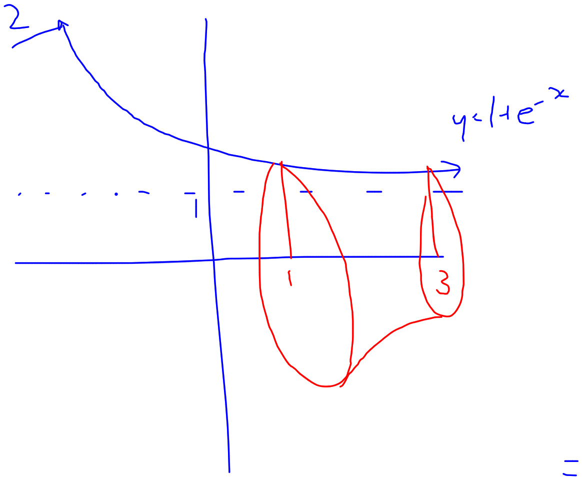
12



$$V=\pi\int_1^3 y^2\,dx$$

$$=\pi\int_1^3\left(1+2e^{-x}+e^{-2x}\right)dx$$

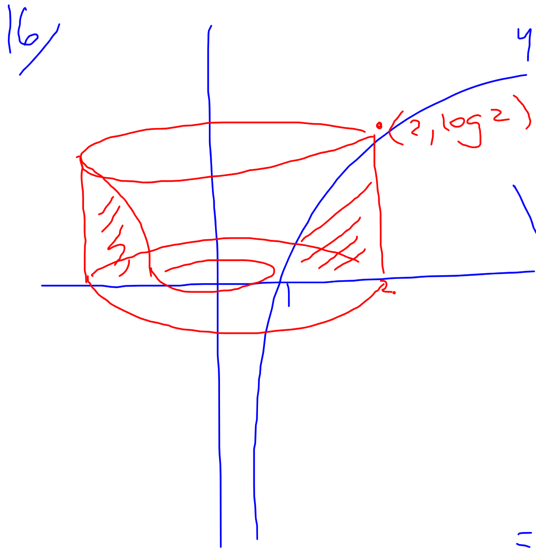
$$=\pi\left[x-2e^{-x}-\frac{1}{2}e^{-2x}\right]_1^3$$

$$=\pi\left(3-2e^{-3}-\frac{1}{2}e^{-6}-1+2e^{-1}+\frac{1}{2}e^{-2}\right)$$

$$=\pi\left(2-2e^{-3}-\frac{1}{2}e^{-6}+2e^{-1}+\frac{1}{2e^{-2}}\right)$$

$= \underline{8.491 \text{ units}^3}$ (to 3dp)

16)



$y = \log x \Rightarrow x = e^y$

$x^2 = e^{2y}$

$$V = \pi(2)^2(\log 2) - \pi\int_0^{\log 2} e^{2y}\,dy$$

$$= 4\pi\log 2 - \frac{\pi}{2}\left[e^{2y}\right]_0^{\log 2}$$

$$= 4\pi\log 2 - \frac{\pi}{2}\left(e^{2\log 2} - e^0\right)$$

$$= 4\pi\log 2 - \frac{\pi}{2}(4-1)$$

$$= \left(4\pi\log 2 - \frac{3\pi}{2}\right)\text{units}^3$$

18c) $\displaystyle\int_0^N 2xe^{-x^2}\,dx$

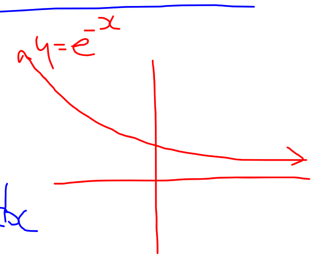
$= \left[-e^{-x^2}\right]_0^N$

$= -e^{-N^2} + e^0$

$= \underline{1 - e^{-N^2}}$

(ii) $y = xe^{-x^2}$ is $\underline{\text{odd}}$.

(iii)



$A = \displaystyle\lim_{N\to\infty} 2\int_0^N xe^{-x^2}\,dx$

$= \displaystyle\lim_{N\to\infty}\left[1 - e^{-N^2}\right]$ $\left(\displaystyle\lim_{N\to\infty} e^{-N^2} \to 0\right)$

$= \underline{1}$

21/ a) $f(x) = xe^{-x}$

$$f'(x) = (x)(-e^{-x}) + (e^{-x})(1)$$

$$= -xe^{-x} + e^{-x}$$

$$\int_0^N \left(-xe^{-x} + e^{-x}\right) dx = \left[xe^{-x}\right]_0^N$$

$$\int_0^N xe^{-x}\,dx = \int_0^N e^{-x}\,dx - \left[xe^{-x}\right]_0^N$$

$$= \left[-e^{-x} - xe^{-x}\right]_0^N$$

$$= -e^{-N} - Ne^{-N} + 1$$

$$= 1 - e^{-N} - Ne^{-N}$$

b) $\lim_{N \to \infty} 1 - e^{-N} - Ne^{-N}$

$= \underline{1.}$

c) $y = x^2 e^{-x}$

$\frac{dy}{dx} = (x^2)(-e^{-x}) + (e^{-x})(2x)$

$= -x^2 e^{-x} + 2xe^{-x}$

$\int (-x^2 e^{-x} + 2xe^{-x}) dx = x^2 e^{-x}$

$\int x^2 e^{-x} dx = \int 2xe^{-x} dx - x^2 e^{-x}$

$= -2e^{-x} - 2xe^{-x} - x^2 e^{-x}$

$$\lim_{N\to\infty} \int_0^N x^2 e^{-x}\,dx$$

$$= \lim_{N\to\infty} \left[-2e^{-x} - 2xe^{-x} - x^2 e^{-x} \right]_0^N$$

$$= \lim_{N\to\infty} \left(-2e^{-N} - 2Ne^{-N} - N^2 e^{-N} + 2e^0 + 0 + 0 \right)$$

$$= \underline{2}$$