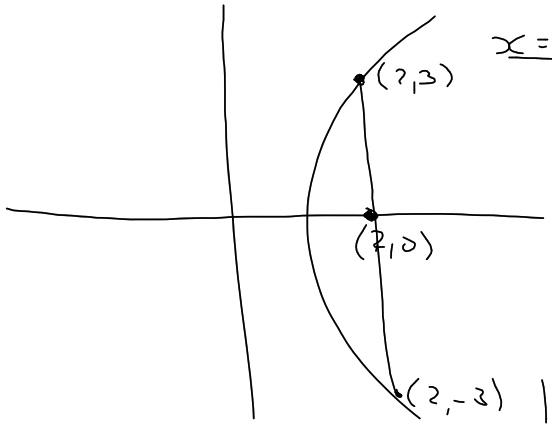
4d)

$x = \sec\theta$

$y = \sqrt{3}\tan\theta$ $\Rightarrow \frac{x^2}{1} - \frac{y^2}{3} = 1$



$x = 2$

$4 - \frac{y^2}{3} = 1$

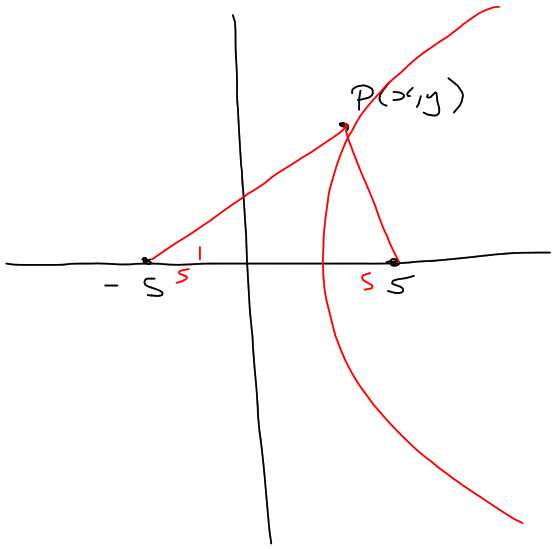
$\frac{y^2}{3} = 3$

$y^2 = 9$

$y = \pm 3$

latus rectum = 6 units

6/



$|PS - PS'| = 2a$

$2a = 6$

$a = 3$

14/

$y = mx + c$ is tangent to $2x^2 - y^2 = 1$

$$2x^2 - (mx+c)^2 = 1$$

$$(2 - m^2)x^2 - 2mcx - (c^2+1) = 0$$

$$\Delta = 0$$

$$4m^2c^2 + 4(2-m^2)(c^2+1) = 0$$

$$m^2c^2 + 2 + 2c^2 - c^2m^2 - m^2 = 0$$

$$m^2 = 2 + 2c^2$$

$$= 2(c^2+1)$$

tangent passes through $(2,3)$

$\therefore \quad 3 = 2m + c$

$c = 3 - 2m$

$m^2 = 2(4m^2 - 12m + 10)$

$7m^2 - 24m + 20 = 0$

$(7m - 10)(m - 2) = 0$

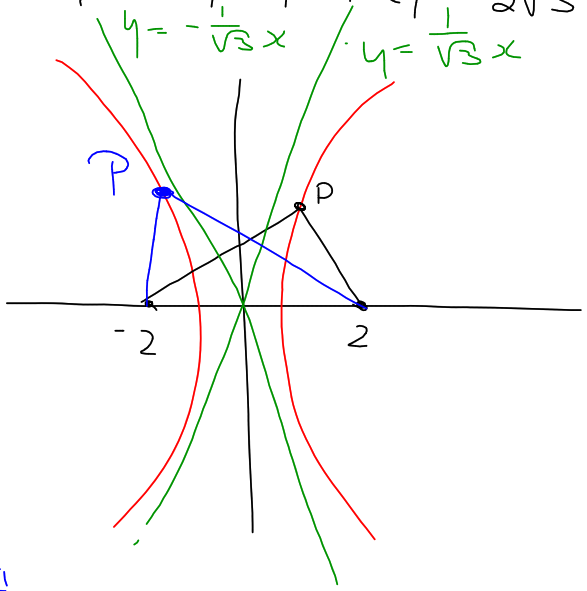
$m = \frac{10}{7}$ or $m = 2$

$\therefore$ tangents are

$y = \frac{10}{7}x + \frac{1}{7}$ $\qquad$ $y = 2x - 1$

16/

$|z-2|-|z+2|=2\sqrt{3}$



$\frac{5\pi}{6}<\arg z\leq\pi$ or $-\pi<\arg z<-\frac{5\pi}{6}$

$|PS-PS'|=2a$

locus is left branch of hyperbola

$2a=2\sqrt{3}$ $\quad ae=2$

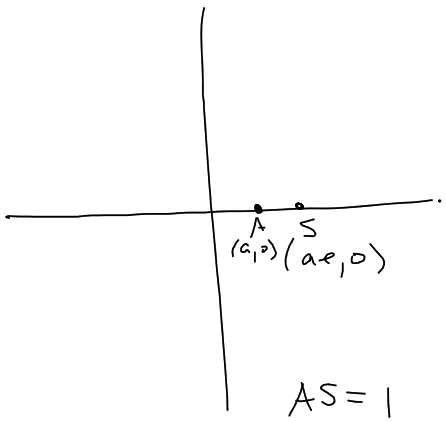
$a=\sqrt{3}$ $\quad e=\frac{2}{\sqrt{3}}$

$b^2=a^2(e^2-1)$

$=3\left(\frac{4}{3}-1\right)$

$=1$

17a)



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$AS = 1$

$ae - a = 1$

$a(e-1) = 1$

$a^2 = \frac{1}{(e-1)^2}$

$b^2 = a^2(e^2-1)$

$= a^2(e-1)(e+1)$

$= \frac{(e-1)(e+1)}{(e-1)^2}$

$= \frac{e+1}{e-1}$