a(t_1^2 + t_2^2 + 1)\right) = a t_1^3 + 2a t_1$

$x + a t_1^3 + a t_1 t_2^2 + a t_1 = a t_1^3 + 2a t_1$

$x = -a t_1 t_2^2 + a t_1$

$= a t_2 + a t_1 \quad (\because t_1 t_2 = -1)$

$= a(t_1 + t_2)$

N is $\underline{\left\{ a(t_1 + t_2), a(t_1^2 + t_2^2 + 1) \right\}}$

c) $x = a(t_1 + t_2)$

$t_1 + t_2 = \frac{x}{a}$

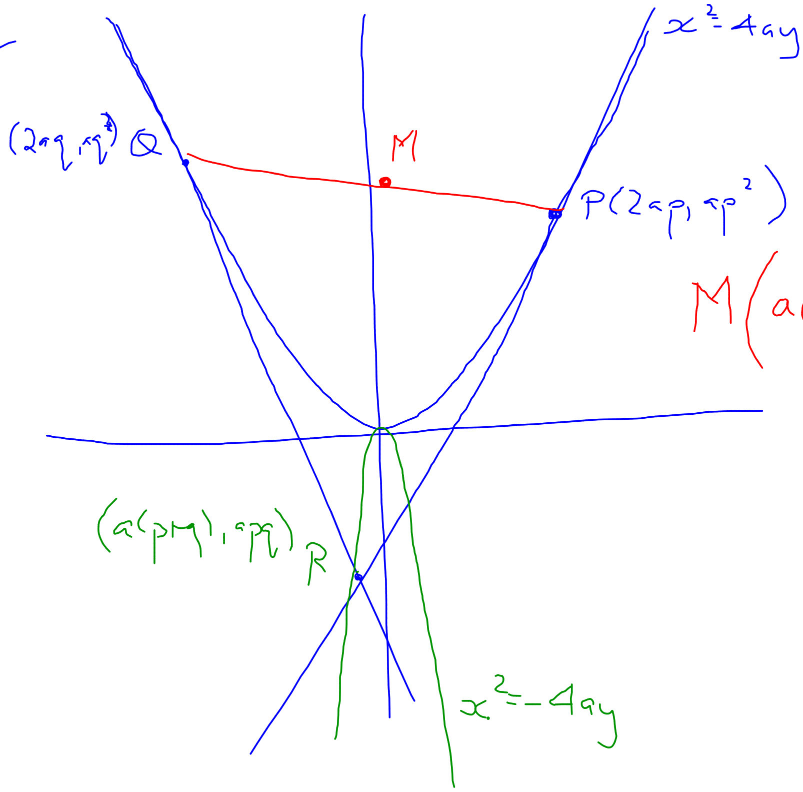
$y = a(t_1^2 + t_2^2 + 1)$

$= a\left[(t_1 + t_2)^2 - 2t_1 t_2 + 1\right]$

$= a\left[\left(\frac{x}{a}\right)^2 + 2 + 1\right]$

$\underline{y = \frac{x^2}{a} + 3a}$

19



$$p^2 + q^2 + 6pq = 0$$

$$M\left(a(p+q), \frac{a}{2}(p^2+q^2)\right)$$

19b) $R\left\{a(p+q),\ apq\right\}$

$$x^2 = -4ay$$

$$a^2(p+q)^2 = -4a^2pq$$

$$p^2 + 2pq + q^2 = -4pq$$

$$p^2 + q^2 + 6pq = 0$$

$$\frac{2y}{a} + \frac{3x^2 - 6ay}{a^2} = 0$$

$$3x^2 = 4ay$$

c) $M_{PQ} = \left(a(p+q),\ \frac{a(p^2+q^2)}{2}\right)$

$x = a(p+q)$ $\quad$ $y = \frac{a}{2}(p^2+q^2)$

$p+q = \frac{x}{a}$ $\quad$ $\frac{2y}{a} = p^2+q^2$

$$p^2+q^2 = (p+q)^2 - 2pq$$

$$\frac{2y}{a} = \frac{x^2}{a^2} - 2pq$$

$$2pq = \frac{x^2 - 2ay}{a^2}$$

<u>OR</u>

$$3x^2 = 4ay$$

$$3x^2 = 3\left[a(p+q)^{\cancel{2}}\right]^2$$

$$= 3a^2(p^2+2pq+q^2)$$

$$= 3a^2(-6pq+2pq)$$

