3/ $x^2 - 2y^2 = 2$

$2x - 4y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{x}{2y}$

$x = 2, y = \pm 1$

$x = 2, y = 1$

$\frac{dy}{dx} = 1$

T

$y - 1 = 1(x - 2)$

$y - 1 = x - 2$

$x - y - 1 = 0$

N

$y - 1 = -1(x - 2)$

$y - 1 = -x + 2$

$x + y - 3 = 0$

$x = 2, y = -1$

$\frac{dy}{dx} = -1$

T, $y + 1 = -(x - 2)$

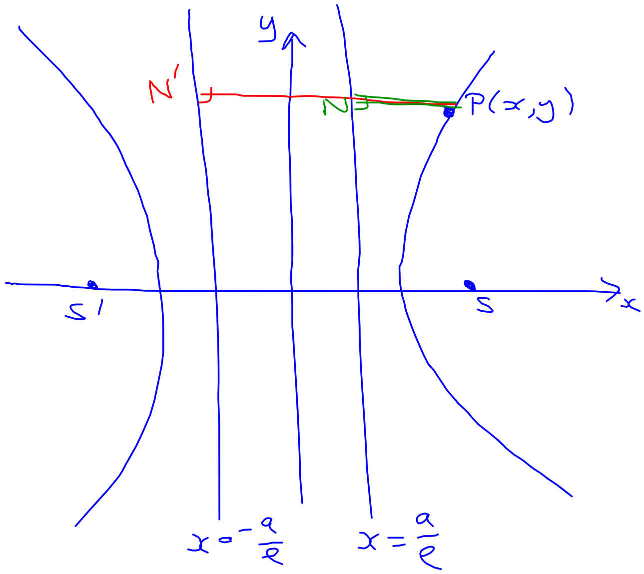
$y + 1 = -x + 2$

$x + y - 1 = 0$

N, $y + 1 = x - 2$

$x - y - 3 = 0$

10/ $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ Prove: $\left| PS' - PS \right| = 2a$



$PS' - PS$

$= ePN' - ePN$

$= e(PN' - PN)$

$= e\left(\dfrac{2a}{e}\right)$

$= \underline{\underline{2a}}$

12/ $16x^2 + 25y^2 = 400$ $\quad // \; y = x+2$

$32x + 50y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{16x}{25y}$

$m = 1, \quad \therefore \quad \frac{-16x}{25y} = 1$

$y = \frac{-16}{25}x$

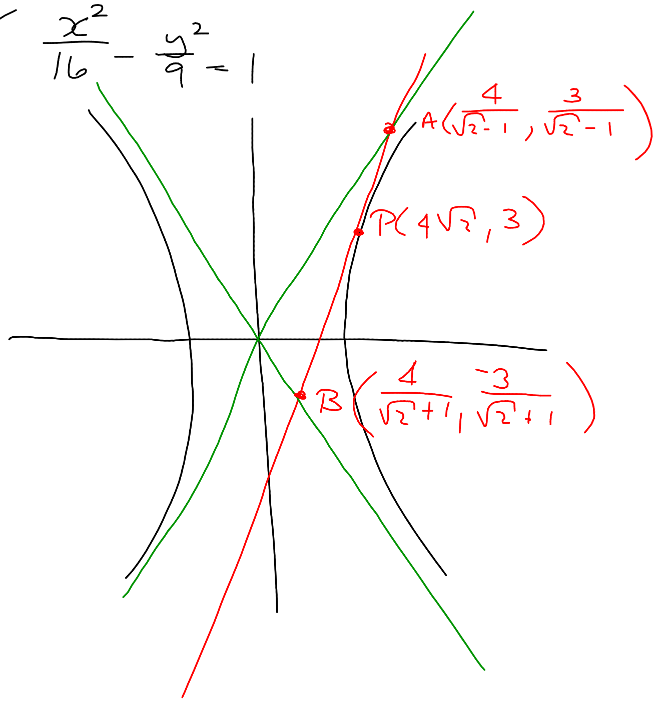
$$16x^2 + 25\left(-\frac{16}{25}x\right)^2 = 400$$

