1h

$v = \cos^2 x$

$\frac{dx}{dt} = \cos^2 x$

$\frac{dt}{dx} = \sec^2 x$

$t = \tan x + c$

$t = 0,\ x = 1$

$0 = \tan 1 + c$

$c = -\tan 1$

$\underline{t = \tan x - \tan 1}$

$\tan x = t + \tan 1$

$\underline{x = \tan^{-1}(t + \tan 1)}$

2h) $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = \ddot{x}$

$\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}\cos^4 x\right)$

$= 2\cos^3 x \cdot -\sin x$

$\underline{= -2\cos^3 x \sin x}$

9c) $v^2 = 2x \log x$

$\ddot{x} = \log x + 1$

$x = 1,\ v = 0$

when $x = 1,\ \ddot{x} = 1$

$\therefore$ move in +ive direction

ie $v > 0$

ie $x > 1$ $\therefore v^2 \neq 0$

ie particle does not stop

thus $v \not< 0$

ie $v > 0$

10/

$$\ddot{x} = \frac{1}{36+x^2}$$

$$\frac{1}{2}v^2 = \frac{1}{6}\tan^{-1}\frac{x}{6} + c$$

$$v^2 = \frac{1}{3}\tan^{-1}\frac{x}{6} + c$$

when $v=0, x=0$

$$c = 0$$

$$v^2 = \frac{1}{3}\tan^{-1}\frac{x}{6}$$

when $x=6$, $v^2 = \frac{1}{3}\tan^{-1} 1$

$$= \frac{\pi}{12}$$

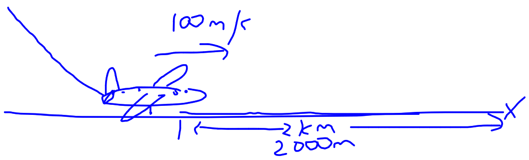
$$v = \frac{\sqrt{\pi}}{2\sqrt{3}} \text{ m/s}$$

$$\lim_{t\to\infty} v^2 = \lim_{t\to\infty} \frac{1}{3}\tan^{-1}\frac{x}{6}$$

$$= \frac{\pi}{6}$$

$\therefore$ as $t\to\infty$, $v \to \sqrt{\frac{\pi}{6}}$ m/s

11)



$\ddot{x} = -k$

$\frac{1}{2}v^2 = -kx + c$

$v^2 = -2kx + c$

when $x = 0$, $v = 100$

$100^2 = c$

$c = 10000$

$x = 2000$, $v = 0$

$0 = -4000k + 10000$

$k = \frac{5}{2}$

$v^2 = -5x + 10000$

b) $x = 1000$, $v^2 = 5000$

$v = 50\sqrt{2}$ m/s

(ii) $v = 50$, $2500 = -5x + 10000$

$5x = 7500$

$x = 1500$ m

c) as we are always travelling in the tve direction until we stop.

