5b)

$\ddot{x} + \frac{1}{4}x = 0$

$x = 0, \ |v| = 4$

$\ddot{x} = -\frac{1}{4}x$

$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = -\frac{1}{4}x$

$\frac{1}{2}v^2 = -\frac{1}{8}x^2 + c$

When $x = 0, \ v = 4$

$8 = 0 + c$

$c = 8$

$v^2 = -\frac{1}{4}x^2 + 16$

$v^2 \geq 0$

$-\frac{1}{4}x^2 + 16 \geq 0$

$\frac{1}{4}x^2 \leq 16$

$x^2 \leq 64$

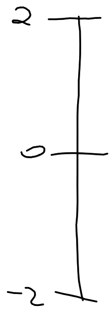
$-8 \leq x \leq 8$

$\therefore$ pulled down 8cm

6b) $T = 3$

$\frac{2\pi}{n} = 3$

$n = \frac{2\pi}{3}$



$$\boxed{\ddot{x} = -\frac{4\pi^2}{9}x}$$

$$v\frac{dv}{dx} = -\frac{4\pi^2}{9}x$$

$$\int_0^v v\,dv = -\frac{4\pi^2}{9}\int_2^x x\,dx$$

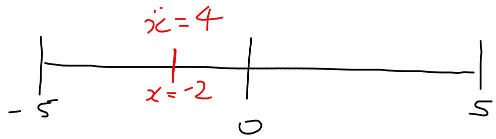
$$v^2 = -\frac{4\pi^2}{9}\left[x^2\right]_2^x$$

$$v^2 = -\frac{4\pi^2}{9}(x^2 - 4)$$

greatest $\ddot{x} = \frac{8\pi^2}{9}$ m/s^2

greatest $v = \frac{4\pi}{3}$ m/s

8/



$\ddot{x} = -n^2 x$

$v \frac{dv}{dx} = -n^2 x$

$\int_0^v v\,dv = -n^2 \int_5^x x\,dx$

$\left[v^2 \right]_0^v = -n^2 \left[x^2 \right]_5^x$

$v^2 - 25 = -n^2 x^2 + 25n^2$

$\underline{v^2 = -2x^2 + 50}$

$\ddot{x} = -n^2 x$

$4 = -n^2(-2)$

$n^2 = 2$

$n = \sqrt{2}$

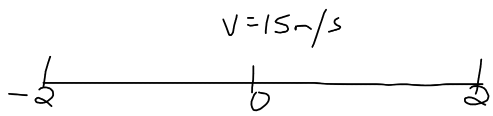
$\underline{\text{when } x = 0}$, $v^2 = 50$

$\underline{v = 5\sqrt{2} \text{ m/s}}$

when $x = 4$, $v^2 = 18$

$\underline{v = 3\sqrt{2} \text{ m/s}}$

10a)



