5b) 5, 6, 7, 8, 9

4 digit numbers = $5 \times 5 \times 5 \times 5$

$= \underline{625}$

13a) Pin #'s = $10 \times 10 \times 10 \times 10$

$= \underline{10000}$.

c) odd digit pins = $5 \times 5 \times 5 \times 5$

$= \underline{625}$

d) ~~start~~/end same = $10 \times 10 \times 10 \times 1$

$= \underline{1000}$

15.

$1, 2, 3, \dots, 9$

a) 4 digit # $= 9 \times 9 \times 9 \times 9$

$= \underline{6561}$

b) end in 1 $= 9 \times 9 \times 9 \times 1$

$= \underline{729}$

c) even $= 4 \times 9 \times 9 \times 9$

$= \underline{2916}$

d) $\div$ by 5 $= 1 \times 9 \times 9 \times 9$

$= \underline{729}$

e) $> 7000 = 3 \times 9 \times 9 \times 9$

$= \underline{2187}$

17d)

a) NUMBER

ways = 6!

= 720

d) N is left of U = 360

(either left of U or right of U equally likely.)

d) P( N is left of U) = $\frac{1}{2}$

Total ways = 6!

Ways N is left of U = $\frac{6!}{2}$

19b)

BEHAVING

$$\text{Ways} = 3! \times 1 \times 5!$$

$$= {}^{3}P_{3} \times {}^{5}P_{5}$$

19c.)

BEHAVING

3 vowels together $= 3!6!$

$= \underline{4320}$

2(b) $Q_S \; Q_L \; Q_1 \, Q_2 \, Q_3 \, Q_4$

Ways shortest/longest next $= 2!\,5!$

23)

$$\text{Ways} = \overset{\text{driver}}{2} \times {}^{4}P_{2}$$

$$= \underline{\underline{24}}.$$

24

$$\text{Letters} = 2 \times 2 \times \ldots \times 2$$
$$= 2^{10}$$

$$\text{Total} = 2^{10} + 2^{9} + 2^{8} + 2^{7} + 2^{6} + 2^{5} + 2^{4} + 2^{3} + 2^{2} + 2^{1}$$
$$= \underline{2046}.$$

25b)

4B, 4G

B, G sit in distinct groups

$= 4! \times 4!\ 2!$

27b) INCLUDE

do not begin with I = $6 \times 6!$

$= \underline{4320}$.

I

$6 \times 6!$

27) INCLUDE

d) 3 vowels, 4 consonants.

V, C alternate $= 1 \times 4! \, 3!$

e) C immediately after D

$\boxed{DC} \leftarrow$ 1 letter

Ways $= 6!$

f) N and D separated by exactly two letters.

_ _ _ _ _ _ _

N _ _ D

_ N _ _ D

_ _ N _ _ D

_ _ _ N _ _ D

$4 \times 2 = \underline{\underline{8}}$

$\text{Ways} = 8 \times 5!$

9) N and D separated by > 2 letters

| _ | _ | _ | _ | _ | _ | _ |
|---|---|---|---|---|---|---|
| N | | | | D | D | D |
| | N | | | | D | D |
| | | N | | | | D |

$6 \times 2 = \underline{\underline{12}}$

Ways $= 12 \times 5!$

29 a) 10 people in a line

(iii) A and B must be together.

$$\text{Ways} = 2!\,9!$$

(iv) A must sit at either end.

$$\text{Ways} = 2 \times 9!$$

(iv) If A and B can sit at end.

$$\text{Ways} = {}^{8}P_{2} \times 8!$$

29 b) n people

(i) if A must be on either end.

Ways $= 2 \times (n-1)!$

(ii) A and B must sit together

Ways $= 2!(n-1)!$

(iii) A and B cannot be at ends

Ways $= {}^{n-2}P_2 \times (n-2)!$

**OR**

(iii) two are not allowed at either end

Ways $= (n-2)(n-3)(n-2)!$

31 b) Jill only stand in left queue

Ways $= 4 \times 7!$

31 c) 8 people, 2 queues of 4

S and L in same queue

Ways $= 2 \times {}^4P_2 \times 6!$

33b)

8 swimmers

Lane 2 finishes after Lane 5.

No restrictions $= 8!$

$$\text{Ways} = \frac{1}{2} \times 8!$$

31)

a) No restrictions = $8!$

b) J left = $4 \times 7!$

c) S, Liam same = $2 \times 4 \times 3 \times 6!$

33) $\_\ \_\ \_\ \_\ \_\ \_\ \_\ \_$

a) $7 \times 1 \times 6!$

b) $(7+6+\ldots+1)\,6!$

$= \dfrac{7 \times 8}{2} \times 6!$

3(c) 8 people

$4p$

$4p$

S, L in same queue

Ways $= 2 \times {}^{4}P_{2} \times 6!$

35b)

0, 1, 2, 3, 4

5 digit #'s.

$$\text{odd } \# = \underset{\substack{\uparrow \\ \text{last}\#}}{2} \times \underset{\substack{\uparrow \\ \text{first}\#}}{3} \times 3!$$

37/ $\# < 4000$ $1, 3, 5, 8, 9$

a) No restrictions =

1 digit # = 5

2 digit # = ${}^5P_2 = 20$

3 digit # = ${}^5P_3 = 60$

4 digits # = $\underset{\text{1st digit}}{2} \times {}^4P_3 = 48$

$$\underline{\underline{133}}$$

b) 1, 3, 5, 8, 9 $< \underline{4000}$ ∴

$\underline{\text{ODD}}$

1 digit = 4

2 digits = $4 \times 4 = 16$

3 digits = $4 \times {}^{4}P_{2} = 48$

4 digits = $\underset{\text{1st \#}}{2} \times \underset{\text{last \#}}{3} \times {}^{3}P_{2} = 36$

$\overline{\underline{104}}$

c) 1, 3, 5, 8, 9 $< 4000$

$\div 5$

1 digit = 1

2 digits = $1 \times 4 = 4$

3 digits = $1 \times {}^4P_2 = 12$

4 digits = $\underset{\text{last \#}}{1} \times \underset{\text{1st \#}}{2} \times {}^3P_2 = 12$

29

d) $1, 3, 5, \cancel{8}, 9$ $\quad < 4000$

$\div 3$

1 digit = 2

2 digits = $3 \times 2! = 6$

3 digits = $4 \times 3! = 24$

4 digits = $\underset{\text{group of \#}}{2} \times \underset{\text{1st \#}}{2} \times 3! = 24$

$\overline{\underline{56}}$

| |
|---|
| 1389 |
| 1359 |

| |
|---|
| 15 |
| 18 |
| 39 |

| |
|---|
| 135 |
| 138 |
| 159 |
| 189 |

39

4123 deragment of 1234

4132 is not

$D(1) = 0$

$D(2) = 1$

$D(3) = 2$

~~123~~ ~~213~~ 312

~~132~~ 231 ~~321~~

$\underline{n=4}$

Ways $= 4!$
$= 24$

Total $= 24 - 24 + 12 - 1$
$= \underline{\underline{11}}$

(ii) # ways so that 1 digit unmoved

$= 4 \times 3!$

$= 24$

(iii) # ways so that 2 unmoved

$= \dfrac{{}^4P_2 \times 2!}{2!}$

$= 12$

(iv) # ways 3 unmoved

$= 1$