$$\frac{dx}{dt} = (10000 - 5x)^{\frac{1}{2}}$$

$$\frac{dt}{dx} = (10000 - 5x)^{-\frac{1}{2}}$$

$$t = -\frac{2}{5}\sqrt{10000-5x} + c$$

when $t = 0,\ x = 0$

$$0 = -\frac{2}{5}\sqrt{10000} + c$$

$$c = 40$$

$$t = -\frac{2}{5}\sqrt{10000 - 5x} + 40$$

$$t - 40 = -\frac{2}{5}\sqrt{10000 - 5x}$$

$$\frac{5}{2}t - 100 = -\sqrt{10000 - 5x}$$

$$10000 - 5x = \left(\frac{5}{2}t - 100\right)^2$$

$$x = 2000 - \frac{1}{5}\left(\frac{5}{2}t - 100\right)^2$$

stops when $x = 2000$

ie $\frac{1}{5}\left(\frac{5}{2}t - 100\right)^2 = 0$

$$t = 40$$

13/ $v = \cos^2 2x$

$x = \frac{1}{2}\tan^{-1} 2t$

$v = \dfrac{1}{1+4t^2}$

$\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}\cos^4 2x\right)$

$= 2\cos^3 2x \cdot -2\sin 2x$

$= -4\cos^3 2x \sin 2x$

$\underline{x = \frac{\pi}{8}}$

$\frac{\pi}{8} = \frac{1}{2}\tan^{-1} 2t$

$\tan^{-1} 2t = \frac{\pi}{4}$

$2t = 1$

$\underline{t = \frac{1}{2}}$

$v = \cos^2 \frac{\pi}{4}$

$= \left(\frac{1}{\sqrt{2}}\right)^2$

$\underline{= \frac{1}{2}}$

$\ddot{x} = -4\cos^3\frac{\pi}{4}\sin\frac{\pi}{4}$

$= -4\left(\frac{1}{2\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)$

$\underline{= -1}$

15a) $v = x^2 e^{-x^2}$

$t = 0,\ x = \frac{1}{2}$

$\ddot{x} = \frac{d}{dx}\left(\frac{1}{2} x^4 e^{-2x^2}\right)$

$= \frac{1}{2}\left[(x^4)(-4x e^{-2x^2}) + (e^{-2x^2})(4x^3)\right]$

$= 2x^3 e^{-2x^2}(1 - x^2)$

max v occurs when $\ddot{x} = 0$

ie $4x^3 e^{-2x^2}(1 - x^2) = 0$

$x = 0$ or $x = \pm 1$

$\therefore x = 1$

max $v = \frac{1}{e}$ m/s

b) $\dfrac{dx}{dt} = x^2 e^{-x^2}$

$dt = \dfrac{e^{x^2}}{x^2}\,dx$

$\displaystyle\int_0^T dt = \int_{\frac{1}{2}}^{1} \frac{e^{x^2}}{x^2}\,dx$

$\displaystyle T = \int_{\frac{1}{2}}^{1} \frac{e^{x^2}}{x^2}\,dx$

$\doteq \dfrac{\frac{1}{4}}{3}\left\{4e^{\frac{1}{4}} + \dfrac{64}{9}e^{\frac{9}{16}} + e\right\}$

$= \underline{1.695\,s}$ (to 4 sig figs)

	1	4	1
x	$\frac{1}{2}$	$\frac{3}{4}$	1
$\frac{e^{x^2}}{x^2}$	$4e^{\frac{1}{4}}$	$\frac{16}{9}e^{\frac{9}{16}}$	e

17

$\ddot{x} = 2x - 1$

$t = 0, \ \dot{x} = 0, \ x = 5$

a) $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = 2x - 1$

$\frac{1}{2}v^2 = x^2 - x + c$

when $x = 5, v = 0$

$0 = 25 - 5 + c$

$c = -20$

$\underline{v^2 = 2x^2 - 2x - 40}$

$\underline{x = 0}$

$v^2 = -40$

but $v^2 \geq 0$

$\therefore \underline{x \neq 0}$

17b) $\ddot{x} = 2x - 1$

$\frac{1}{2}v^2 = x^2 - x + c$

$v^2 = 2x^2 - 2x + c$

when $x = 5$, $v = 0$

$0 = 40 + c$

$c = -40$

$v^2 = 2x^2 - 2x - 40$

$|v| = 2\sqrt{5}$

$20 = 2x^2 - 2x - 40$

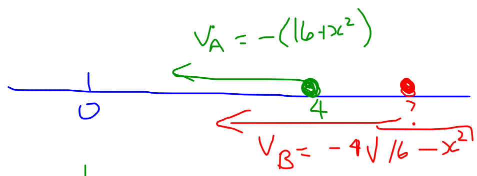
$x^2 - x - 30 = 0$

$(x - 6)(x + 5) = 0$

$x = 6$ or $x = -5$

$\therefore \underline{x = 6}$ ($x \neq 0$ $\therefore$ particle cannot travel to -5)

18/



A: $\frac{dx}{dt} = -(16+x^2)$

$$t = -\int_4^0 \frac{dx}{16+x^2}$$

$$= \left[\frac{1}{4}\tan^{-1}\frac{x}{4}\right]_0^4$$

$$= \frac{1}{4}\left(\frac{\pi}{4} - 0\right)$$

$$= \frac{\pi}{16}$$

B. $\frac{dx}{dt} = -4\sqrt{16-x^2}$

$$t = -\frac{1}{4}\int_x^0 \frac{dx}{\sqrt{16-x^2}}$$

$$\frac{\pi}{16} = \frac{1}{4}\left[\sin^{-1}\frac{x}{4}\right]_0^x$$

$$\frac{\pi}{4} = \sin^{-1}\frac{x}{4}$$

$$\frac{x}{4} = \frac{1}{\sqrt{2}}$$

$$x = 2\sqrt{2}$$

OR

$$\frac{dx}{dt} = -(16+x^2) \qquad \frac{dx}{dt} = -4\sqrt{16-x^2}$$

$$-\int_4^0 \frac{dx}{16+x^2} = t \qquad -\frac{1}{4}\int_x^0 \frac{dx}{\sqrt{16-x^2}} = t$$

