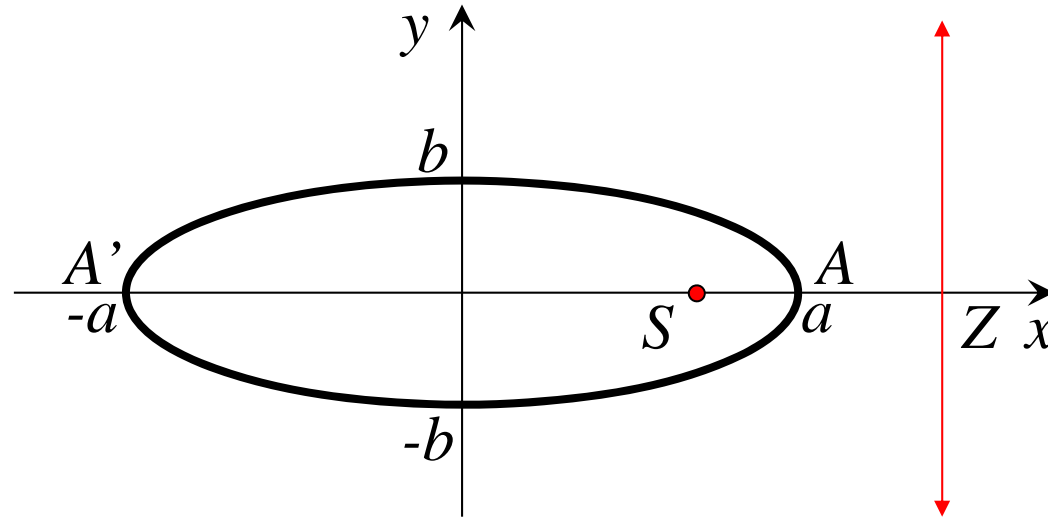


# *Ellipse* ( $e < 1$ )



$$SA = eAZ \quad \text{and} \quad SA' = eA'Z$$

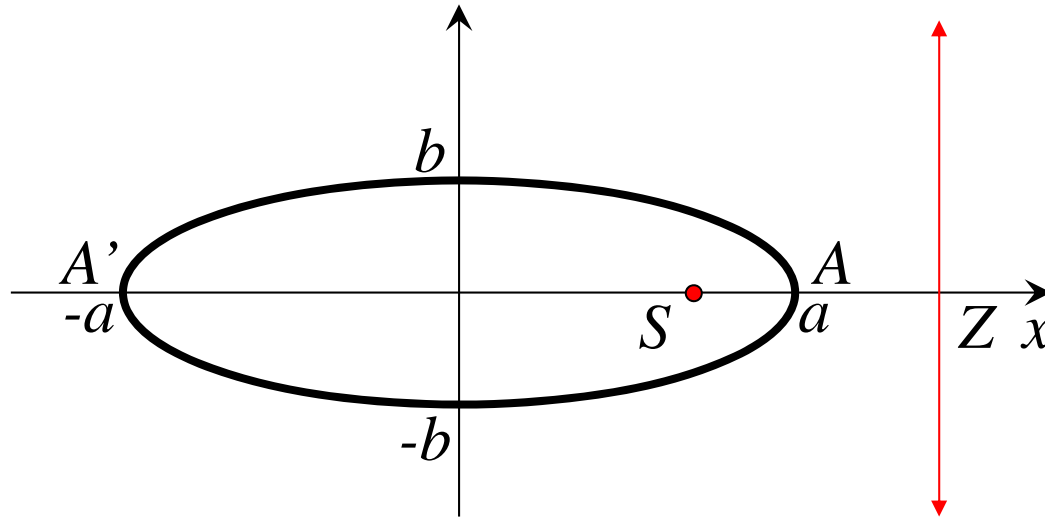
$$(1) \quad SA' + SA = 2a$$

$$(2) \quad SA' - SA = e(A'Z - AZ)$$

$$= e(AA')$$

$$= e(2a)$$

$$= 2ae$$



$$(1) + (2); \quad 2SA' = 2a(1 + e)$$

$$SA' = a(1 + e)$$

$$(1) - (2); \quad 2SA = 2a(1 - e)$$

$$SA = a(1 - e)$$

### Focus

$$OS = OA - SA$$

$$= a - a(1 - e)$$

$$= ae$$

$$\therefore S(\pm ae, 0)$$


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### Directrix

$$OZ = OA + AZ$$

$$= OA + \frac{SA}{e} \quad (\because SA = eAZ)$$

$$= \frac{ae}{e} + \frac{a(1 - e)}{e}$$

$$= \frac{a}{e}$$

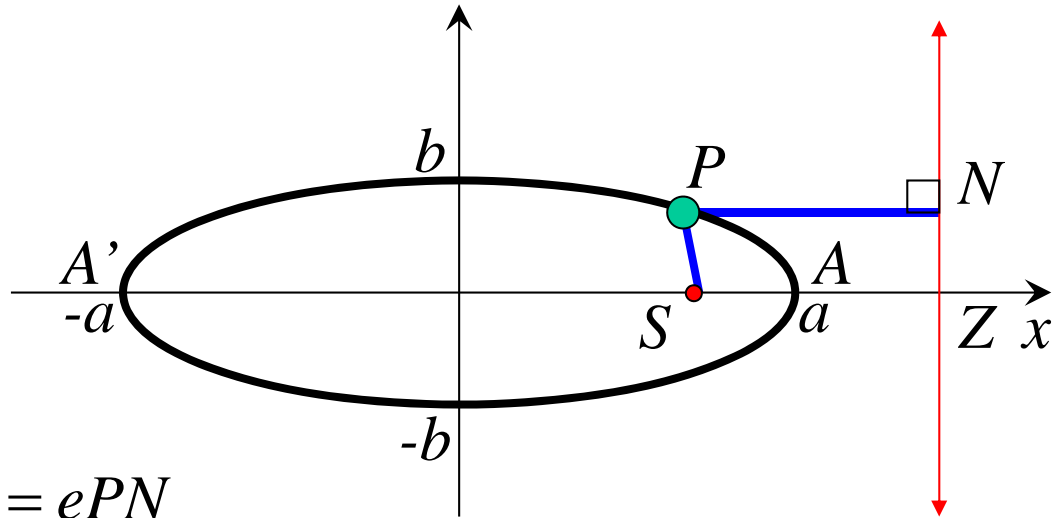
$$\therefore \text{directrices } x = \pm \frac{a}{e}$$


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$$S(ae, 0)$$

$$P(x, y)$$

$$N\left(\frac{a}{e}, y\right)$$



$$SP = ePN$$

$$\sqrt{(x - ae)^2 + (y - 0)^2} = e \sqrt{\left(x - \frac{a}{e}\right)^2 + (y - y)^2}$$