$$x^2 + \frac{16}{25}x^2 = 25$$

$$41x^2 = 625$$

$$x = \pm \frac{25}{\sqrt{41}}$$

13/ $\dfrac{x^2}{16} - \dfrac{y^2}{9} = 1$



$$AB = \sqrt{\left(\frac{4}{\sqrt{2}+1} - \frac{4}{\sqrt{2}-1}\right)^2 + \left(\frac{3}{\sqrt{2}-1} + \frac{3}{\sqrt{2}+1}\right)^2}$$

$$= \sqrt{\frac{(4\sqrt{2}-4-4\sqrt{2}-4)^2 + (3\sqrt{2}+3+3\sqrt{2}-3)^2}{2-1}}$$

$$= \sqrt{64 + 72}$$

$$= \sqrt{136}$$

$$= \underline{2\sqrt{34}}$$

19

$y = mx + c$ tangent

to $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\frac{x^2}{a^2} + \frac{(mx+c)^2}{b^2} = 1$$

$$b^2x^2 + a^2m^2x^2 + 2a^2mcx + a^2c^2 - a^2b^2 = 0$$

$$(a^2m^2 + b^2)x^2 + 2a^2cmx + a^2(c^2 - b^2) = 0$$

$$\Delta = 0$$

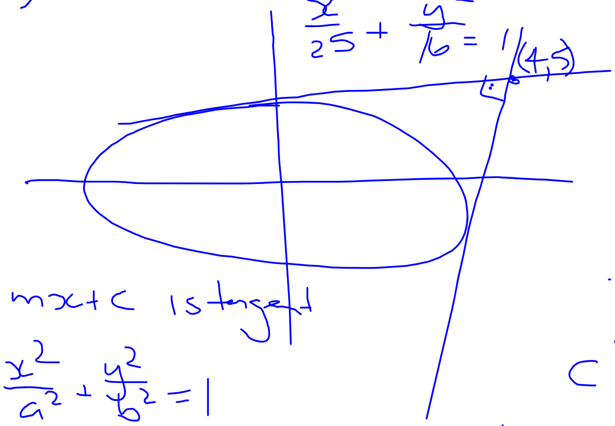
$$4a^4c^2m^2 - 4a^2(a^2m^2 + b^2)(c^2 - b^2) = 0$$

$$\cancel{a^2c^2m^2} - \cancel{a^2m^2c^2} + a^2b^2m^2 - b^2c^2 + b^4 = 0$$

$$a^2m^2 - c^2 + b^2 = 0$$

$$c^2 = a^2m^2 + b^2$$

$(4,5)$



If $y=mx+c$ is tangent to $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ then

$$c^2=a^2m^2+b^2$$

$y=mx+c$ passes through $(4,5)$

$$\therefore\ 5=4m+c$$

$$c^2=25m^2+16$$

$$(5-4m)^2=25m^2+16$$

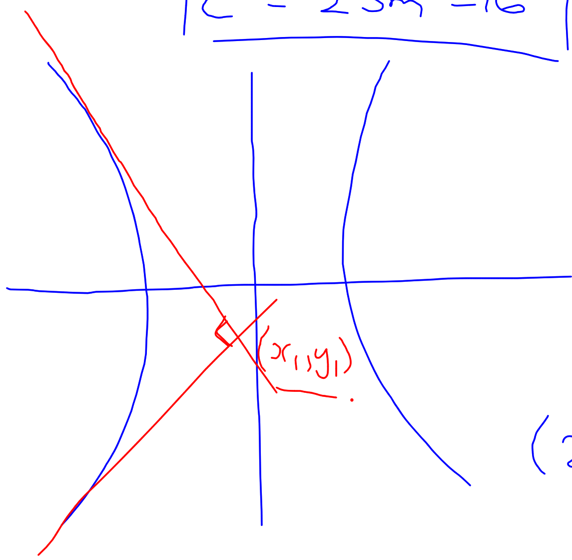
$$25-40m+16m^2=25m^2+16$$

$$9m^2+40m-9=0$$

$$\alpha\beta=-1$$

21/ $y = mx + c$ tangent to $\frac{x^2}{25} - \frac{y^2}{16} = 1$

$$\boxed{c^2 = 25m^2 - 16}$$



tangents pass through (x_1, y_1)

