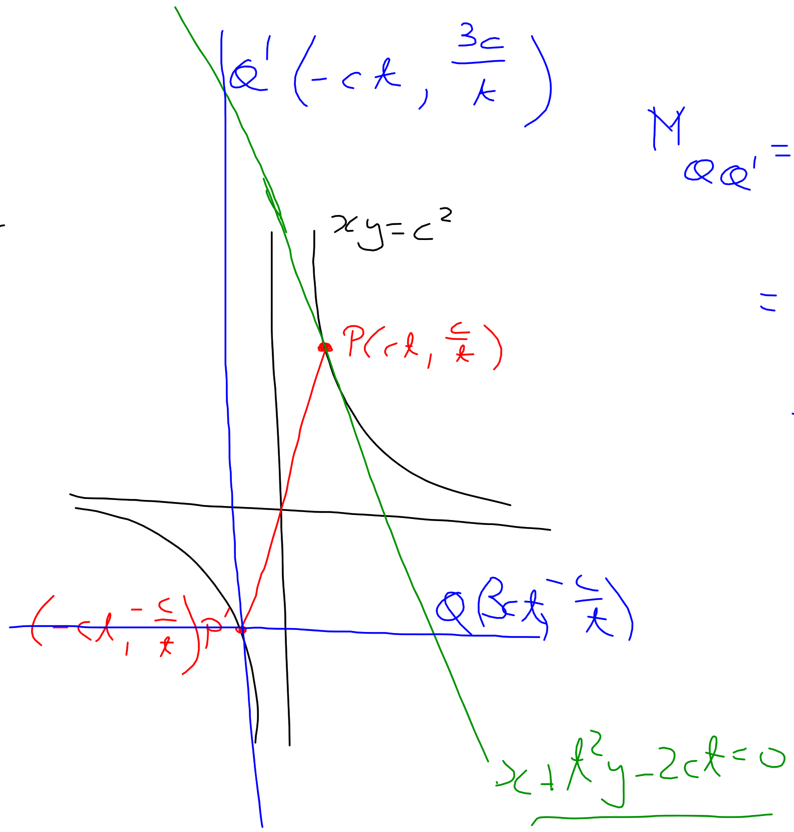
9



$$M_{QQ'} = \left(\frac{-ct+3ct}{2}, \frac{\frac{3c}{t} - \frac{c}{t}}{2} \right)$$

$$= \left(ct, \frac{c}{t} \right)$$

$$= \underline{\underline{P}}$$

$\underline{Q}$:

$$xy = 3ct \times -\frac{c}{t}$$
$$= -3c^2$$

$\underline{Q'}$:

$$xy = -ct \times \frac{3c}{t}$$
$$= -3c^2$$

$\therefore$ Q and Q' lie on $\underline{xy = -3c^2}$

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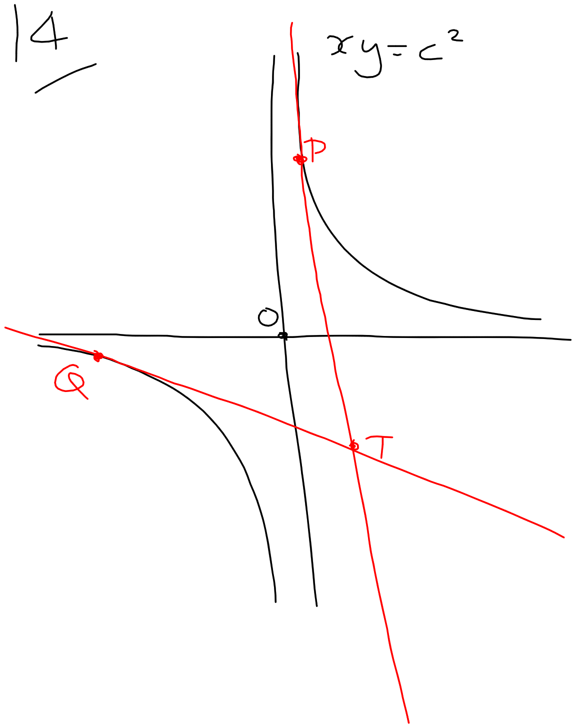


