

5b) 5, 6, 7, 8, 9

$$\begin{aligned} \text{4 digit numbers} &= 5 \times 5 \times 5 \times 5 \\ &= \underline{625} \end{aligned}$$

13a)

$$\begin{aligned} \text{Pin \#}'s &= 10 \times 10 \times 10 \times 10 \\ &= \underline{10000} \end{aligned}$$

$$\begin{aligned} \text{c) odd digit pins} &= 5 \times 5 \times 5 \times 5 \\ &= \underline{625} \end{aligned}$$

$$\begin{aligned} \text{d) start/end same} &= 10 \times 10 \times 10 \times 1 \\ &= \underline{1000} \end{aligned}$$

15

1, 2, 3, ..., 9

$$\begin{aligned} \text{a) 4 digit \#} &= 9 \times 9 \times 9 \times 9 \\ &= \underline{6561} \end{aligned}$$

$$\begin{aligned} \text{b) end in 1} &= 9 \times 9 \times 9 \times 1 \\ &= \underline{729} \end{aligned}$$

$$\begin{aligned} \text{c) even} &= 4 \times 9 \times 9 \times 9 \\ &= \underline{2916} \end{aligned}$$

$$\begin{aligned} \text{d) } \div \text{ by 5} &= 1 \times 9 \times 9 \times 9 \\ &= \underline{729} \end{aligned}$$

$$\begin{aligned} \text{e) } > 7000 &= 3 \times 9 \times 9 \times 9 \\ &= \underline{2187} \end{aligned}$$

17d)

a) NUMBER.
ways = $6!$
 $= 720$

d) N is left of U = 360

(either left of U
or right of U
equally likely)

d) $P(\text{N is left of U}) = \frac{1}{2}$

Total ways = $6!$

ways N is left of U = $\frac{6!}{2}$

19b) BEHAVING

$$\text{Ways} = 3! \times 1 \times 5!$$

$$= {}^3P_3 \times {}^5P_5$$

19c.) BEHAVING

$$\begin{aligned} 3 \text{ vowels together} &= 3!6! \\ &= \underline{4320} \end{aligned}$$

21b) $Q_s Q_L Q_1 Q_2 Q_3 Q_4$

Ways shortest/longest next = $2!5!$

Q3,

$$\begin{aligned} \text{Ways} &= 2 \times \overset{\text{driver}}{\downarrow} {}^4P_2 \\ &= \underline{\underline{24}} \end{aligned}$$

24

$$\begin{aligned}\text{Letters} &= 2 \times 2 \times \dots \times 2 \\ &= 2^{10}\end{aligned}$$

$$\begin{aligned}\text{Total} &= 2^{10} + 2^9 + 2^8 + 2^7 + 2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2^1 \\ &= \underline{2046}.\end{aligned}$$

25b) $4B, 4G$

B, G sit in distinct groups

$$= 4! \times 4! \cdot 2!$$

27b) INCLUDE

do not begin with I = $6 \times 6!$
= 4320.

— — — $\frac{I}{1}$ — — —
| | | | | |

$6 \times 6!$

27

INCLUDE

d) 3 vowels, 4 consonants.

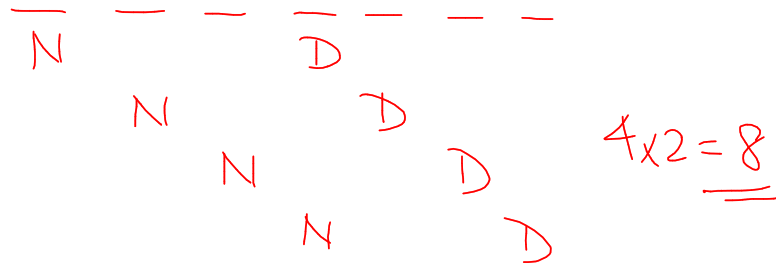
V, C alternate = $1 \times 4! \cdot 3!$

e) C immediately after D

\boxed{DC} ← 1 letter

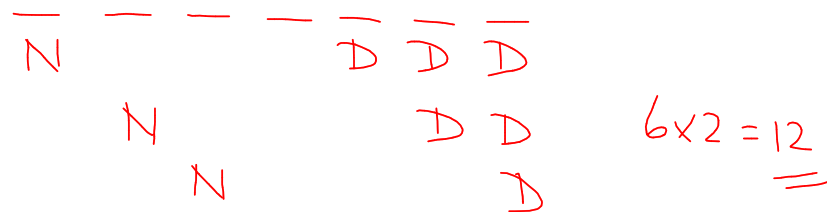
ways = $6!$

f) N and D separated by exactly two letters.