$\therefore y_1 = mx_1 + c$

$c = y_1 - mx_1$

$(y_1 - mx_1)^2 = 25m^2 - 16$

$y_1^2 - 2mx_1y_1 + m^2x_1^2 = 25m^2 - 16$

$(25 - x_1^2)m^2 + 2x_1y_1m - (16 + y_1^2) = 0$

solutions are m_1 and m_2

$$m_1 \times m_2 = -1$$

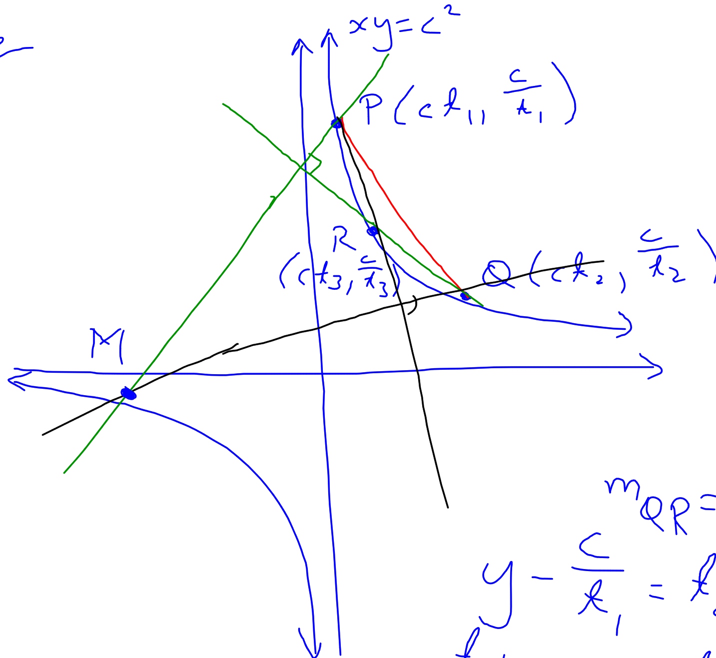
$$\frac{-(16+y_1^2)}{(25-x_1^2)} = -1 \qquad \text{(product of roots)}$$