$$(x - ae)^2 + y^2 = e^2 \left(x - \frac{a}{e}\right)^2$$

$$x^2 - 2aex + a^2e^2 + y^2 = e^2x^2 - 2aex + a^2$$

$$x^2(1 - e^2) + y^2 = a^2(1 - e^2)$$

$$\frac{x^2}{a^2} + \frac{y^2}{a^2(1 - e^2)} = 1$$

when  $x = 0, y = \pm b$  i.e.  $\frac{b^2}{a^2(1-e^2)} = 1$

$$b^2 = a^2(1-e^2)$$

**Ellipse:** ( $a > b$ )

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

where;  $b^2 = a^2(1-e^2)$

focus :  $(\pm ae, 0)$

directrices :  $x = \pm \frac{a}{e}$

$e$  is the eccentricity

$$\text{note: } e^2 = \frac{a^2 - b^2}{a^2}$$

major semi-axis =  $a$  units

minor semi-axis =  $b$  units

Note: If  $b > a$

foci on the  $y$  axis

$$a^2 = b^2(1-e^2)$$

focus :  $(0, \pm be)$

directrices :  $y = \pm \frac{b}{e}$

$$\text{Area} = \pi ab$$

e.g. Find the eccentricity, foci and directrices of the ellipse

$\frac{x^2}{9} + \frac{y^2}{5} = 1$  and sketch the ellipse showing all of the important features.