$$\text{Ways} = 8 \times 5!$$

9) N and D separated by > 2 letters



$$\text{ways} = 12 \times 5!$$

29 a) 10 people in a line

(iii) A and B must be together.

$$\text{Ways} = 2! \cdot 9!$$

(iv) A must sit at either end.

$$\text{Ways} = 2 \times 9!$$

(v) If A and B can sit at end.

$$\text{Ways} = {}^8 P_2 \times 8!$$

29/b)

n people

(i) if A must be on either end.

$$\text{Ways} = 2 \times (n-1)!$$

(ii) A and B must sit together

$$\text{Ways} = 2! \cdot (n-1)!$$

(iii) A and B cannot be at ends

$$\text{Ways} = {}^{n-2}P_2 \times (n-2)!$$

OR

(iii) two are not allowed at either end

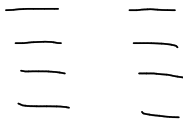
$$\text{Ways} = (n-2)(n-3)(n-2)!$$

31b) Jill only stand in left queue

$$\text{Ways} = 4 \times 7!$$

31c) 8 people, 2 queues of 4

S and L in same queue
↓



$$\text{Ways} = 2 \times {}^4P_2 \times 6!$$

33b) 8 swimmers

Lane 2 finishes after Lane 5.

No restrictions = $8!$

$$\text{Ways} = \frac{1}{2} \times 8!$$

31 a) Nb restrictions = $8!$

b) J left = $4 \times 7!$

c) S, Liam same = $2 \times 4 \times 3 \times 6!$

33, — — — — — — — — — —

a) $7 \times 1 \times 6!$

b) $(7+6+\dots+1)6!$
 $= \frac{7 \times 8}{2} \times 6!$

3(c) 8 people $\begin{matrix} \nearrow \\ \searrow \end{matrix}$ $\begin{matrix} 4p \\ 4p \end{matrix}$

S, L in same queue

$$\text{ways} = 2 \times {}^4P_2 \times 6!$$

35b) 0, 1, 2, 3, 4

5 digit #'s.

$$\text{odd \#} = 2 \times 3 \times 3!$$

↑ ↑
last# first#

37, # < 4000

1, 3, 5, 8, 9

a) No restrictions =

$$1 \text{ digit \#} = 5$$

$$2 \text{ digit \#} = {}^5P_2 = 20$$

$$3 \text{ digit \#} = {}^5P_3 = 60$$

$$4 \text{ digits \#} = 2 \times {}^4P_3 = 48$$

↑
1st digit

133

b) $1, 3, 5, 8, 9$ $< \underline{4000}$
ODD

$$1 \text{ digit} = 4$$

$$2 \text{ digits} = 4 \times 4 = 16$$

$$3 \text{ digits} = 4 \times {}^4P_2 = 48$$

$$4 \text{ digits} = 2 \times 3 \times {}^3P_2 = 36$$

\uparrow \uparrow
1st # 1st #

104

$$c) \quad 1, 3, 5, 8, 9 < 4000$$

$$\underline{\underline{\div 5}}$$

$$1 \text{ digit} = 1$$

$$2 \text{ digits} = 1 \times 4 = 4$$

$$3 \text{ digits} = 1 \times {}^4P_2 = 12$$

$$4 \text{ digits} = 1 \times 2 \times {}^3P_2 = 12$$

1st # 1st #

$$\underline{\underline{29}}$$

d) 1, 3, 5, 8, 9 < 4000

$$\div 3$$

$$1 \text{ digit} = 2$$

$$2 \text{ digits} = 3 \times 2! = 6$$

$$3 \text{ digits} = 4 \times 3! = 24$$

$$4 \text{ digits} = 2 \times 2 \times 3! = 24$$

↑ group of #
↑ 1st #

1389
1359

15
18
39

135
138
159
189

$$= 56 =$$

39

4 1 2 3

4 1 3 2
↑

derangement of 1234

is not

$$D(1) = 0$$

$$D(2) = 1$$

$$D(3) = 2$$

~~123~~ ~~231~~ 312

~~132~~ 231 ~~321~~

$$\underline{n=4}$$

$$\begin{aligned}\text{Ways} &= 4! \\ &= 24\end{aligned}$$

$$\begin{aligned}\text{Total} &= 24 - 24 + 12 - 1 \\ &= \underline{\underline{11}}\end{aligned}$$

(iii) # ways so that 1 digit unmoved

$$= 4 \times 3!$$

$$= 24$$

(iii) # ways so that 2 unmoved

$$= \frac{{}^4P_2}{2!}$$

$$= 12$$

(iv) # ways 3 unmoved

$$= 1$$