5/ $3x^2 - 4y^2 = 24$ normal at $(4, -\sqrt{6})$

$$6x - 8y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{3x}{4y}$$

at $(4, -\sqrt{6})$, $\frac{dy}{dx} = \frac{12}{-4\sqrt{6}}$

$$= \frac{3}{-\sqrt{6}}$$

$\therefore$ slope of normal is $\frac{\sqrt{6}}{3}$

$$y + \sqrt{6} = \frac{\sqrt{6}}{3}(x - 4)$$

$$3y + 3\sqrt{6} = \sqrt{6}x - 4\sqrt{6}$$

$$\sqrt{6}x - 3y - 7\sqrt{6} = 0$$

11

$x^2 + 6y^2 = 15$

$m_T = \frac{1}{2}$

$2x + 12y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{x}{6y}$

$\therefore -\frac{x}{6y} = \frac{1}{2}$

$-x = 3y$

$9y^2 + 6y^2 = 15$

$y^2 = 1$

$y = \pm 1$

$\therefore$ pts of contact

$(-3, 1), (3, -1)$

$y - 1 = \frac{1}{2}(x+3)$

$2y - 2 = x + 3$

$\underline{x - 2y + 5 = 0}$

$y + 1 = \frac{1}{2}(x-3)$

$2y + 2 = x - 3$

$\underline{x - 2y - 5 = 0}$

15

$$x + y = 5 \Rightarrow y = 5 - x$$

$$9x^2 + 16y^2 = 144$$

$$9x^2 + 16(5-x)^2 = 144$$

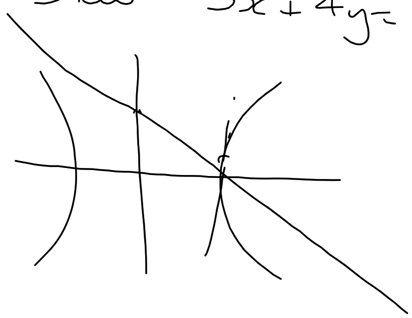
$$9x^2 + 400 - 160x + 16x^2 = 144$$

$$25x^2 - 160x + 256 = 0$$

$$(5x - 16)^2 = 0$$

$$x = \frac{16}{5} \qquad \therefore \text{pt contact } \underline{\left(\frac{16}{5}, \frac{9}{5}\right)}$$

16/ Show $3x + 4y = 10$ is normal to $2x^2 - 3y^2 = 5$



$$2x^2 - 3y^2 = 5$$

$$4x - 6y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{2x}{3y}$$

$\therefore$ normal has slope $-\frac{3y}{2x}$

$$-\frac{3y}{2x} = -\frac{3}{4}$$

$$6y = 3x$$

$$2y = x$$

$$2(2y)^2 - 3y^2 = 5$$

$$8y^2 - 3y^2 = 5$$

$$5y^2 = 5$$

$$y = \pm 1$$

possible pts : $(2,1)$ and $(-2,-1)$

$(2,1)$

$3x+4y = 3(2)+4(1)$

$= 10$ ✓

$(-2,-1)$

$3x+4y = -10$ ✗

$(2,1)$ is on hyperbola, and line $3x+4y=10$ and it is $\perp$ to tangent at that pt.

16

$3x + 4y = 10 \Rightarrow y = \frac{5}{2} - \frac{3}{4}x$

$2x^2 - 3y^2 = 5$

$2x^2 - 3\left(\frac{5}{2} - \frac{3}{4}x\right)^2 = 5$

$2x^2 - \frac{75}{4} + \frac{45}{4}x - \frac{27}{16}x^2 = 5$

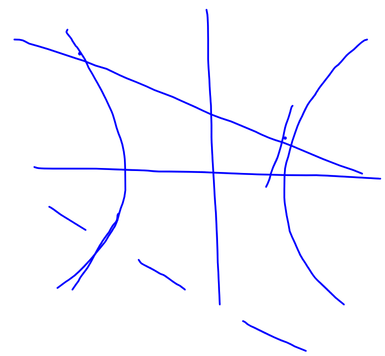
$32x^2 - 300 + 180x - 27x^2 = 80$

$5x^2 + 180x - 380 = 0$

$x^2 + 36x - 76 = 0$

$(x + 38)(x - 2) = 0$

$x = -38, \; x = 2$



$(-38, 31)$ or $(2, 1)$

when $x = -38$, $m_N = \frac{-93}{-76} = \frac{93}{76}$ ✗

when $x = 2$, $m_N = \frac{-3}{4}$

17

$2x^2 - 3y^2 = 6$

tangents $\parallel$ $x + y = 0$

$4x - 6y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{2x}{3y}$

$\therefore \frac{2x}{3y} = -1$

$2x = -3y$

$2x^2 - 3\left(-\frac{2x}{3}\right)^2 = 6$

$2x^2 - \frac{4x^2}{3} = 6$

$2x^2 = 18$

$x^2 = 9$

$x = \pm 3$

$\therefore$ pts $(-3, 2)$ and $(3, -2)$

$\therefore$ tangents $x + y = \pm 1$

18a) $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$, $a^2l^2+b^2m^2=n^2$

show $lx+my+n=0$ is a tangent.

