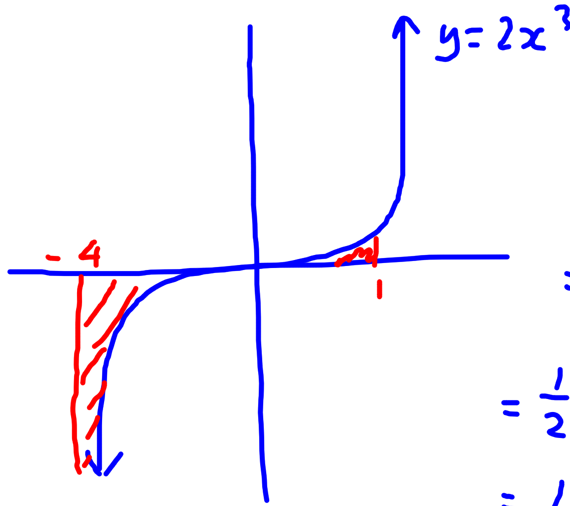
7b)



$$A = -\int_{-4}^{0} 2x^3\,dx + \int_{0}^{1} 2x^3\,dx$$

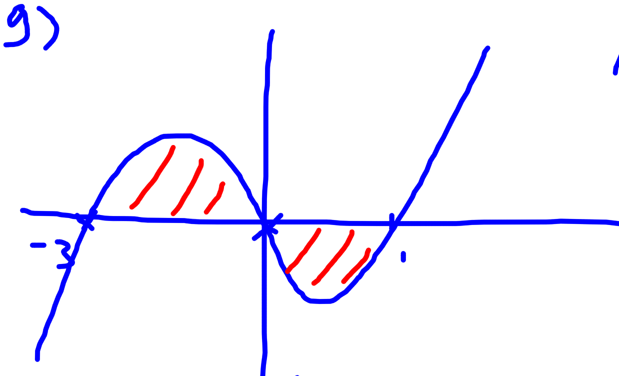
$$= \left[\frac{1}{2}x^4\right]_{0}^{-4} + \left[\frac{1}{2}x^4\right]_{0}^{1}$$

$$= \frac{1}{2}(-4)^4 + \frac{1}{2}(1)^4 - (0)^4$$

$$= 128 + \frac{1}{2}$$

$$= \frac{257}{2} \text{ units}^2$$

7g)



$$A = \int_{-3}^{0} (x^3 + 2x^2 - 3x)\,dx - \int_{0}^{1} (x^3 + 2x^2 - 3x)\,dx$$

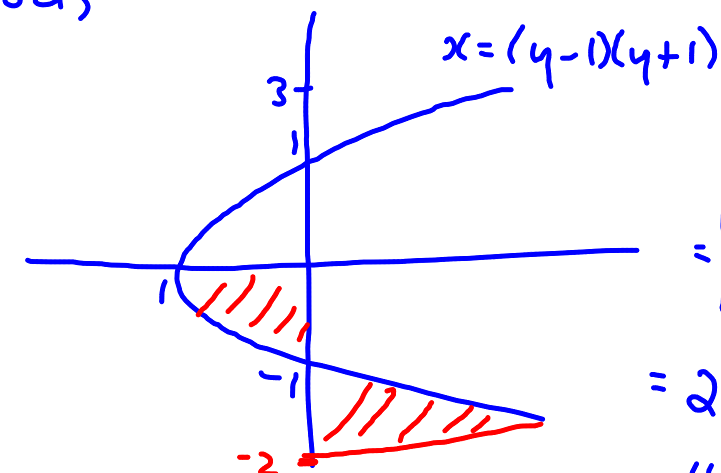
$$= \left[\frac{1}{4}x^4 + \frac{2}{3}x^3 - \frac{3}{2}x^2\right]_{-3}^{0} + \left[\frac{1}{4}x^4 + \frac{2}{3}x^3 - \frac{3}{2}x^2\right]_{1}^{0}$$

$$= 2(0) - \left(\frac{1}{4}(-3)^4 + \frac{2}{3}(-3)^3 - \frac{3}{2}(-3)^2\right) - \left(\frac{1}{4} + \frac{2}{3} - \frac{3}{2}\right)$$

