

$$14b) \quad f(x) = \frac{\log x}{x}$$
$$f'(x) = \frac{(x)\left(\frac{1}{x}\right) - (\log x)(1)}{x^2}$$
$$= \frac{1 - \log x}{x^2}$$

$$\begin{aligned} 14L) \quad f(x) &= \log_x 3^x \\ &= \frac{\log 3^x}{\log x} \\ &= \frac{x \log 3}{\log x} \end{aligned}$$

$$\begin{aligned} f'(x) &= \frac{(\log x)(\log 3) - (x \log 3)(\frac{1}{x})}{(\log x)^2} \\ &= \frac{(\log 3)\log x - \log 3}{(\log x)^2} \\ &= \frac{\log 3(\log x - 1)}{(\log x)^2} \end{aligned}$$

$$18d) f(x) = \log(x^{\frac{1}{2}} + \log x)$$

$$f'(x) = \frac{\frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{x}}{x^{\frac{1}{2}} + \log x}$$

$$= \frac{x^{\frac{1}{2}} + 2}{2x(x^{\frac{1}{2}} + \log x)}$$

$$\begin{aligned} 18f) \quad f(x) &= \log\left(\frac{(x-3)^4 \sqrt{x}}{x+1}\right) \\ &= 4\log(x-3) + \frac{1}{2}\log x - \log(x+1) \\ &= \frac{4}{x-3} + \frac{1}{2x} - \frac{1}{x+1} \end{aligned}$$

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$$y = \frac{x}{\log x}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{(\log x)(1) - (x)\left(\frac{1}{x}\right)}{(\log x)^2} \\ &= \frac{\log x - 1}{(\log x)^2} \end{aligned}$$

$$\begin{aligned} \frac{dy}{dx} &= \left(\frac{y}{x}\right) - \left(\frac{y}{x}\right)^2 \\ &= \frac{1}{\log x} - \frac{1}{(\log x)^2} \\ &= \frac{\log x - 1}{(\log x)^2} \\ &= \frac{dy}{dx} \end{aligned}$$

$$19b) \quad y = \log(\log x)$$

$$\frac{dy}{dx} = \frac{\frac{1}{x}}{\log x}$$

$$= \frac{1}{x \log x}$$

$$\frac{d^2y}{dx^2} = \frac{-\left((x)\left(\frac{1}{x}\right) + (\log x)(1)\right)}{(x \log x)^2}$$

$$= \frac{-\log x - 1}{(x \log x)^2}$$

$$\begin{aligned}
& x \frac{d^2y}{dx^2} + x \left(\frac{dy}{dx} \right)^2 + \frac{dy}{dx} \\
= & x \left(\frac{-\log x - 1}{(x \log x)^2} \right) + x \left(\frac{1}{x \log x} \right)^2 + \frac{1}{x \log x} \\
= & \frac{-x \log x - x + x + x \log x}{(x \log x)^2} \\
= & \frac{0}{1}
\end{aligned}$$

20 a)

$$y = \frac{(x+1)\sqrt{x-1}}{x+2}$$

$$\log y = \log(x+1) + \frac{1}{2}\log(x-1) - \log(x+2)$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{1}{x+1} + \frac{1}{2(x-1)} - \frac{1}{x+2}$$

$$\frac{dy}{dx} = \left(\frac{1}{x+1} + \frac{1}{2(x-1)} - \frac{1}{x+2} \right) \frac{(x+1)\sqrt{x-1}}{x+2}$$

f)

$$y = \sqrt{x} \sqrt{x+1} \sqrt{x+2}$$

$$\log y = \frac{1}{2} \log x + \frac{1}{2} \log(x+1) + \frac{1}{2} \log(x+2)$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{1}{2x} + \frac{1}{2(x+1)} + \frac{1}{2(x+2)}$$

$$\frac{dy}{dx} = \frac{1}{2} \left(\frac{1}{x} + \frac{1}{x+1} + \frac{1}{x+2} \right) \sqrt{x} \sqrt{x+1} \sqrt{x+2}$$

2/a)

$$y = x^x$$

$$\log y = x \log x$$

$$\frac{1}{y} \frac{dy}{dx} = (x) \left(\frac{1}{x} \right) + (\log x)(1)$$

$$= 1 + \log x$$

$$\frac{dy}{dx} = (1 + \log x) x^x$$

$$\frac{d(\log y)}{dx} = \frac{d(\log y)}{dy} \times \frac{dy}{dx}$$