$$\frac{1}{4}\int_0^x \frac{dx}{\sqrt{16-x^2}} = \int_0^4 \frac{dx}{16+x^2}$$

$$\frac{1}{4}\left[\sin^{-1}\frac{x}{4}\right]_0^x = \frac{1}{4}\left[\tan^{-1}\frac{x}{4}\right]_0^4$$

$$\sin^{-1}\frac{x}{4} = \frac{\pi}{4}$$

$$\frac{x}{4} = \sin\frac{\pi}{4}$$

$$\frac{x}{4} = \frac{1}{\sqrt{2}}$$

$$x = 2\sqrt{2}$$

20)

$v^2 = 14x - x^2$

$v^2 \geq 0$

$x(14-x) \geq 0$

$0 \leq x \leq 14$

$\ddot{x} = \frac{d}{dx}\left(7x - \frac{1}{2}x^2\right)$

$= 7 - x$

max speed occurs when $\ddot{x} = 0$

$x = 7$, $v^2 = 49$

$v = \pm 7$

$\therefore$ max speed is 7m/s

b) $|7-x| \leq 3$

$7 - x \leq 3$ or $-(7-x) \leq 3$

$-x \leq -4$ $\quad -7 + x \leq 3$

$x \geq 4$ $\quad x \leq 10$

$4 \leq x \leq 10$

Q1a)

$\ddot{x} = 3x^2 - 14x$

$\frac{1}{2}v^2 = x^3 - 7x^2 + c$

$v^2 = 2x^3 - 14x^2 + c$

$x = 0, \; v = 6\sqrt{2}$

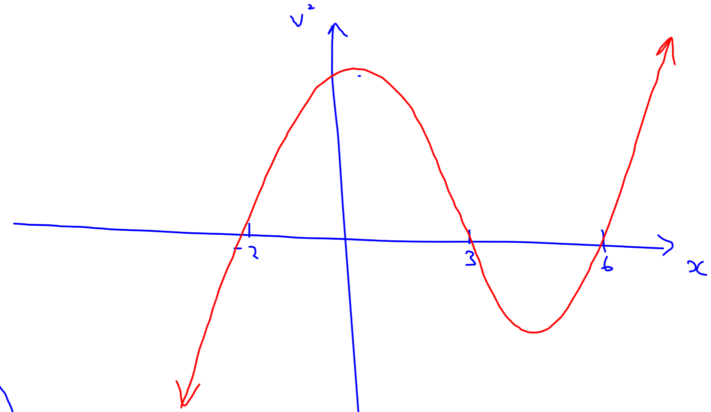
$72 = c$

$v^2 = 2x^3 - 14x^2 + 72$

$v^2 = 2(x^3 - 7x^2 + 36)$

$v^2 = 2(x-6)(x^2 - x - 6)$

$v^2 = 2(x-6)(x-3)(x+2)$



b) $x = 3, \; v = 0, \; \ddot{x} = -15$

moves in negative direction

c) max speed when $\ddot{x} = 0$

$3x^2 - 14x = 0$

$x(3x - 14) = 0$

$x = 0$ or $x = \frac{14}{3}$

$v^2 = 72$

$v = 6\sqrt{2}$

24

$$\ddot{x} = -kx^{-2}$$

$$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = -kx^{-2}$$

$$\frac{1}{2}v^2 = kx^{-1} + c$$

$$v^2 = 2kx^{-1} + c$$

$x = D,\ v = 0$

$$0 = \frac{2k}{D} + c$$

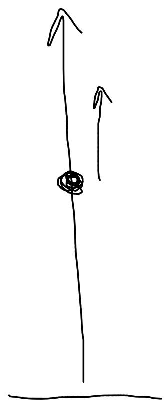
$$c = -\frac{2k}{D}$$

$$v^2 = \frac{2k}{x} - \frac{2k}{D}$$

$$= \frac{2k(D-x)}{Dx}$$

$$v = \sqrt{\frac{2k(D-x)}{Dx}}$$

25



$x = R,\ v = V,\ \ddot{x} = -g$

$$\ddot{x} = \frac{-k}{x^2}$$

$$-g = \frac{-k}{R^2}$$

$$\underline{k = gR^2}$$