$$\frac{x^2}{9} + \frac{y^2}{5} = 1 \quad e^2 = \frac{a^2 - b^2}{a^2}$$

$$a^2 = 9 \quad e^2 = \frac{9 - 5}{9}$$

$$a = 3$$

$$= \frac{4}{9}$$

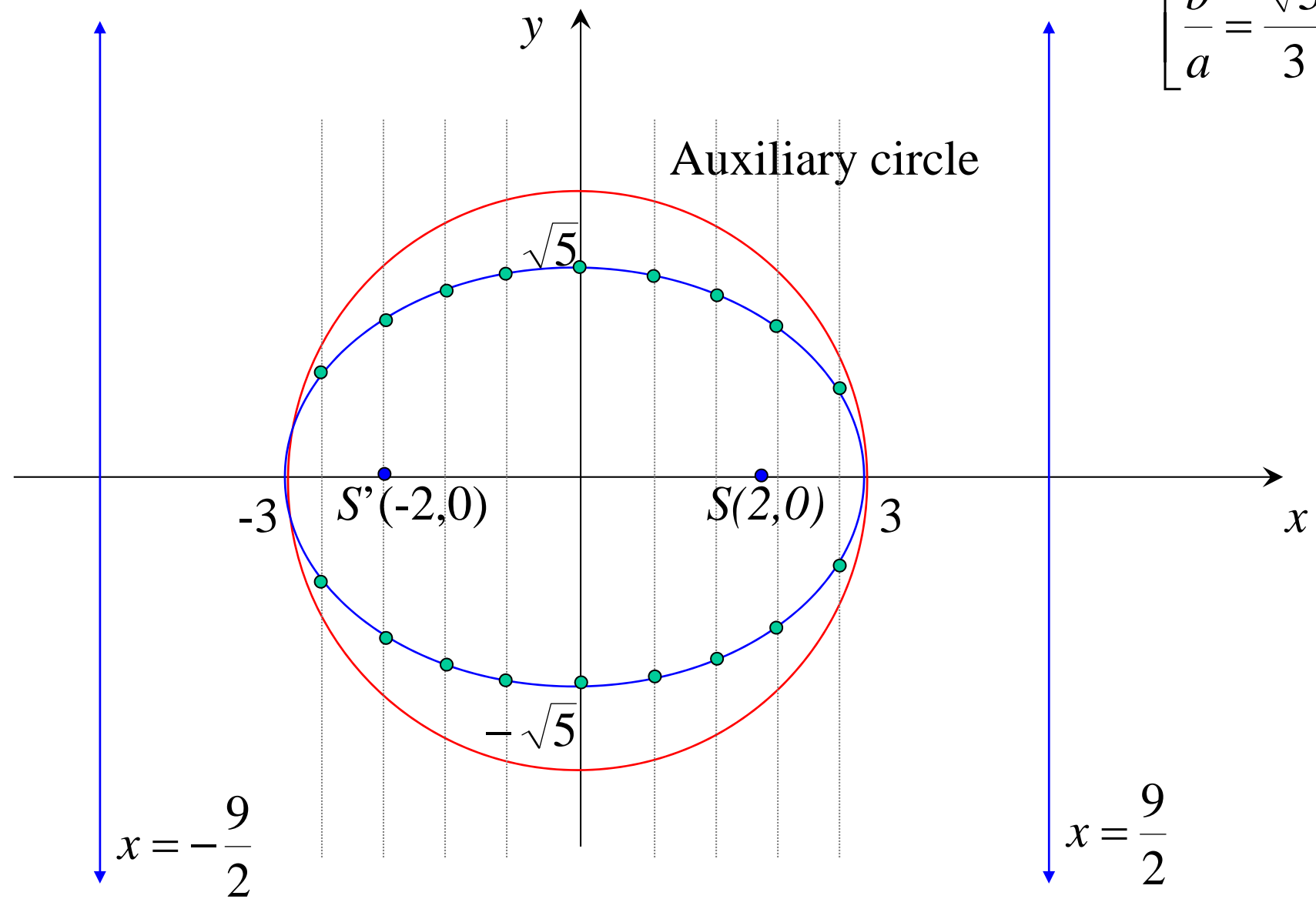
$$\therefore \text{eccentricity} = \frac{2}{3}$$

$$\text{foci : } (\pm 2, 0)$$

$$\text{directrices : } x = \pm 3 \times \frac{3}{2}$$

$$x = \pm \frac{9}{2}$$

$$\left[ \frac{b}{a} = \frac{\sqrt{5}}{3} \right]$$



*Major axis = 6 units*

*Minor axis =  $2\sqrt{5}$  units*

$$(ii) \quad 9x^2 + 4y^2 + 18x - 16y - 11 = 0$$

$$\frac{x^2 + 2x}{4} + \frac{y^2 - 4y}{9} = \frac{11}{36}$$

$$\frac{(x+1)^2}{4} + \frac{(y-2)^2}{9} = \frac{11}{36} + \frac{1}{4} + \frac{4}{9}$$

$$\frac{(x+1)^2}{4} + \frac{(y-2)^2}{9} = 1$$

centre :  $(-1, 2)$

$$b^2 = 9$$

$$b = 3$$

$$e^2 = \frac{b^2 - a^2}{b^2}$$

$$e^2 = \frac{9 - 4}{9}$$

$$e = \frac{\sqrt{5}}{3}$$

foci :  $(-1, 2 \pm \sqrt{5})$

directrices :  $y = 2 \pm \frac{9}{\sqrt{5}}$

**Patel : Exercise 6A;  
1, 2, 3, 5, 7,  
8, 9, 11, 13, 15**