8/

$$z^2 - (c-2i)z + 3 + ib = 0$$

$\alpha = 1 - i$

$\alpha + \beta = c - 2i$

$1 - i + \beta = c - 2i$

$\beta = c - 1 - i$

$\alpha\beta = 3 + ib$

$(1-i)\beta = 3 + ib$

$(1-i)(c-1-i) = 3 + ib$

$c - 1 - i - ci + i - 1 = 3 + ib$

$c - 2 - ci = 3 + ib$

$c - 2 = 3 \qquad -c = b$

$c = 5 \qquad b = -5$

9

$$Z = \cos\theta + i\sin\theta$$

$$Z^n + Z^{-n} = \cos n\theta + i\sin n\theta + \cos(-n\theta) + i\sin(-n\theta)$$

$$\cos(-n\theta) = \cos n\theta$$

$$\sin(-n\theta) = -\sin n\theta$$

$$Z^n + Z^{-n} = \cos n\theta + i\sin n\theta + \cos n\theta - i\sin n\theta$$

$$= \underline{2\cos n\theta}$$

$$2z^4 + 3z^3 + 5z^2 + 3z + 2 = 0$$

$$2z^2 + 3z + 5 + \frac{3}{z} + \frac{2}{z^2} = 0$$

$$2\left(z^2 + \frac{1}{z^2}\right) + 3\left(z + \frac{1}{z}\right) + 5 = 0$$

$$4\cos 2\theta + 6\cos\theta + 5 = 0 \qquad (z = \operatorname{cis}\theta)$$

$$8\cos^2\theta - 4 + 6\cos\theta + 5 = 0$$

$$8\cos^2\theta + 6\cos\theta + 1 = 0$$

$$(4\cos\theta + 1)(2\cos\theta + 1) = 0$$

$$\cos\theta = -\frac{1}{4} \quad \text{or} \quad \cos\theta = -\frac{1}{2}$$

$$\sin\theta = \pm\frac{\sqrt{15}}{4} \qquad \sin\theta = \pm\frac{\sqrt{3}}{2}$$

$$z = -\frac{1}{4} + \frac{\sqrt{15}}{4}i,\ -\frac{1}{4} - \frac{\sqrt{15}}{4}i,\ -\frac{1}{2} + \frac{\sqrt{3}}{2}i,\ -\frac{1}{2} - \frac{\sqrt{3}}{2}i$$

10/ $mx^3 + nx^2 + p = 0$

$27m^2p = -4n^3$

$P'(x) = 3mx^2 + 2nx$

$P'(x) = 0$

$3mx^2 + 2nx = 0$

$x(3mx + 2n) = 0$

$x = 0$ or $x = -\frac{2n}{3m}$

$\therefore x = -\frac{2n}{3m}$ is double root

$m\left(\frac{-2n}{3m}\right)^3 + n\left(\frac{-2n}{3m}\right)^2 + p = 0$

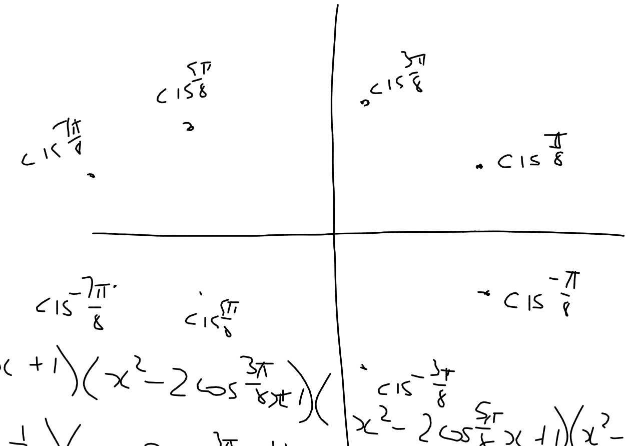
$\frac{-8n^3}{27m^2} + \frac{4n^3}{9m^2} + p = 0$

