5/ a) $y' = \frac{1}{4x}$

$y = \frac{1}{4}\log x + c$

$x = e^2, y = 1$

$1 = \frac{1}{4}\log e^2 + c$

$1 = \frac{1}{2} + c$

$c = \frac{1}{2}$

$\underline{y = \frac{1}{4}\log x + \frac{1}{2}}$

x int when $y = 0$

$0 = \frac{1}{4}\log x + \frac{1}{2}$

$\frac{1}{4}\log x = -\frac{1}{2}$

$\log x = -2$

$\underline{x = e^{-2}}$

c) $f'(x) = \frac{2x+5}{x^2+5x+4}$

$f(x) = \log(x^2+5x+4) + c$

$x=1,\ y=1$

$1 = \log 10 + c$

$c = 1 - \log 10$

$f(x) = \log(x^2+5x+4) + 1 - \log 10$

$= \log\left(\frac{x^2+5x+4}{10}\right) + 1$

e)

$y' = \frac{2+x}{x}$

$= \frac{2}{x} + 1$

$\underline{y = 2\log x + x + c}$

$\underline{(1,1)}$: $1 = 2\log 1 + 1 + c$

$c = 0$

$\underline{y = 2\log x + x}$

when $x = 2$,

$\underline{y = 2\log 2 + 2}$

7 a) $\displaystyle\int_{-e}^{-2} \frac{1-3x^2}{x-x^3}\,dx$

$= \left[\log(x-x^3)\right]_{-e}^{-2}$

$= \log(6) - \log(e-e^3)$

$= \log\left(\frac{6}{e-e^3}\right)$

$= \log\left(\frac{6}{1-e^2}\right) - \log e$

$= \log\left(\frac{6}{1-e^2}\right) - 1$

7c) $$\int_1^e \frac{2x+e}{x^2+ex}\,dx$$

$$= \Big[\log(x^2+ex)\Big]_1^e$$

$$= \underline{\log\left(\frac{2e^2}{1+e}\right)}$$

8/ $f(x) = x\log x$

$f'(x) = (x)\left(\frac{1}{x}\right) + (\log x)(1)$

$= 1 + \log x$

(ii) $\int_{\sqrt{e}}^{e} \log x \, dx = \int_{\sqrt{e}}^{e} \left[(1+\log x) - 1\right] dx$

$= \left[x\log x - x\right]_{\sqrt{e}}^{e}$

$= e\log e - e - \sqrt{e}\log\sqrt{e} + \sqrt{e}$

$= e - e - \frac{1}{2}\sqrt{e} + \sqrt{e}$

$= \underline{\frac{1}{2}\sqrt{e}}$

8c)

$$f(x) = x^2 \log x$$

$$f'(x) = (x^2)\left(\frac{1}{x}\right) + (\log x)(2x)$$

$$= x + 2x\log x$$

$$\int_{\sqrt{e}}^{e} x\log x \, dx = \frac{1}{2}\int_{\sqrt{e}}^{e} \Big((x + 2x\log x) - x \Big) dx$$

$$= \frac{1}{2}\left[x^2 \log x - \frac{1}{2}x^2 \right]_{\sqrt{e}}^{e}$$

$$= \frac{1}{2}\left(e^2 \log e - \frac{1}{2}e^2 - e\log\sqrt{e} + \frac{1}{2}e \right)$$

$$= \frac{1}{2}\left(\frac{1}{2}e^2 - \frac{1}{2}e + \frac{1}{2}e \right)$$

$$= \underline{\frac{1}{4}e^2}$$

9b) $\displaystyle\int \frac{dx}{t(s+tx)}$

$$= \frac{1}{t^2}\int \frac{t^2\,dx}{t(s+tx)}$$

$$= \frac{1}{t^2}\log\left[t(s+tx)\right] + c$$

10a) $\dfrac{x}{(x+4)^2} = \dfrac{x+4-4}{(x+4)^2}$

$$\int_0^1 \frac{x}{(x+4)^2}\,dx = \int_0^1 \left[\frac{1}{x+4} - \frac{4}{(x+4)^2}\right] dx$$

$$= \left[\log(x+4) + \frac{4}{x+4}\right]_0^1$$

$$= \log\frac{5}{4} + \frac{4}{5} - \frac{4}{4}$$

$$= \log\frac{5}{4} - \frac{1}{5}$$

11

a) $\int_4^{16} \frac{dx}{\sqrt{x}(\sqrt{x}-1)}$

$u = \sqrt{x} - 1$

$du = \frac{dx}{2\sqrt{x}}$

$$= 2\int_4^{16} \frac{dx}{2\sqrt{x}(\sqrt{x}-1)} = 2\int_4^{16} \frac{\frac{dx}{2\sqrt{x}}}{(\sqrt{x}-1)}$$

$$= 2\left[\log(\sqrt{x}-1)\right]_4^{16}$$

$$= 2(\log 3 - \log 1)$$

$$= \underline{2\log 3}$$

11(c) $\displaystyle\int_e^{e^e} \frac{1}{x\log x^2}\,dx = \frac{1}{2}\int_e^{e^e} \frac{\frac{1}{x}}{\log x}\,dx$

$$= \frac{1}{2}\Big[\log(\log x)\Big]_e^{e^e}$$

$$= \frac{1}{2}\log\left(\frac{\log e^e}{\log e}\right)$$

$$= \frac{1}{2}\log e$$

$$= \underline{\frac{1}{2}}$$

11d)
$$\int_1^{27} \frac{3}{(\sqrt[3]{x}+1)\,x^{\frac{2}{3}}}\,dx$$

$$= 9\int_1^{27} \frac{dx}{3(\sqrt[3]{x}+1)\,x^{\frac{2}{3}}}$$

$$= 9\int_1^{27} \frac{\frac{1}{3}x^{-\frac{2}{3}}}{(\sqrt[3]{x}+1)}\,dx$$

$$= 9\left[\log(\sqrt[3]{x}+1)\right]_1^{27}$$

$$= 9\log\left(\frac{4}{2}\right)$$

$$= \underline{9\log 2}$$

$$f(x) = x^{\frac{1}{3}} + 1$$

$$f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$$
$$= \frac{1}{3x^{\frac{2}{3}}}$$

12.

$$y = \log\left(x + \sqrt{x^2+1}\right)$$

$$y' = \frac{1 + \frac{1}{2}(x^2+1)^{-\frac{1}{2}}(2x)}{x + \sqrt{x^2+1}}$$

$$= \frac{1 + x(x^2+1)^{-\frac{1}{2}}}{x + \sqrt{x^2+1}}$$

$$= \frac{\sqrt{x^2+1} + x}{\sqrt{x^2+1}\left(x + \sqrt{x^2+1}\right)}$$

$$= \frac{1}{\sqrt{x^2+1}}$$

$$\int \frac{dx}{\sqrt{x^2+1}}$$

$$= \underline{\log\left(x + \sqrt{x^2+1}\right) + c}$$

14

a)

$f(x) = \log(ax+b)$

$f'(x) = \dfrac{a}{ax+b}$

$f(x) = \log(-ax-b)$

$f'(x) = \dfrac{-a}{-ax-b}$

$= \dfrac{a}{ax+b}$

$\therefore \displaystyle\int \frac{a}{ax+b}\,dx = \underline{\log|ax+b| + c}$.

$$\int_{-5}^{-1} \frac{dx}{2x+1}$$

$$= \frac{1}{2}\left[\log(2x+1)\right]_{-5}^{-1}$$

$$= \frac{1}{2}\log\left(\frac{-1}{-9}\right)$$

$$= \frac{1}{2}\log\frac{1}{9}$$