$$= 3a^2(-4pq)$$

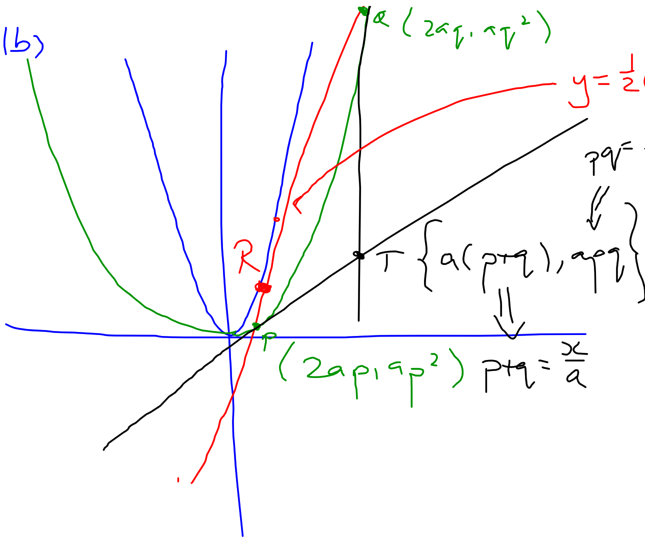
$$= -12a^2pq$$

$$4ay = 2a^2(p^2+q^2)$$

$$= 2a^2(-6pq)$$

$$= -12a^2pq$$

2(b)



$$y = \tfrac{1}{2}(p+q)x - apq$$

$$x^2 = 2ay$$

$$y = \frac{x^2}{2a}$$

$$\frac{x^2}{2a} = \frac{1}{2}(p+q)x - apq$$

$$x^2 = a(p+q)x - 2a^2pq$$

$$x^2 - a(p+q)x + 2a^2pq = 0$$

$$\Delta = 0$$

$$a^2(p+q)^2 - 8a^2pq = 0$$

$$(p+q)^2 - 8pq = 0$$

$$\underline{(p+q)^2 = 8pq}$$

c)

$$T\{a(p+q),\ apq\}$$

$$(p+q)^2 = 8pq$$

$$x = a(p+q)$$

$$\frac{x}{a} = p+q$$

$$y = apq$$

$$= \frac{a(p+q)^2}{8}$$

$$= \frac{a\left(\frac{x}{a}\right)^2}{8}$$

$$\underline{y = \frac{x^2}{8a}}$$

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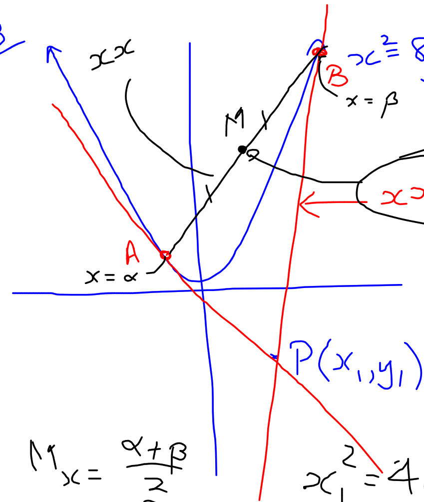

$xx_1 = 2a(y + y_1)$

$\frac{xx_1}{2a} = y + y_1$

$y = \frac{xx_1}{2a} - y_1$

$x^2 = \frac{4xx_1}{a} - 8y_1$

$ax^2 - 4x_1x + 8ay_1 = 0$

but $a = 2$.

$2x^2 - 4x_1x + 16y_1 = 0$

$x^2 - 2x_1x + 8y_1 = 0$

$x_1 - y_1 = 2$

$M_x = \frac{\alpha + \beta}{2}$

$= \frac{2x_1}{2}$

$= x_1$

$x_1^2 = 4y + 4y_1$.

$y = \frac{x_1^2}{4} - y_1$

M is $\{x_1, \frac{1}{4}x_1^2 - y_1\}$!

$X = x_1$

$Y = \frac{1}{4}X^2 - y_1$

$= \frac{1}{4}X^2 - x_1 + 2$

$Y = \frac{1}{4}X^2 - X + 2$