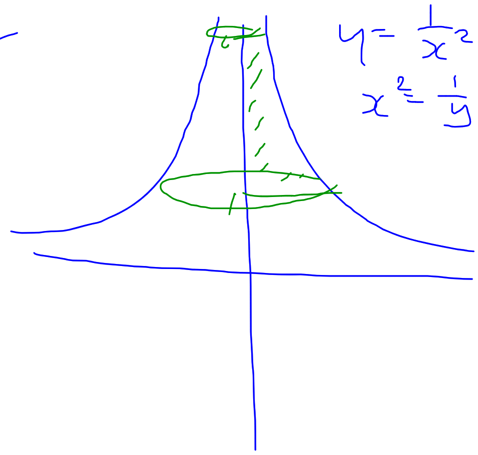
8.

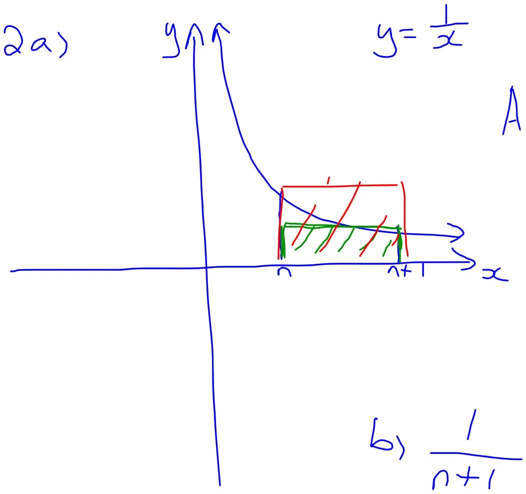


$y = \frac{1}{x^2}$

$x^2 = \frac{1}{y}$

$$V = \pi \int_1^{16} \frac{1}{y}\, dy$$

22a)



$$A_{\text{inner rectangle}} < \int_n^{n+1} \frac{dx}{x} < A_{\text{outer rectangle}}$$

$$(1)\times\left(\frac{1}{n+1}\right) < \int_n^{n+1} \frac{dx}{x} < (1)\times\left(\frac{1}{n}\right)$$

$$\frac{1}{n+1} < \int_n^{n+1} \frac{dx}{x} < \frac{1}{n}$$

b) $$\frac{1}{n+1} < \Big[\log x\Big]_n^{n+1} < \frac{1}{n}$$

$$\frac{1}{n+1} < \log\frac{n+1}{n} < \frac{1}{n}$$

$$\frac{1}{n+1} < \log\left(1+\frac{1}{n}\right) < \frac{1}{n}$$

$$\frac{n}{n+1} < n\log\left(1+\frac{1}{n}\right) < 1$$

$$\frac{n}{n+1} < \log\left(1+\frac{1}{n}\right)^n < 1$$

c) $$\lim_{n\to\infty} \frac{n}{n+1} < \lim_{n\to\infty} \log\left(1+\frac{1}{n}\right)^n < \lim_{n\to\infty} 1$$

$$1 < \lim_{n\to\infty} \log\left(1+\frac{1}{n}\right)^n < 1$$

$$\therefore \lim_{n\to\infty} \log\left(1+\frac{1}{n}\right)^n = 1$$

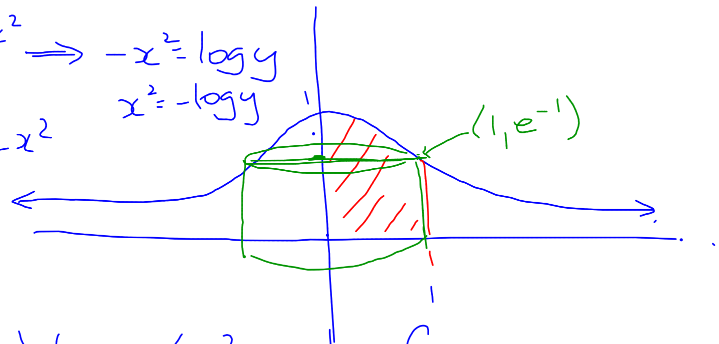
$$\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n = e$$

23

$$y = e^{-x^2} \Longrightarrow -x^2 = \log y$$

$$x^2 = -\log y$$

$$y' = -2x e^{-x^2}$$



$$V = V = \pi(1)^2\left(\frac{1}{e}\right) - \pi \int_{\frac{1}{e}}^{1} \log y \, dy$$

$$= \frac{\pi}{e} - \pi\Big[y\log y - y\Big]_{\frac{1}{e}}^{1}$$

$$= \frac{\pi}{e} - \pi\left(-1 - \frac{1}{e}\log\frac{1}{e} + \frac{1}{e}\right)$$

$$= \frac{\pi}{e} + \pi - \frac{2\pi}{e}$$

$$= \pi - \frac{\pi}{e}$$