$$16+y_1^2 = 25-x_1^2$$

$$x_1^2+y_1^2 = 9$$

$\therefore$ locus of P is circle $x^2+y^2=9$

2b



$$m_{PQ} = \frac{\frac{c}{t_2} - \frac{c}{t_1}}{ct_2 - ct_1}$$

$$= \frac{\frac{t_1 - t_2}{t_1 t_2}}{t_2 - t_1}$$

$$= \frac{-1}{t_1 t_2}$$

$$m_{QR} = \frac{-1}{t_2 t_3}$$

$$y - \frac{c}{t_1} = t_2 t_3 (x - ct_1)$$

$$t_1 y - c = t_1 t_2 t_3 x - c t_1^2 t_2 t_3$$

PM: $t_1 t_2 t_3 x - t_1 y = c(t_1^2 t_2 t_3 - 1)$

QM: $t_1 t_2 t_3 x - t_2 y = c(t_1 t_2^2 t_3 - 1)$

$$(t_1 - t_2)y = c(t_1 t_2^2 t_3 - t_1^2 t_2 t_3)$$

$$= c t_1 t_2 t_3 (t_2 - t_1)$$

$$\underline{y = -c t_1 t_2 t_3}$$

$$t_1 t_2 t_3 x + c t_1^2 t_2 t_3 = c(t_1^2 t_2 t_3 - 1)$$

$$t_1 t_2 t_3 x = -c$$

$$x = \frac{-c}{t_1 t_2 t_3}$$

$$\therefore M\left(\frac{-c}{t_1 t_2 t_3}, -c t_1 t_2 t_3\right)$$

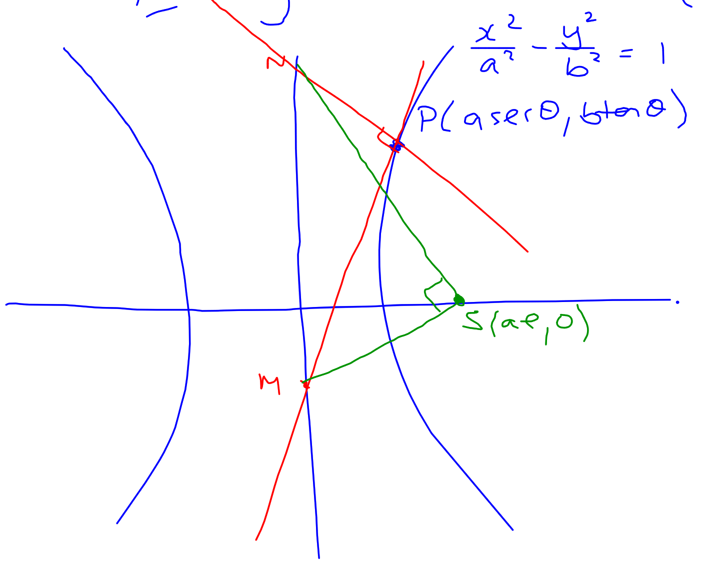
$$xy = c^2$$

$\therefore$ M is on hyperbola

29

$T:\ bx\sec\theta - ay\tan\theta = ab$

$N:\ by\sec\theta + ax\tan\theta = (a^2+b^2)\sec\theta\tan\theta$



$\underline{\underline{N}}$: $by\sec\theta = (a^2+b^2)\sec\theta\tan\theta$

$$y = \frac{(a^2+b^2)\tan\theta}{b}$$

$\underline{\underline{M}}$: $-ay\tan\theta = ab$

$$y = \frac{-b}{\tan\theta}$$

$$m_{SN} \times m_{SM} = \frac{\frac{(a^2+b^2)\cancel{\tan\theta}}{\cancel{b}}}{-ae} \times \frac{\frac{\cancel{b}}{\cancel{\tan\theta}}}{ae}$$

$$= -\frac{a^2+b^2}{a^2e^2}$$

$$= \frac{-a^2e^2}{a^2e^2}$$

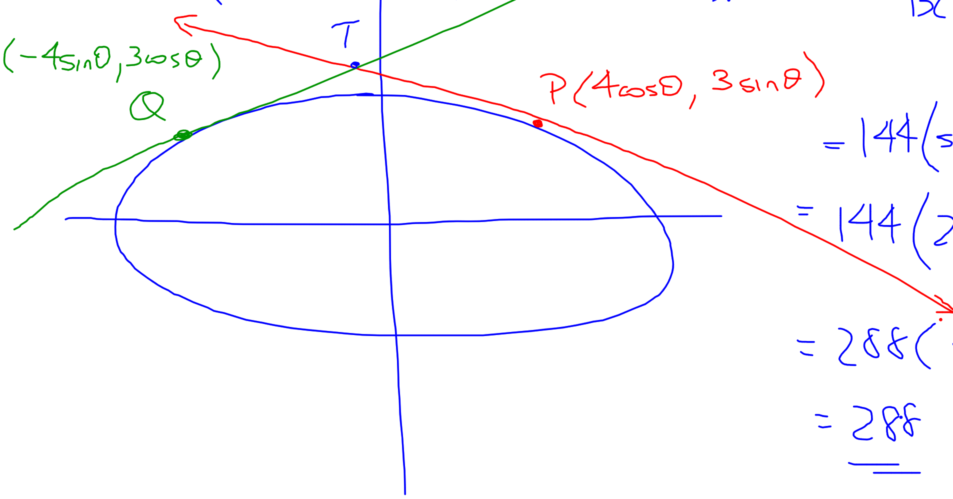
$$= -1$$

$\therefore SN \perp SM$

$\therefore$ circle with MN diameter passes through focus ($\angle$ in semicircle)

