4/

$$x^4 + x^3 + x + 1 = 0$$

$$(x+1)^2(x^2 - x + 1) = 0$$

$(x+1)^2 = 0$ or $x^2 - x + 1 = 0$.

no real solutions

$\underline{\underline{x = -1}}$

$$P(x) = x^4 + x^3 + x + 1$$

$$P'(x) = 4x^3 + 3x^2 + 1$$

$P'(-1) = 0$, $P(-1) = 0$

$\therefore -1$ is double root

5b) $P(x) = x^4 - 6x^3 + 12x^2 - 10x + 3$

$P'(x) = 4x^3 - 18x^2 + 24x - 10$

$P''(x) = 12x^2 - 36x + 24$

$P''(1) = 0, \quad P'(1) = 0, \quad P(1) = 0$

$\therefore$ 1 is triple root

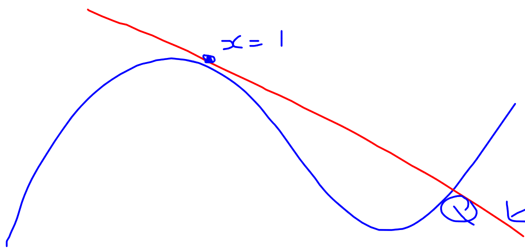
$(x-1)^3(x-3) = 0$

$x = 1$ or $x = 3$

roots are 1, 1, 1, 3

6b)

$y = x^3 - 2x^2 + 1$ at $x = 1$



$\frac{dy}{dx} = 3x^2 - 4x$

$x = 1, \ \frac{dy}{dx} = -1$

$y - 0 = -1(x - 1)$

Q ← $y = x - 1$

$x - 1 = x^3 - 2x^2 + 1$

$x^3 - 2x^2 - x + 2 = 0$

$(x-1)^2(x+2) = 0$

$x = 1$ or $x = -2$

$Q(-2, 1)$

7b) $P(x) = x^3 + 3x^2 - 24x + k = 0$

$P'(x) = 3x^2 + 6x - 24$

$= 3(x-2)(x+4)$

possible double roots are $x = 2$, $x = -4$

$\underline{x = 2}$

$2^3 + 3(2)^2 - 24(2) + k = 0$

$\underline{k = 28}$

$\underline{x = -4}$

$(-4)^3 + 3(-4)^2 - 24(-4) + k = 0$

$\underline{k = -80}$

8g)

$$4x^3 - 11x^2 + x + 1 = 0$$

$$(4x+1)(x^2 - 3x + 1) = 0$$

$$4x + 1 = 0 \qquad x^2 - 3x + 1 = 0$$

$$x = -\frac{1}{4} \qquad x = \frac{3 \pm \sqrt{5}}{2}$$