$$T\left(\frac{2ct_1t_2}{t_1+t_2}, \frac{2c}{t_1+t_2}\right)$$

$$t_1t_2 = k^2$$

locus is: $x = k^2 y$

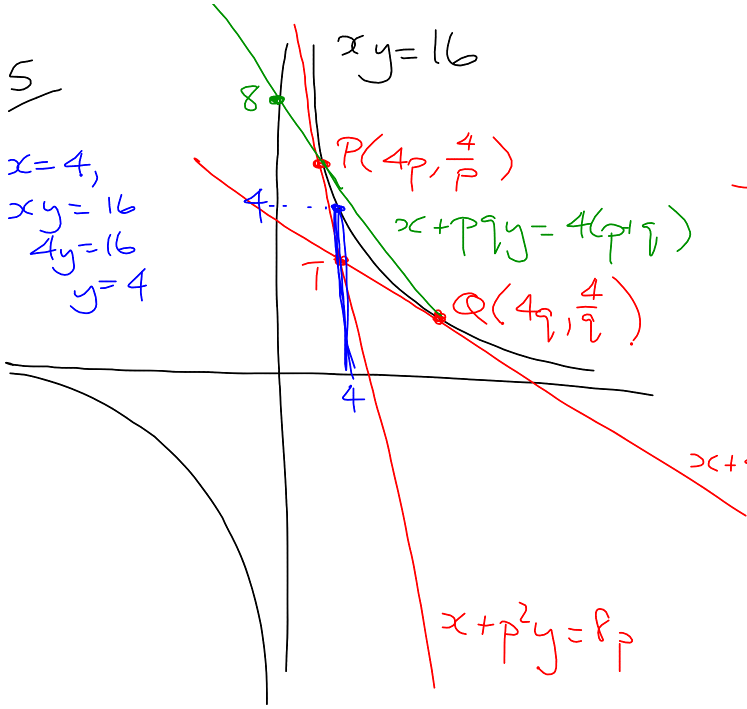
$$y = \frac{x}{k^2}$$

excluding $(0,0)$

as $\frac{2c}{t_1+t_2} \neq 0$

15

$x=4,$
$xy=16$
$4y=16$
$y=4$



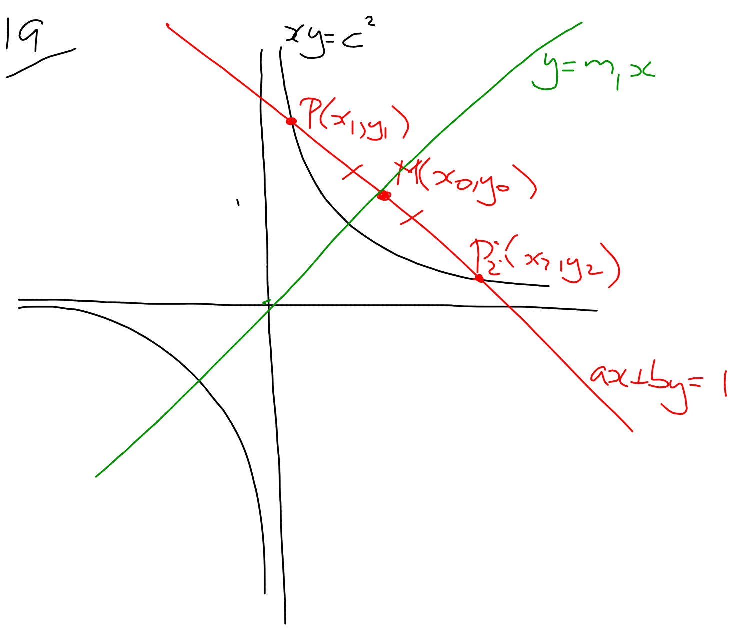
$$x+p^2y=8p$$
$$x+q^2y=8q$$
$$(p^2-q^2)y=8(p-q)$$
$$y=\frac{8(p-q)}{(p+q)(p-q)}$$
$$y=\frac{8}{p+q}$$
$$\therefore x+\frac{8p^2}{p+q}=8p$$
$$x=\frac{8p^2+8pq-8p^2}{p+q}$$
$$=\frac{8pq}{p+q}$$
$$\therefore T \text{ is } \left(\frac{8pq}{p+q}, \frac{8}{p+q}\right)$$

$$x+pqy=4(p+q)$$
$$(0,8):\quad 8pq=4(p+q)$$
$$pq=\frac{p+q}{2}$$
$$\frac{pq}{p+q}=\frac{1}{2}$$