31/

$\left(-4(\sin\theta-\cos\theta),\ 3(\cos\theta+\sin\theta)\right)$



$9x^2+16y^2$

$= 144(\sin\theta-\cos\theta)^2 + 144(\cos\theta+\sin\theta)^2$

$= 144(2\sin^2\theta + 2\cos^2\theta)$

$= 288(\sin^2\theta+\cos^2\theta)$

$= \underline{\underline{288}}$

31. $P(4\cos\theta, 3\sin\theta)$

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

T_P: $3x\cos\theta + 4y\sin\theta = 12$

$Q(-4\sin\theta, 3\cos\theta)$

T_Q: $3x\sin\theta - 4y\cos\theta = -12$

$$3x\cos^2\theta + 4y\sin\theta\cos\theta = 12\cos\theta$$

$$3x\sin^2\theta - 4y\sin\theta\cos\theta = -12\sin\theta$$

$$3x = 12(\cos\theta - \sin\theta)$$

$$x = 4(\cos\theta - \sin\theta)$$

$$12\cos\theta(\cos\theta - \sin\theta) + 4y\sin\theta = 12$$

$$4y\sin\theta = 12 - 12\cos^2\theta + 12\sin\theta\cos\theta$$

$$= 12\sin^2\theta + 12\sin\theta\cos\theta$$

$$y = 3(\sin\theta + \cos\theta)$$

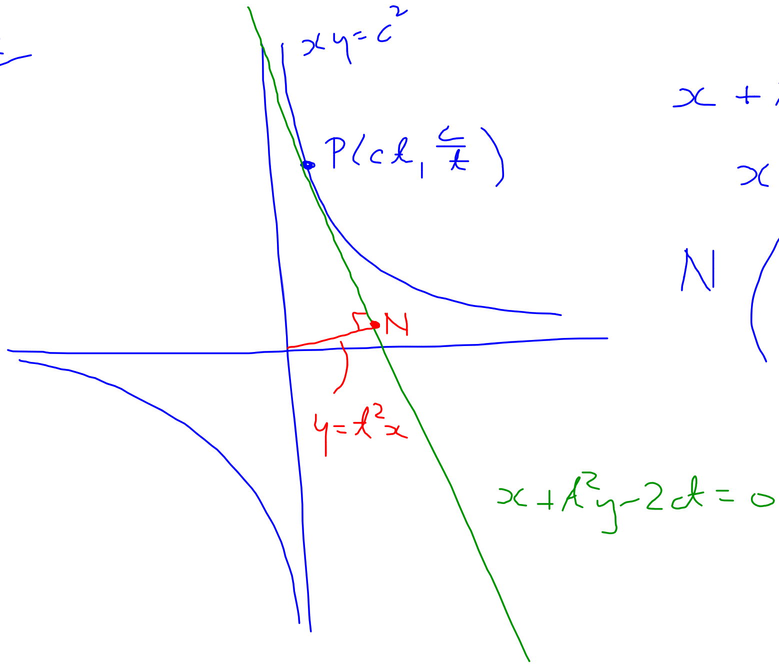
$$9x^2 + 16y^2 = 144(\cos\theta - \sin\theta)^2 + 144(\sin\theta + \cos\theta)^2$$

$$= 288\cos^2\theta + 288\sin^2\theta$$

$$= 288$$

$\therefore$ locus of T is $9x^2 + 16y^2 = 288$

42



$$x + t^4 x - 2ct = 0$$

$$x = \frac{2ct}{t^4+1}$$

$$N\left(\frac{2ct}{t^4+1}, \frac{2ct^3}{t^4+1}\right)$$

$$\boxed{(x^2+y^2)^2 = 4c^2xy}$$

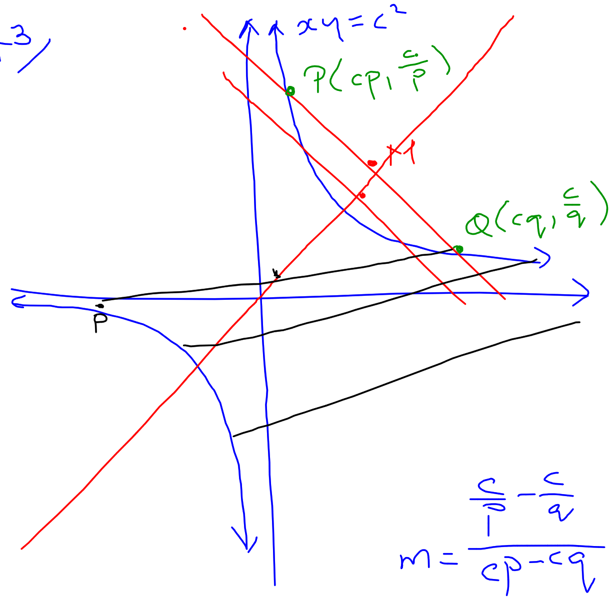
$$\frac{(x^2+y^2)^2}{xy} = \frac{\left[\left(\frac{2ct}{t^4+1}\right)^2 + \left(\frac{2ct^3}{t^4+1}\right)^2\right]^2}{\left(\frac{2ct}{t^4+1}\right)\left(\frac{2ct^3}{t^4+1}\right)}$$

