10/

$$P(x) = x^4 + 7x^3 + 9x^2 - 27x + c$$

$$P'(x) = 4x^3 + 21x^2 + 18x - 27$$

$$P''(x) = 12x^2 + 42x + 18$$

$$= 6(2x+1)(x+3)$$

13/

$P(x) = ax^{n+1} + bx^n + 1$

$a = n$
$b = -(1+n)$

$(x-1)^2$ is a factor

$P'(x) = (n+1)ax^n + nbx^{n-1}$

$P'(1) = 0$ $\qquad$ $P(1) = 0$

$a(n+1) + bn = 0$ $\qquad$ $a + b + 1 = 0$

$(-b-1)(n+1) + bn = 0$ $\qquad$ $a = -b - 1$

$-bn - b - n - 1 + bn = 0$

$-b = n + 1$

$\underline{b = -(n+1)}$

14

$$Ax^3 + Bx^2 + D = 0$$

$$P'(x) = 3Ax^2 + 2Bx$$

$$= x(3Ax + 2B)$$

$\therefore$ double root is $x = -\frac{2B}{3A}$

$$A\left(\frac{-2B}{3A}\right)^3 + B\left(\frac{-2B}{3A}\right)^2 + D = 0$$

$$-8AB^3 + 4B^3 \times 3A + D \times 27A^3 = 0$$

$$-8AB^3 + 12AB^3 + 27A^3D = 0$$

$$4AB^3 + 27A^3D = 0$$

$$\underline{4B^3 + 27A^2D = 0}$$

15/ $P(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!}$, $n \geq 2$

$P'(x) = 1 + x + \frac{x^2}{2!} + \dots + \frac{x^{n-1}}{(n-1)!}$

for a multiple root to exist

$$P(x) = P'(x)$$

$$\frac{x^n}{n!} = 0$$

$$x = 0$$

but $x = 0$ is not a root as $P(0) = 1 \neq 0$

$\therefore$ no multiple roots

16a) $x^4 + mx^2 + n = 0 \quad , \quad m \neq 0$

$P'(x) = 4x^3 + 2mx$

$= 2x(2x^2 + m)$

$P''(x) = 12x^2 + 2m$

$= 2(6x^2 + m)$

$P''(x) = 0$

$x^2 = -\frac{m}{6}$

$$P'(x) = 4x^3 + 2mx$$
$$= 2x(2x^2 + m)$$
$$P'(x) = 0$$
$$x^2 = -\frac{m}{2}$$

for a root of multiplicity > 2 to exist

$$P'(x) = P''(x) = 0$$

$$\therefore -\frac{m}{2} = -\frac{m}{6}$$

but $m \neq 0$

$\therefore$ root cannot have multiplicity > 2

16b (ii) as $P(x)$ is an even function

if α is a double root so is $-\alpha$

(iii) double root so $P'(\alpha) = 0$

$$\alpha^2 = -\frac{m}{2}$$

$$m < 0$$

16 b(iii) $P(\alpha) = 0$

$$\alpha^4 + m\alpha^2 + n = 0$$

$$\frac{m^2}{4} - \frac{m^2}{2} + n = 0$$

$$-\frac{m^2}{4} + n = 0$$

$$n = \frac{m^2}{4}$$

roots of $\alpha, -\alpha$

$$\sqrt{-\frac{m}{2}}, -\sqrt{-\frac{m}{2}}$$

19

$$P(x) = x^3 + 3px^2 + 3qx + r$$

$$P'(x) = 3x^2 + 6px + 3q$$

$$\boxed{\begin{gathered} P'(\alpha) = 0 \\ 3\alpha^2 + 6p\alpha + 3q = 0 \end{gathered}}$$

$$\alpha^2 + 2p\alpha + q = 0$$

$$\alpha^2\beta = -r \qquad 2\alpha + \beta = -3p$$

$$\beta = -3p - 2\alpha$$

$$\alpha^2 = \frac{r}{3p + 2\alpha}$$

$$\boxed{\alpha^3 + 3p\alpha^2 + 3q\alpha + r = 0}$$

$$\alpha^3 + 2p\alpha^2 + q\alpha = 0$$

$$p\alpha^2 + 2q\alpha + r = 0$$

$$p\alpha^2 + 2p^2\alpha + pq = 0$$

$$2(p^2 - q)\alpha + pq - r = 0$$

$$\alpha = \frac{r - pq}{2(p^2 - q)}$$

$$\boxed{\alpha = \frac{pq - r}{2(q - p^2)}}$$

$$px^2 + 2qx + r = 0$$

$$x^2 + 2px + q = 0$$

$$p^2x^2 + 2pqx + pr = 0$$

$$qx^2 + 2pqx + q^2 = 0$$

$$(p^2 - q)x^2 \quad + (pr - q^2) = 0$$

$$(p^2 - q)x^2 = q^2 - pr$$

$$x^2 = \frac{q^2 - pr}{p^2 - q} = \frac{(pq - r)^2}{4(q - p^2)^2}$$

$$4(p^2 - q) = (pq - r)^2$$