$$\ddot{x} = -n^2 x$$

$$\int_0^{15} v\,dv = -n^2 \int_2^0 x\,dx$$

$$\left[v^2\right]_0^{15} = -n^2 \, 2 \left[x^2\right]_2^0$$

$$15^2 = 4n^2$$

$$n = \frac{15}{2}$$

$$\therefore \ddot{x} = -\frac{225}{4} x$$

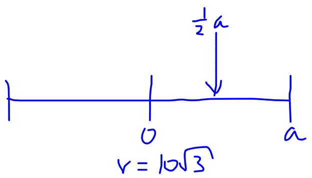
when $x = \frac{2}{3}$

$$\ddot{x} = -\frac{225}{4} \times \frac{2}{3}$$

$$= -\frac{75}{2}$$

$$\left[v^2\right]_0^{v} = -\frac{225}{4}\left[x^2\right]_2^{\frac{2}{3}}$$

13/



$\ddot{x} = -n^2 x$

$\frac{1}{2}v^2 = -\frac{1}{2}n^2x^2 + c$

when $x = 0$, $v = 10\sqrt{3}$

$150 = c$

$v^2 = -n^2x^2 + 300$

when $x = a$, $v^2 = 0$

$-a^2n^2 = -300$

$an = \sqrt{300}$

$v^2 = -\frac{300}{a^2}x^2 + 300$

$x = \frac{1}{2}a$

$v^2 = -\frac{300}{a^2}\left(\frac{1}{4}a^2\right) + 300$

$= 225$

$v = \pm 15$

$\therefore$ speed is 15 m/s

14a) $v^2 = -9x^2 + 18x + 27$

$\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

$= \frac{d}{dx}\left(-\frac{9}{2}x^2 + 9x + \frac{27}{2}\right)$

$= -9x + 9$

$= -9(x-1)$

centre of motion $x = 1$

$T = \frac{2\pi}{3}$

$v^2 = -9(x^2 - 2x - 3)$

$= -9(x+1)(x-3)$

$v^2 \geq 0$

$-1 \leq x \leq 3$

$\therefore$ amplitude $= 2$

14b)(iv) $V^2 = 8 - 10x - 3x^2$

$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = -5 - 3x$

$\ddot{x} = -3\left(x + \frac{5}{3}\right)$

$\therefore$ particle is in SHM as $\ddot{x} = -n^2 X$

centre $= -\frac{5}{3}$

$V^2 \geq 0$

$T = \frac{2\pi}{\sqrt{3}}$

$8 - 10x - 3x^2 \geq 0$

$3x^2 + 10x - 8 \leq 0$

$(3x - 2)(x + 4) \leq 0$

$-4 \leq x \leq \frac{2}{3}$

amplitude $= \frac{7}{3}$

16)

$\ddot{x} = -9(x-7)$

$\frac{1}{2}v^2 = -\frac{9}{2}x^2 + 63x + c$

when $x = 0, v = 0$

$c = 0$

$v^2 = -9x^2 + 126x$

$= -9(x^2 - 14x)$

$= -9(x-7)^2 + 441$

$\therefore$ max speed $= 441$ m/s

18/

$x = 3 + \sin 4t + \sqrt{3}\cos 4t$

$x = 3 + 2\sin\left(4t + \frac{\pi}{3}\right)$

$\dot{x} = 8\cos\left(4t + \frac{\pi}{3}\right)$

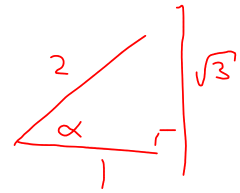
$\ddot{x} = -32\sin\left(4t + \frac{\pi}{3}\right)$

$= -16(x-3)$

centre : $x = 3$

$T = \frac{2\pi}{4}$

$= \frac{\pi}{2}$



c) $x = 3$

$2\sin\left(4t + \frac{\pi}{3}\right) = 0$

$\sin\left(4t + \frac{\pi}{3}\right) = 0$

$4t + \frac{\pi}{3} = \pi k + (-1)^k \sin^{-1} 0$

$4t = \pi k - \frac{\pi}{3}$

$t = \frac{3\pi k - \pi}{12}$ seconds

speed $= 8$ m/s

19 $x = 10 + 8\sin 2t + 6\cos 2t$

$x = 10 + 10\sin(2t + \alpha)$

$\dot{x} = 20\cos(2t + \alpha)$

$\ddot{x} = -40\sin(2t + \alpha)$

$= -4(x - 10)$

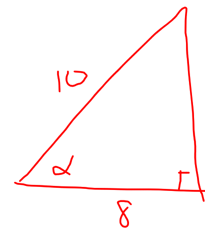
$\therefore$ SHM, in the form $-n^2x$

centre: $x = 10$ m

$T = \frac{2\pi}{2}$

$= \pi$ seconds

amplitude $= 10$m.



$\alpha = \tan^{-1}\frac{3}{4}$

b) $x = 0$

$\sin(2t + \alpha) = -1$

$2t + \alpha = \frac{3\pi}{2}$

$2t = \frac{3\pi}{2} - \alpha$

$t = \frac{3\pi}{4} - \frac{\alpha}{2}$

$t = 2.034$ s