Dynamics

Newton's Laws Of Motion

Law 1

" *Corpus omne perseverare in statu suo quiescendi vel movendi uniformiter, nisi quatenus a viribus impressis cogitur statum illum mutare."*

Everybody continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed upon it.

Law 2

" *Mutationem motus proportionalem esse vi motrici impressae, et fieri secundum rectam qua vis illa impritmitur."*

The change of motion is proportional to the motive force impressed, and it is made in the direction of the right line in which that force is impressed.

$$\text{Motive force} \propto \text{change in motion}$$

$$F \propto \text{acceleration}$$

$$F = \text{constant} \times \text{acceleration}$$

$$\therefore F = ma \qquad \text{(constant = mass)}$$

Law 3

“ Actioni contrariam semper et aequalem esse reactionem: sive corporum duorum actions in se mutuo semper esse aequales et in partes contrarias dirigi.”

To every action there is always opposed an equal reaction: or, the mutual actions of two bodies upon each other are always equal, and directed in contrary parts.

Acceleration

$$\ddot{x} = \frac{d^2x}{dt^2}$$ (acceleration is a function of *t*)

$$= \frac{dv}{dt}$$

$$= \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$ (acceleration is a function of *x*)

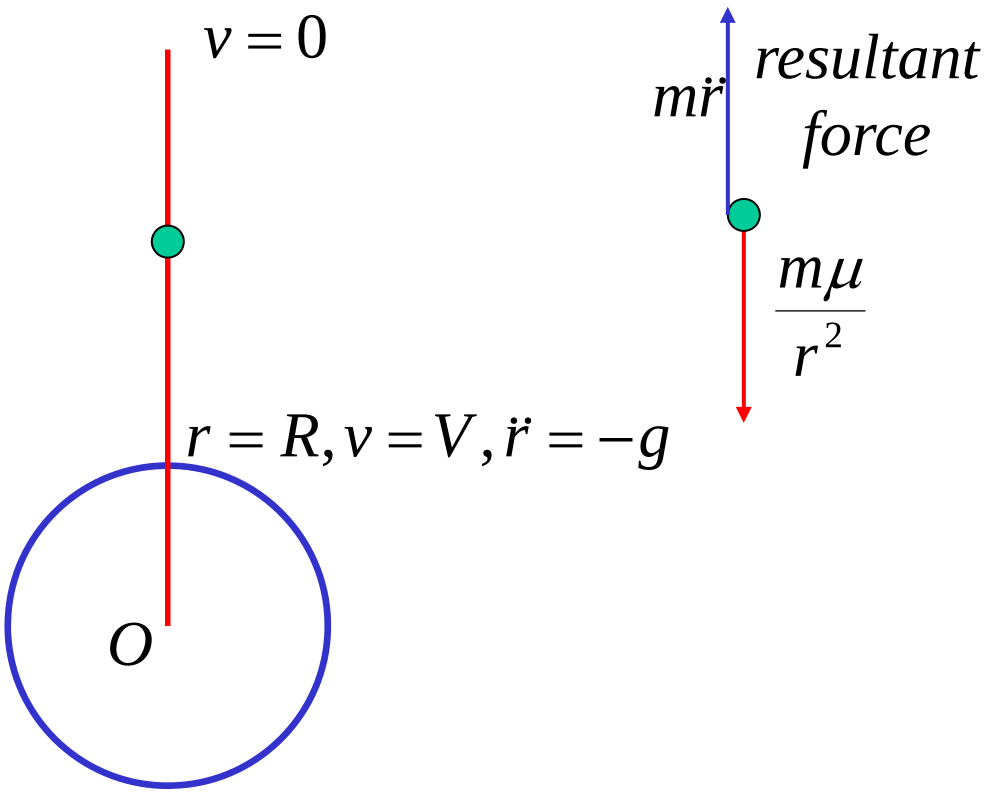
$$= v\frac{dv}{dx}$$ (acceleration is a function of *v*)

e.g. (1981)

Assume that the earth is a sphere of radius R and that, at a point $r\ (\geq R)$ from the centre of the earth, the acceleration due to gravity is proportional to $\frac{1}{r^2}$ and is directed towards the earth's centre.

A body is projected vertically upwards from the surface of the earth with initial speed V.

(i) Prove that it will escape the earth if and only if $V \geq \sqrt{2gR}$ where g is the magnitude of the acceleration due to gravity at the earth's surface.



$$m\ddot{r} = -\frac{m\mu}{r^2}$$

$$\ddot{r} = \frac{-\mu}{r^2}$$

$$\text{when } r = R, \ddot{r} = -g$$

$$\text{i.e.} -g = \frac{-\mu}{R^2}$$

$$\mu = gR^2$$

$$\ddot{r} = \frac{-gR^2}{r^2}$$

$$v\frac{dv}{dr} = \frac{-gR^2}{r^2}$$

$$\int_V^v v\,dv = -gR^2 \int_R^r \frac{dr}{r^2}$$

$$\left[v^2\right]_V^v = 2gR^2\left[\frac{1}{r}\right]_R^r$$

$$v^2 - V^2 = \frac{2gR^2}{r} - \frac{2gR^2}{R}$$

$$v^2 = \frac{2gR^2}{r} + V^2 - 2gR$$

If particle never stops then $r \to \infty$

$$\lim_{r\to\infty} v^2 = \lim_{r\to\infty}\left(\frac{2gR^2}{r} + V^2 - 2gR\right)$$

$$= V^2 - 2gR$$

Now $v^2 \geq 0$

$$\therefore V^2 - 2gR \geq 0$$

$$V^2 \geq 2gR$$

Thus if $V \geq \sqrt{2gR}$ the particle never stops,

i.e. the particle escapes the earth.

(ii) If $V=\sqrt{2gR}$,prove that the time taken to rise to a height R above the earth's surface is;

$$\frac{1}{3}\left(4-\sqrt{2}\right)\sqrt{\frac{R}{g}}$$

$$v^2=\frac{2gR^2}{r}+V^2-2gR$$

If $V=\sqrt{2gR}$, $\quad v^2=\dfrac{2gR^2}{r}+2gR-2gR$

$$v^2=\frac{2gR^2}{r}$$

$$v=\sqrt{\frac{2gR^2}{r}}$$ (cannot be –ve, as it does not return)

$$\frac{dr}{dt}=\sqrt{\frac{2gR^2}{r}}$$

$$\frac{dt}{dr}=\sqrt{\frac{r}{2gR^2}}$$

$$\int_0^t dt = \frac{1}{\sqrt{2gR^2}} \int_R^{2R} \sqrt{r}\,dr$$

$\left\{ \int_R^{2R} \text{ as travelling from } R \text{ to } 2R \right\}$

$$t = \frac{1}{\sqrt{2gR^2}} \left[\frac{2}{3} r^{\frac{3}{2}} \right]_R^{2R}$$

$$= \frac{\sqrt{2}}{3\sqrt{gR^2}} \left(2R\sqrt{2R} - R\sqrt{R} \right)$$

$$= \frac{\sqrt{2}}{3R\sqrt{g}} \left(2\sqrt{2} - 1 \right) R\sqrt{R}$$

$$= \frac{\sqrt{R}}{3\sqrt{g}} \left(4 - \sqrt{2} \right)$$

$$= \frac{1}{3} \left(4 - \sqrt{2} \right) \sqrt{\frac{R}{g}}$$

$\therefore$ time taken to rise to a height R above earth's surface is $\frac{1}{3}\left(4-\sqrt{2}\right)\sqrt{\frac{R}{g}}$ seconds

Exercise 8C; 3, 15, 19