3/ a, b, c, d are integers

$b \neq 0,\ c \neq 0$

$$\frac{\frac{a}{b} + \frac{c}{d}}{2} = \frac{ad + bc}{2bd}$$

Since a, b, c, d are integers

so are $ad+bc$ and $2bd$

$\therefore$ fraction is rational

4

Assume $\sqrt{3}$ is rational

$\sqrt{3} = \frac{a}{b}$ $\quad$ a, b integers with no common factor

$b\sqrt{3} = a$

$3b^2 = a^2$

$\therefore$ 3 is a factor of a^2

thus 9 is a factor of a^2

$3b^2 = 9k$ $\quad$ k is an integer

$b^2 = 3k$

$\therefore$ 3 is a factor of b^2

however a and b have no common factor

$\therefore \sqrt{3}$ is not rational

thus $\sqrt{3}$ is irrational

14 a, b irrational $a < b$

choose integer n, $\frac{1}{n} < b - a$

p is greatest integer $\frac{p}{n} < a$

a)

$$\frac{1}{n} < b - a$$

$$\frac{p}{n} < a$$

$$\frac{p}{n} + \frac{1}{n} < a + b - a$$

$$\frac{p+1}{n} < b$$

$\frac{p}{n} < a$ but p is greatest integer such that $\frac{p}{n} < a$.

$\frac{p+1}{n} \nless a$ ($\because$ $p+1$ is a greater integer than p)

$\therefore \frac{p+1}{n} > a$

$$a < \frac{p+1}{n} < b$$

14/ $a < b$ (a, b are +ve irrational)

$\frac{1}{n} < b - a$ (n positive integer)

$\frac{p}{n} < a$ (p is greatest integer)

$a = \sqrt{5}$ $b = \sqrt{7}$

$\frac{1}{n} < \sqrt{7} - \sqrt{5}$

$n > \frac{1}{\sqrt{7} - \sqrt{5}} = 2.44$

$n = 3$

$\frac{p}{3} < \sqrt{5}$

$p < 3\sqrt{5} = 6.7$

$p = 6$

$\frac{p+1}{n} = \frac{6+1}{3}$

$= \frac{7}{3}$

$= 2.\dot{3}$

$$a = \frac{1}{\sqrt{1001}} \qquad b = \frac{1}{\sqrt{1000}}$$

$$\frac{1}{n} < \frac{1}{\sqrt{1000}} - \frac{1}{\sqrt{1001}}$$

$$\frac{1}{n} < \frac{\sqrt{1001} - \sqrt{1000}}{\sqrt{(1000)(1001)}}$$

$$n > \frac{\sqrt{(1000)(1001)}}{\sqrt{1001} - \sqrt{1000}} \qquad 63292.9\ldots$$

$$\underline{n = 63293}$$

$$\frac{p}{n} < a$$

$$\frac{p}{63293} < \frac{1}{\sqrt{1001}}$$

$$p < \frac{63293}{\sqrt{1001}} \qquad (2000.5..)$$

$$p = 2000$$

$$\frac{1}{\sqrt{1001}} < \frac{p+1}{n} < \frac{1}{\sqrt{1000}}$$

$$\frac{1}{\sqrt{1001}} < \frac{2001}{63293} < \frac{1}{\sqrt{1000}}$$

$$0.031607\ldots \qquad 0.031615.. \qquad 0.0316228$$