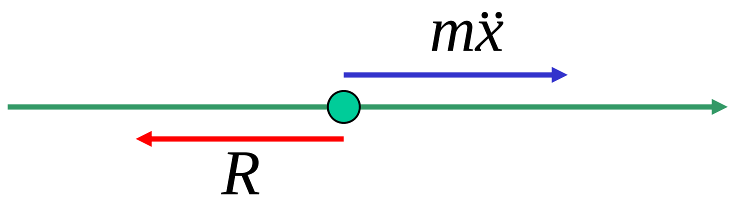
Resisted Motion

Resistance is <u>ALWAYS</u> in the <u>OPPOSITE</u> direction to the motion.

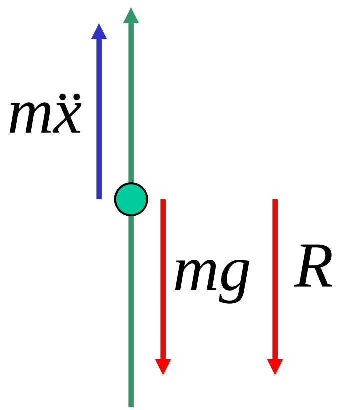
(Newton's 3rd Law)

Case 1 (horizontal line)



$$m\ddot{x} = -R$$
$$\ddot{x} = -\frac{R}{m}$$

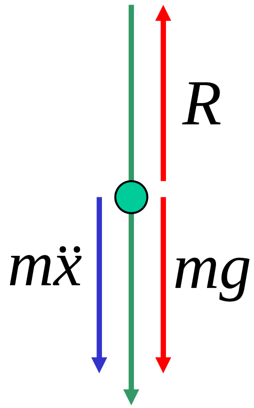
Case 2 (upwards motion)



$$m\ddot{x} = -mg - R$$
$$\ddot{x} = -g - \frac{R}{m}$$

NOTE: greatest height still occurs when $v = 0$

Case 3 (downwards motion)

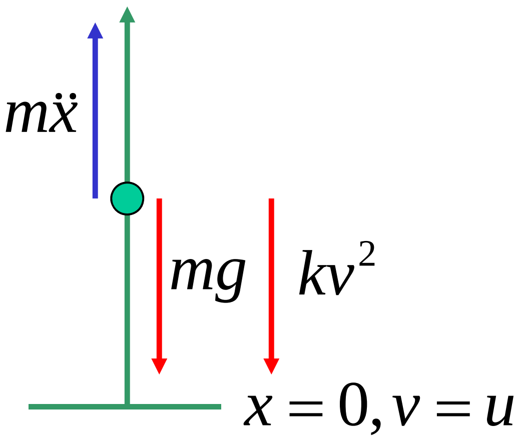


$$m\ddot{x} = mg - R$$

$$\ddot{x} = g - \frac{R}{m}$$

NOTE: terminal velocity occurs when $\ddot{x} = 0$

e.g. (i) A particle is projected vertically upwards with a velocity of u m/s in a resisting medium.
Assuming that the retardation due to this resistance is equal to kv^2 find expressions for the greatest height reached and the time taken to reach that height.



$$m\ddot{x} = -mg - kv^2$$

$$\ddot{x} = -g - \frac{k}{m}v^2$$

$$v\frac{dv}{dx} = \frac{-mg - kv^2}{m}$$

$$\frac{dx}{dv} = \frac{mv}{-mg - kv^2}$$

$$\int_0^x dx = -m\int_u^o \frac{vdv}{mg + kv^2}$$

$$x = \frac{m}{2k}\int_0^u \frac{2kvdv}{mg + kv^2}$$

$$= \frac{m}{2k}\left[\log\left(mg + kv^2\right)\right]_0^u$$

$$= \frac{m}{2k}\left\{\log\left(mg + ku^2\right) - \log\left(mg\right)\right\}$$