$$= \frac{16c^4(t^2+t^6)^2}{(t^4+1)^4} \times \frac{(t^4+1)^2}{4c^2t^4}$$

$$= \frac{16c^4t^4(1+t^4)^2}{(t^4+1)^4} \times \frac{(t^4+1)^2}{4c^2t^4}$$

$$= \underline{\underline{4c^2}}$$

43)



chords // $lx + my = 0$

ie $m = -\frac{l}{m}$

$$M = \left(\frac{c(p+q)}{2}, \frac{c\left(\frac{1}{p}+\frac{1}{q}\right)}{2} \right)$$

$$= \left(\frac{c(p+q)}{2}, \frac{c(p+q)}{2pq} \right)$$

$$m = \frac{\frac{c}{p} - \frac{c}{q}}{cp - cq}$$

$$-\frac{l}{m} = \frac{q-p}{pq(p-q)}$$

$$= \frac{-1}{pq}$$

$$pq = \frac{m}{l}$$

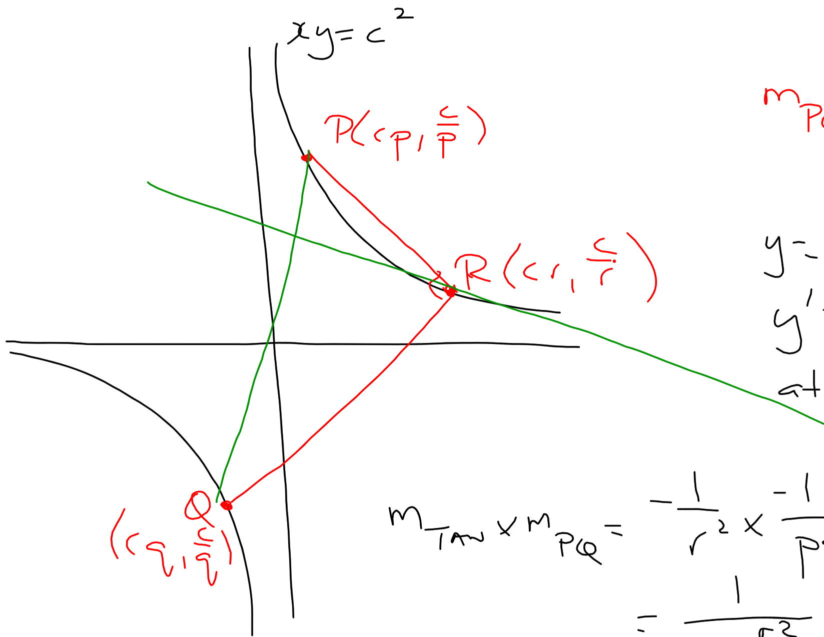
$$y = \frac{x}{\frac{m}{l}}$$

$$= \frac{lx}{m}$$

$$my = lx$$

$$\underline{lx - my = 0}$$

47



$m_{PQ} = -\frac{1}{pq}$

$y = \frac{c^2}{x}$

$y' = -\frac{c^2}{x^2}$

at R, $y' = \frac{-c^2}{c^2r^2}$

$= -\frac{1}{r^2}$

$m_{PR} \times m_{QR} = -1$

$-\frac{1}{pr} \times \frac{-1}{qr} = -1$

$\frac{1}{pqr^2} = -1$

$pqr^2 = -1$

$m_{TAN} \times m_{PQ} = -\frac{1}{r^2} \times \frac{-1}{pq}$

$= \frac{1}{pqr^2}$

$= -1$

$\therefore$ tangent $\perp$ PQ