$\therefore$ locus of T is $x=\frac{1}{2}\times 8$

$x=4,\ 0<y<4$

19



$$ax+by=1 \Rightarrow y=\frac{1-ax}{b}$$

$$x\left(\frac{1-ax}{b}\right)=c^2$$

$$x-ax^2=bc^2$$

$$ax^2-x+bc^2=0$$

$$x_1+x_2=\alpha+\beta$$

$$=\frac{1}{a}$$

but $x_0=\dfrac{x_1+x_2}{2}$

$$x_0=\frac{1}{2a}$$

$$a=\frac{1}{2x_0}$$

$ax + by = 1$

$\frac{x}{2x_0} + by = 1$

(x_0, y_0) lies on this line

$\frac{x_0}{2x_0} + by_0 = 1$

$by_0 = \frac{2x_0 - x_0}{2x_0}$

$by_0 = \frac{1}{2}$

$b = \frac{1}{2y_0}$

$\therefore l$ is

$\frac{x}{2x_0} + \frac{y}{2y_0} = 1$

$y_0 x + x_0 y = 2x_0 y_0$

$x_0 y = -y_0 x + 2x_0 y_0$

$y = \frac{-y_0}{x_0} x + 2y_0$

$\therefore m_l = \frac{-y_0}{x_0}$

$$m_\ell = \frac{-y_0}{x_0}$$

$$m_L = \frac{y_0 - 0}{x_0 - 0}$$

$$= \frac{y_0}{x_0}$$

$$= -m_\ell$$

$$= -m_1$$

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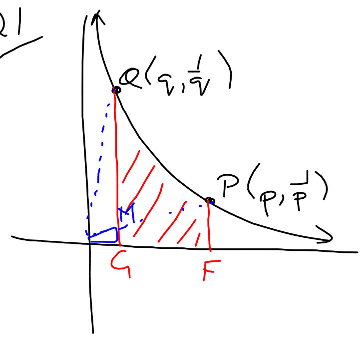
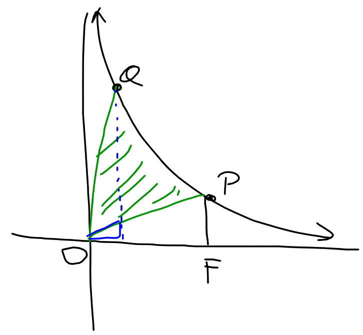
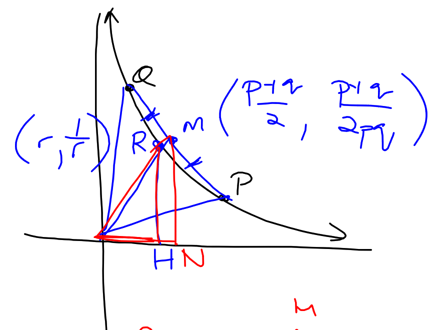
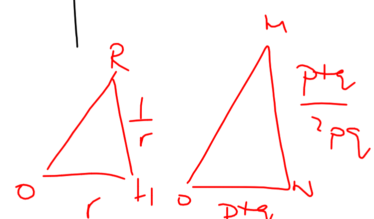






$$\text{Area } \triangle OPF = \frac{1}{2} \times p \times \frac{1}{p}$$
$$= \frac{1}{2}$$

$$\text{Red area} = \text{Area } OQPFGO - \triangle OGQ$$
$$= \text{Area } OQPFGO - \frac{1}{2}$$
$$= \text{Area } OQPFGO - \triangle POF$$
$$= \text{Green area.}$$



$$\frac{r}{\frac{p+q}{2}} = \frac{\frac{1}{r}}{\frac{p+q}{2pq}}$$

$$r = \frac{pq}{r}$$

$$r^2 = pq$$

$$\text{Area } PQGF = \int_q^p \frac{1}{x}\,dx$$

$$= \Big[\ln x\Big]_q^p$$

$$= \ln\left(\frac{p}{q}\right)$$

$$= 2\ln\left(\sqrt{\frac{p}{q}}\right)$$

$$= 2\ln\left(\sqrt{\frac{pq}{q^2}}\right)$$

$$= 2\ln\left(\sqrt{\frac{r^2}{q^2}}\right)$$

$$= 2\ln\left(\frac{r}{q}\right)$$

$$\text{Area } GQRH = \int_q^r \frac{1}{x}\,dx$$

$$= \ln\left(\frac{r}{q}\right)$$

Area ORQ = Area GQRH (by part ii)

Area OPQ = Area GQPH

= 2 Area GQRH

= 2 Area ORQ