7d)

$$f(x) = a^x$$

$$= e^{\log a^x}$$

$$= e^{x \log a}$$

$$f'(x) = (\log a) e^{x \log a}$$

$$= (\log a)\, a^x$$

$a = e^{\log a}$

10 a) $y = e^{-x}$

$y' = -e^{-x}$

$y'' = e^{-x}$

$y'' + 2y' + y$

$= e^{-x} - 2e^{-x} + e^{-x}$

$= 0$

$y = xe^{-x}$

$y' = (x)(-e^{-x}) + (e^{-x})(1)$

$= -xe^{-x} + e^{-x}$

$y'' = xe^{-x} - e^{-x} - e^{-x}$

$= xe^{-x} - 2e^{-x}$

$y'' + 2y' + y$

$= xe^{-x} - 2e^{-x} - 2xe^{-x} + 2e^{-x} + xe^{-x}$

$= 0$

10c)

$y = 5e^{\lambda x}$

$y' = 5\lambda e^{\lambda x}$

$y'' = 5\lambda^2 e^{\lambda x}$

$y'' + 3y' - 10y = 0$

$5\lambda^2 e^{\lambda x} + 15\lambda e^{\lambda x} - 50e^{\lambda x} = 0$

$5e^{\lambda x}(\lambda^2 + 3\lambda - 10) = 0$

$\lambda^2 + 3\lambda - 10 = 0$

$(\lambda + 5)(\lambda - 2) = 0$

$\lambda = -5$ or $\lambda = 2$

12c)

$$y = 3^{-\frac{1}{x}}$$

$$y' = \left(\frac{1}{x^2}\right)\left(3^{-\frac{1}{x}}\right)\log 3$$

$$= \frac{\log 3}{x^2\, 3^{\frac{1}{x}}}$$

20. $\lim_{u \to 1} \frac{\log u}{u-1} = 1$

$$\lim_{h \to 0} \frac{e^h - 1}{h} \qquad h = \log u$$

$$u = e^h$$

$$= \lim_{u \to 1} \frac{u-1}{\log h}$$

$$= \frac{1}{\quad}$$