$$= -\frac{81}{4} + 18 + \frac{27}{2} - \frac{1}{4} - \frac{2}{3} + \frac{3}{2}$$

$$= \frac{71}{6} \text{ units}^2$$

8d)



$$A = \int_{-2}^{-1} (y^2 - 1)\,dy - \int_{-1}^{0} (y^2-1)\,dy$$

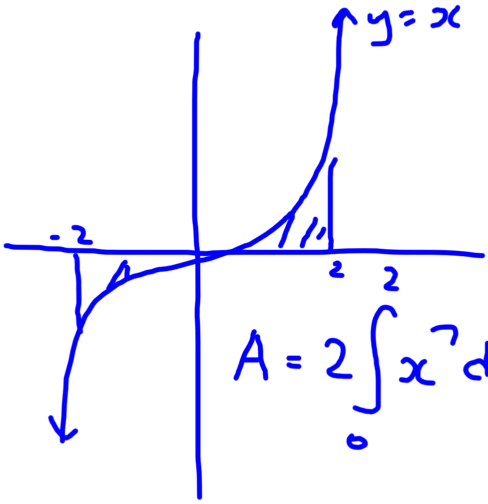
$$= \left[\tfrac{1}{3}y^3 - y\right]_{-2}^{-1} + \left[\tfrac{1}{3}y^3 - y\right]_{0}^{-1}$$

$$= 2\left\{\tfrac{1}{3}(-1)^3 - (-1)\right\} - \left\{\tfrac{1}{3}(-2)^3 - (-2)\right\} - 0$$

$$= \frac{4}{3} + \frac{8}{3} - 2$$

$$= \underline{2 \text{ units}^2}$$

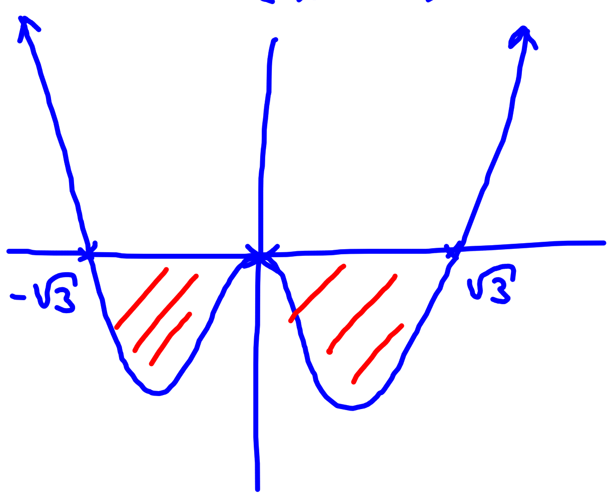
9a) ii)



$$A = 2\int_0^2 x^7\,dx$$

(iii) $y = x^4 - 3x^2$

$= x^2(x^2 - 3)$



$$A = -2\int_0^{\sqrt{3}} (x^4 - 3x^2)\,dx$$

$$= 2\left[\frac{1}{5}x^5 - x^3\right]_{\sqrt{3}}^{0}$$

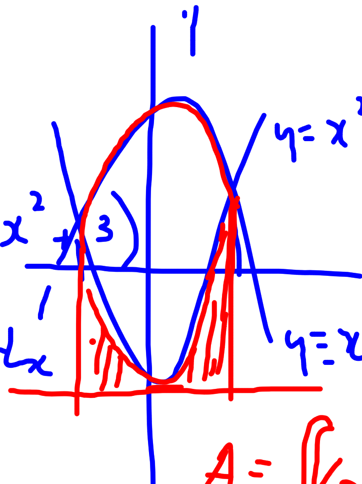
$$= 2\left(0 - \left(\frac{1}{5}(\sqrt{3})^5 - (\sqrt{3})^3\right)\right)$$

