5/

$$x^4 + 2x^3 - 21x^2 - 22x + 40 = 0$$

roots be $a-3d$, $a-d$, $a+d$, $a+3d$

$\sum \alpha = -2$

$4a = -2$

$a = -\frac{1}{2}$

$\therefore x = -5, -2, 1, 4$

$(a^2-d^2)(a^2-9d^2) = 40$

$\left(\frac{1}{4}-d^2\right)\left(\frac{1}{4}-9d^2\right) = 40$

$(1-4d^2)(1-36d^2) = 640$

$144d^4 - 40d^2 - 639 = 0$

$d^2 = \frac{40 \pm \sqrt{369664}}{288}$

$= \frac{40 \pm 608}{288}$

$d^2 = \frac{9}{4}$

$d = \pm\frac{3}{2}$

7

$x^3 + px^2 + qx + r = 0$

let roots $a-d$, a, $a+d$

$\sum \alpha = -p$

$3a = -p$

$a = -\frac{p}{3}$

$\sum \alpha\beta = q$

$a^2 - d^2 + 2a^2 = q$

$3a^2 - d^2 = q$

$d^2 = 3a^2 - q$

$\alpha\beta\gamma = -r$

$a(a^2 - d^2) = -r$

$a\left(a^2 - 3a^2 + q\right) = -r$

$-\frac{p}{3}\left(-2\left(\frac{p}{3}\right)^2 + q\right) = -r$

$\frac{2p^3}{27} - \frac{pq}{3} = -r$

$2p^3 - 9pq = -27r$

8/ $6x^3 - 17x^2 - 5x + 6 = 0$

roots, $\alpha, \frac{-2}{\alpha}, \beta$

$-2\beta = -1$

$\beta = \frac{1}{2}$

$(2x-1)(3x^2 - 7x - 6) = 0$

$(2x-1)(3x+2)(x-3) = 0$

$x = \frac{1}{2}, -\frac{2}{3}, 3$

9/

$$x^4 + 2x^3 - 3x^2 - 4x + 4 = 0$$

$P'(x) = 4x^3 + 6x^2 - 6x - 4$

double root $x = 1$ $\quad P(1) = P'(1) = 0$

$(x^2 - 2x + 1)(x^2 + 2x + 4) = 0$

$x = 1$ $\qquad$ no real solutions

$\underline{\underline{x = 1}}$

10/ α, β, γ roots of $x^3 + bx^2 + 12x + 4 = 0$

$\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$ are AP

$y = \frac{1}{x}$

$x = \frac{1}{y}$

$$\frac{1}{y^3} + \frac{b}{y^2} + \frac{12}{y} + 4 = 0$$

$$4y^3 + 12y^2 + by + 1 = 0$$

let roots $a-d, a, a+d$

$\sum \alpha = 3a = -3$

$a = -1$

$$\alpha\beta\delta = a(a^2 - d^2) = -\frac{1}{4}$$

$$-1(1 - d^2) = -\frac{1}{4}$$

$$1 - d^2 = \frac{1}{4}$$

$$d^2 = \frac{3}{4}$$

$$d = \pm\frac{\sqrt{3}}{2}$$

$$\Sigma\alpha\beta = (-1)\left(\frac{-2+\sqrt{3}}{2}\right) + (-1)\left(\frac{-2-\sqrt{3}}{2}\right) + \left(\frac{-2+\sqrt{3}}{2}\right)\left(\frac{-2-\sqrt{3}}{2}\right) = \frac{b}{4}$$

$$\frac{2-\sqrt{3}}{2} + \frac{2+\sqrt{3}}{2} + \frac{1}{4} = \frac{b}{4}$$

$$\frac{9}{4} = \frac{b}{4}$$

$$b = 9$$

$$x^3 + 9x^2 + 12x + 4 = 0$$

$$x = -1, \frac{1}{-1-\frac{\sqrt{3}}{2}}, \frac{1}{-1+\frac{\sqrt{3}}{2}}$$

$$x = -1, \frac{2}{-2-\sqrt{3}}, \frac{2}{-2+\sqrt{3}}$$