6c) $y' = 2^{-x}$ $\qquad y = \dfrac{1}{2\log 2}$ when $x = 1$

$$y = \frac{-2^{-x}}{\log 2} + c$$

$$y = \frac{-1}{2^x \log 2} + c$$

$x = 1$, $y = \dfrac{1}{2\log 2}$,

$$\frac{1}{2\log 2} = \frac{-1}{2\log 2} + c$$

$$\frac{2}{2\log 2} = c$$

$$y = \frac{-1}{2^x \log 2} + \frac{1}{\log 2}$$

$$= \frac{-2^{-x} + 1}{\log 2}$$

8c)
$$f'(x) = \frac{1}{\sqrt{x}} - x e^{-x^2}$$
$$= x^{-\frac{1}{2}} + \frac{1}{2}\left(-2x e^{-x^2}\right)$$
$$f(x) = 2x^{\frac{1}{2}} + \frac{1}{2} e^{-x^2}$$
$$= \underline{2\sqrt{x} + \frac{1}{2} e^{-x^2}}$$

8(ii)

$$f'(x) = \log(e^{2kx})$$

$$= 2kx \log e$$

$$= 2kx$$

$$f(x) = kx^2 + c$$

9a) $f(x) = xe^{x}$

$f'(x) = (x)(e^{x}) + (e^{x})(1)$

$= \underline{xe^{x} + e^{x}}$

$\int (xe^{x} + e^{x})\,dx = xe^{x} + c$

$\int xe^{x}\,dx = xe^{x} - \int e^{x}\,dx + c$

$= \underline{xe^{x} - e^{x} + c}$

10a) $f(x) = e^{x} + e^{-x}$

$$f'(x) = e^{x} - e^{-x}$$

b) $$\int_0^2 \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}\,dx \Longleftarrow \int \frac{f'(x)}{f(x)}\,dx$$

$$= \log f(x)$$

$$= \Big[\log(e^{x} + e^{-x})\Big]_0^2$$

$$= \log(e^{2} + e^{-2}) - \log(2)$$

$$= \log\left(\frac{e^{2} + e^{-2}}{2}\right)$$

13b)

$$y = x^2 e^{-x^2}$$

$$\frac{dy}{dx} = (x^2)(-2x\,e^{-x^2}) + (e^{-x^2})(2x)$$

$$= -2x^3 e^{-x^2} + 2x e^{-x^2}$$

$$= \underline{2x e^{-x^2}(1 - x^2)}.$$

$$\therefore \int \left(-2x^3 e^{-x^2} + 2x e^{-x^2}\right) dx = x^2 e^{-x^2} + c$$

$$-2\int x^3 e^{-x^2} dx + \int 2x e^{-x^2} dx = x^2 e^{-x^2} + c$$

$$2\int x^3 e^{-x^2} dx = \int 2x e^{-x^2} dx - x^2 e^{-x^2} + c$$

$$= -e^{-x^2} - x^2 e^{-x^2} + c$$

$$\int x^3 e^{-x^2} dx = \underline{-\frac{1}{2} e^{-x^2}\left(1+x^2\right) + c}$$

17) $$\int_0^1 2^{\log x}\,dx = \int_0^1 2^{\frac{\log_2 x}{\log_2 e}}\,dx$$

$$= \int_0^1 2^{\log_2 x^{\frac{1}{\log_2 e}}}\,dx$$

$$= \int_0^1 x^{\frac{1}{\log_2 e}}\,dx$$

$$= \left[\frac{x^{\left(1+\frac{1}{\log_2 e}\right)}}{1+\frac{1}{\log_2 e}} \right]_0^1$$

$$= \frac{1}{1+\frac{1}{\log_2 e}}$$

$$= \frac{1}{\frac{\log_2 e + 1}{\log_2 e}}$$

$$= \underline{\underline{\frac{\log_2 e}{1+\log_2 e}}}$$

17. $$\int_0^1 2^{\log x}\,dx$$

$$= \int_0^1 2^{\frac{\log_2 x}{\log_2 e}}\,dx$$

$$= \int_0^1 2^{\log_2 x^{\frac{1}{\log_2 e}}}\,dx$$

$$= \int_0^1 x^{\frac{1}{\log_2 e}}\,dx$$

$$= \int_0^1 x^{\log 2}\,dx$$

$$= \left[\frac{x^{\log 2+1}}{\log 2+1}\right]_0^1$$

$$= \underline{\frac{1}{\log 2+1}}$$

$$\log 2 = \frac{\log_2 2}{\log_2 e} = \frac{1}{\log_2 e}$$