$my=-lx-n$

$y=-\frac{l}{m}x-\frac{n}{m}$ $\qquad \dfrac{x^2}{a^2}+\dfrac{\left(\frac{l}{m}x+\frac{n}{m}\right)^2}{b^2}=1$

$b^2m^2x^2+a^2l^2x^2+2a^2lnx+a^2n^2=a^2b^2m^2$

$(b^2m^2+a^2l^2)x^2+2a^2lnx+a^2n^2-a^2b^2m^2=0$

$n^2x^2+2a^2lnx+a^2(n^2-b^2m^2)=0$

$\Delta=4a^4l^2n^2-4a^2n^2(n^2-b^2m^2)$

$=4a^4l^2n^2-4a^2n^2(a^2l^2)$

$=0$

$\therefore$ tangent if $a^2l^2+b^2m^2=n^2$

18a) $lx + my + n = 0$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, if $a^2l^2 + b^2m^2 = n^2$

$x = a\cos\theta$ $\quad$ $y = b\sin\theta$

$\frac{dx}{d\theta} = -a\sin\theta$ $\quad$ $\frac{dy}{d\theta} = b\cos\theta$

if tangent

$\frac{dy}{dx} = \frac{b\cos\theta}{-a\sin\theta}$

$\therefore \frac{b\cos\theta}{-a\sin\theta} = \frac{-l}{m}$

$\underline{bm\cos\theta - al\sin\theta = 0}$ ... (1)

but $(a\cos\theta, b\sin\theta)$ is on line

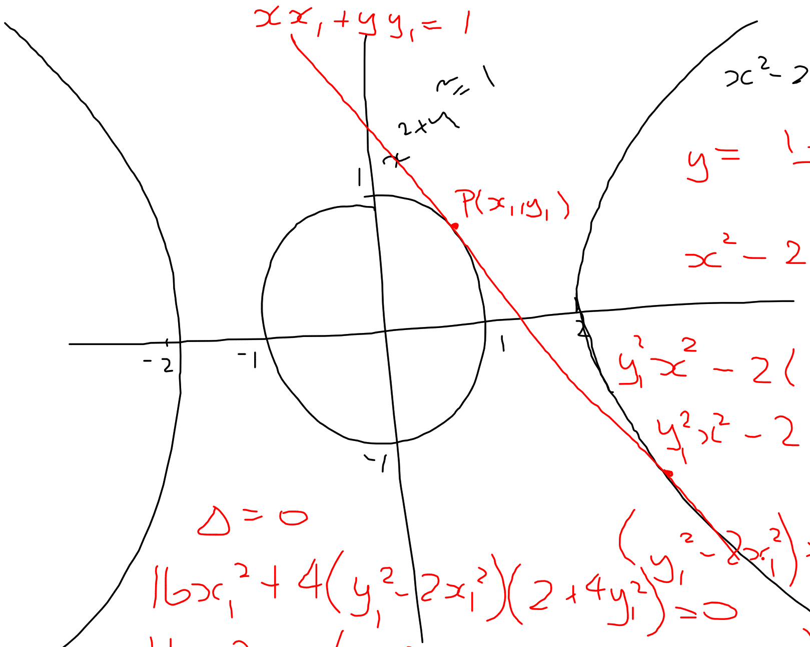
$$al\cos\theta + bm\sin\theta = -n \quad \ldots \quad (2)$$

$$(1)^2: \; b^2m^2\cos^2\theta - 2albm\sin\theta\cos\theta + a^2l^2\sin^2\theta = 0$$

$$(2)^2: \; b^2m^2\sin^2\theta + 2albm\sin\theta\cos\theta + a^2l^2\cos^2\theta = n^2$$

$$b^2m^2 + a^2l^2 = n^2$$

20



$$y = \frac{1 - xx_1}{y_1}$$

$$x^2 - 2\left(\frac{1 - xx_1}{y_1}\right)^2 = 4$$

$$y_1^2 x^2 - 2(1 - xx_1)^2 = 4y_1^2$$

$$y_1^2 x^2 - 2 + 4xx_1 - 2x_1^2 x^2 = 4y_1^2$$

$$(y_1^2 - 2x_1^2)x^2 + 4x_1 x - (2 + 4y_1^2) = 0$$

$$\Delta = 0$$

$$16x_1^2 + 4(y_1^2 - 2x_1^2)(2 + 4y_1^2) = 0$$

$$16x_1^2 + 4(2y_1^2 + 4y_1^4 - 4x_1^2 - 8x_1^2 y_1^2) = 0$$

$$8y_1^2 + 16y_1^4 - 32x_1^2 y_1^2 = 0$$

$$8y_1^2(1 + 2y_1^2 - 4x_1^2) = 0$$

$y_1^2 = 0$ ✗ $\qquad 4x_1^2 - 2y_1^2 = 1$

$4x_1^2 + 4y_1^2 = 4 \qquad x^2 + y^2 = 1$

$$6y_1^2 = 3$$

$$y_1^2 = \frac{1}{2}$$

$$y_1 = \pm\frac{1}{\sqrt{2}} \quad \therefore x_1 = \pm\frac{1}{\sqrt{2}}$$

line

$xx_1 + yy_1 = 1$

$x + y = \sqrt{2}$

$x - y = \sqrt{2}$

$-x + y = \sqrt{2}$

$-x - y = \sqrt{2}$