$$= \frac{m}{2k}\log\left(\frac{mg + ku^2}{mg}\right)$$

$$= \frac{m}{2k}\log\left(1 + \frac{ku^2}{mg}\right)$$

$\therefore$ the greatest height

is $\frac{m}{2k}\log\left(1+\frac{ku^2}{mg}\right)$ metres

$$\ddot{x} = -g - \frac{k}{m}v^2$$

$$\frac{dv}{dt} = \frac{-mg - kv^2}{m}$$

$$\int_0^t dt = -m\int_u^0 \frac{dv}{mg + kv^2}$$

$$t = \frac{m}{k}\int_0^u \frac{dv}{\frac{mg}{k} + v^2}$$

$$= \frac{m}{k}\left[\sqrt{\frac{k}{mg}}\tan^{-1}\left(\sqrt{\frac{k}{mg}}v\right)\right]_0^u$$

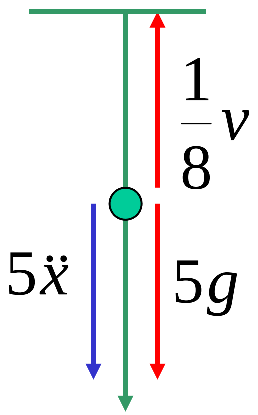
$$= \sqrt{\frac{m}{kg}}\left\{\tan^{-1}\left(\sqrt{\frac{k}{mg}}u\right) - \tan^{-1} 0\right\}$$

$$= \sqrt{\frac{m}{kg}}\tan^{-1}\left(\sqrt{\frac{k}{mg}}u\right)$$

$\therefore$ it takes $\sqrt{\frac{m}{kg}}\tan^{-1}\left(\sqrt{\frac{k}{mg}}u\right)$ seconds to reach the greatest height.

(ii) A body of mass 5kg is dropped from a height at which the gravitational acceleration is g.
Assuming that air resistance is proportional to speed v, the constant of proportion being $\frac{1}{8}$, find;

a) the velocity after time t.



$$5\ddot{x} = 5g - \frac{1}{8}v$$

$$\ddot{x} = g - \frac{1}{40}v$$

$$\frac{dv}{dt} = \frac{40g - v}{40}$$

$$\int_0^t dt = 40\int_0^v \frac{dv}{40g - v}$$

$$t = -40\left[\log\left(40g - v\right)\right]_0^v$$

$$= -40\left\{\log\left(40g - v\right) - \log\left(40g\right)\right\}$$

$$= 40\log\left(\frac{40g}{40g - v}\right)$$

$$\frac{t}{40} = \log\left(\frac{40g}{40g - v}\right)$$

$$\frac{40g}{40g - v} = e^{\frac{t}{40}}$$

$$\frac{40g - v}{40g} = e^{-\frac{t}{40}}$$

$$40g - v = 40ge^{-\frac{t}{40}}$$

$$v = 40g - 40ge^{-\frac{t}{40}}$$

$$v = 40g\left(1 - e^{-\frac{t}{40}}\right)$$

b) the terminal velocity

terminal velocity occurs when $\ddot{x} = 0$

$$\text{i.e. } 0 = g - \frac{1}{40}v$$

$$v = 40g$$

OR

$$\lim_{t \to \infty} v = \lim_{t \to \infty} 40g\left(1 - e^{-\frac{t}{40}}\right)$$

$$= 40g$$

$\therefore$ terminal velocity is $40g$ m/s

c) The distance it has fallen after time t

$$\frac{dx}{dt} = 40g\left(1 - e^{-\frac{t}{40}}\right)$$

$$\int_0^x dx = 40g\int_0^t \left(1 - e^{-\frac{t}{40}}\right)dt$$

$$x = 40g\left[t + 40e^{-\frac{t}{40}}\right]_0^t$$

$$x = 40g\left\{t + 40e^{-\frac{t}{40}} - 0 - 40\right\}$$

$$x = 40gt + 1600ge^{-\frac{t}{40}} - 1600g$$

"*Cambridge*" Exercise 4A;
1 to 4

"*Cambridge*" Exercise 4B;
1 to 9

***Patel* Exercise 8C;**
2, 4, 5, 6, 8, 10, 13, 16