$-8n^3 + 12n^3 + 27m^2p = 0$

$\underline{4n^3 + 27m^2p = 0}$

24/

$x^8+1=0$




$$x^8+1 = \left(x^2-2\cos\tfrac{\pi}{8}x+1\right)\left(x^2-2\cos\tfrac{3\pi}{8}x+1\right)\left(x^2-2\cos\tfrac{5\pi}{8}x+1\right)\left(x^2-2\cos\tfrac{7\pi}{8}x+1\right)$$

$$x^4+\frac{1}{x^4} = \left(x-2\cos\tfrac{\pi}{8}+\tfrac{1}{x}\right)\left(x-2\cos\tfrac{3\pi}{8}+\tfrac{1}{x}\right)\left(x-2\cos\tfrac{5\pi}{8}+\tfrac{1}{x}\right)\left(x-2\cos\tfrac{7\pi}{8}+\tfrac{1}{x}\right)$$

$$2\cos4\theta = \left(2\cos\theta-2\cos\tfrac{\pi}{8}\right)\left(2\cos\theta-2\cos\tfrac{3\pi}{8}\right)\left(2\cos\theta-2\cos\tfrac{5\pi}{8}\right)\left(2\cos\theta-2\cos\tfrac{7\pi}{8}\right)$$

$$\cos4\theta = 8\left(\cos\theta-\cos\tfrac{\pi}{8}\right)\left(\cos\theta-\cos\tfrac{3\pi}{8}\right)\left(\cos\theta-\cos\tfrac{5\pi}{8}\right)\left(\cos\theta-\cos\tfrac{7\pi}{8}\right)$$

32/

$$x^3 + 3mx^2 + 3nx + r = 0$$

$$P'(x) = 3x^2 + 6mx + 3n$$

$$= 3(x^2 + 2mx + n)$$

$$x = \frac{-2m \pm \sqrt{4m^2 - 4n}}{2}$$

$$= -m \pm \sqrt{m^2 - n}$$

$$\left(-m \pm \sqrt{m^2-n}\right)^3 + 3m\left(-m \pm \sqrt{m^2-n}\right)^2 + 3n\left(-m \pm \sqrt{m^2-n}\right) + r = 0$$

$$-m^3 \pm 3m^2\sqrt{m^2-n} - \cancel{3m(m^2-n)} \pm (m^2-n)\sqrt{m^2-n}$$

$$+3m^3 \mp 6m^2\sqrt{m^2-n} + \cancel{3m(m^2-n)} - 3mn \pm 3n\sqrt{m^2-n} + r = 0$$

$$2m^3 - 3mn + r \mp 2m^2\sqrt{m^2-n} \pm 2n\sqrt{m^2-n} = 0$$

$$2m^3 - 3mn + r = \pm 2(m^2-n)\sqrt{m^2-n}$$

$$\underline{4m^6} + 9m^2n^2 + r^2 - \underline{12m^4n} + \underline{4m^3r} - 6mnr = 4(m^2-n)^3$$

$$m^2n^2 - 2mnr + r^2 = 4(m^2-n)^3 - 4m^6 - 8m^2n^2 + 12m^4n - 4m^3r + 4mnr$$

$$= 4\left[(m^2-n)^3 - m^6 + 3nm^4 - rm^3 - 2n^2m^2 + nrm\right]$$

$$= 4\left[(m^2-n)^3 - m\left(m^5 + 3nm^3 - rm^2 - 2n^2m + nr\right)\right]$$

$$= 4\left[(m^2-n)^3 - m(m^2-n)\left(m^3 \qquad + 2nm - r\right)\right]$$

35/ $P(3) = 5$ $\quad$ $P(4) = 9$

$$P(x) = (x-3)(x-4)A(x) + \underbrace{\cancel{R(x)}}_{ax+b}$$

$P(3) = 5$ $\quad$ $P(4) = 9$

$5 = 3a + b$ $\quad$ $9 = 4a + b$

$5 = 3a + b$

$4 = a \quad \therefore b = -7$

$\therefore R(x) = 4x - 7$