$$= -2\left(\frac{9\sqrt{3}}{5} - 3\sqrt{3}\right)$$

$$= \frac{12\sqrt{3}}{5} \text{ units}^2$$

$A = \int \left((x^2 - 4) - (-x^2 + 3) \right)$

$= -\int (2x^2 - 7)\,dx$



$y = x^2 \cancel{-4} + 6$

$y = x^2 \cancel{+3} + 13$

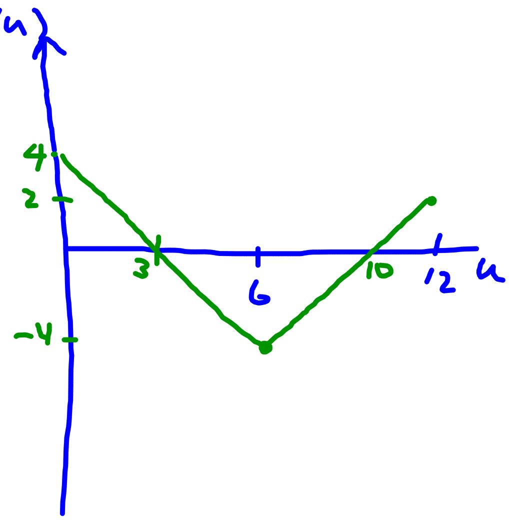
$A = \int \left[(x^2 + 6) - (-x^2 + 13) \right] dx$

$= \int (2x^2 - 7)\,dx$

18 $G(x) = \int_0^x g(u)\,du$

$g(u) = \begin{cases} 4 - \frac{4}{3}u, & 0 \le u < 6 \\ u - 10, & 6 \le u \le 12 \end{cases}$

a)



$y = \int_0^x g(u)\,du$

$y' = \frac{d}{dx} \int_0^x g(u)\,du$

$= g(x)$

stationary pts occur when $y' = 0$

$g(x) = 0$

$x = 3$ or $x = 10$

$x = 3, \quad y = \int_0^3 g(u)\,du$

$= 6$

$x = 10, \quad y = \int_0^{10} g(u)\,du$

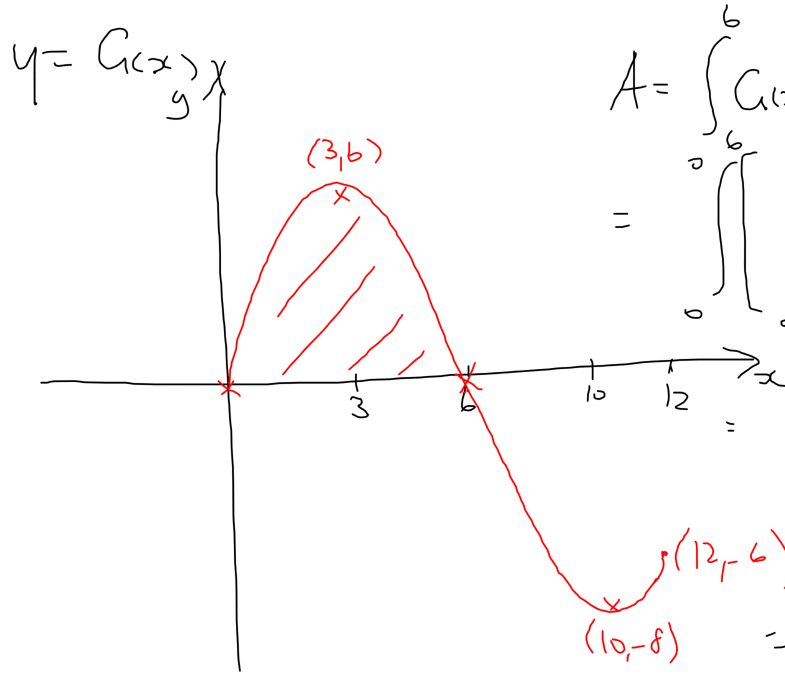
$= -8$

(3, 6) is a max tp

$G(x) = 0$

$x = 6$

(10, -8) is a min tp



$$A = \int_0^6 G(x)\,dx$$

$$= \int_0^6 \left[\int_0^x \left(4 - \frac{4}{3}u\right) du \right] dx$$

$$= \int_0^6 \left[4u - \frac{2}{3}u^2 \right]_0^x dx$$

$$= \int_0^6 \left(4x - \frac{2}{3}x^2\right) dx$$

$$= \left[2x^2 - \frac{2}{9}x^3 \right]_0^6$$

$$= 2(6)^2 - \frac{2}{9}(6)^3 - 0$$

$$= \underline{24 \text{ units}^2}$$