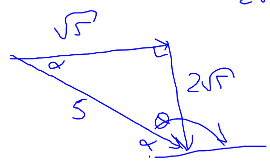
4f)

$\dot{x} = \sqrt{5}$

$\dot{y} = -10t + 2\sqrt{5}$

when $t = \frac{2}{\sqrt{5}}$, $\dot{y} = \frac{-20}{\sqrt{5}} + 2\sqrt{5}$

$= -2\sqrt{5}$



9 b(ii)

(i) $H = \frac{V^2 \sin^2\alpha}{2g}$

(ii) max $H = \frac{V^2}{2g}$ $\quad (-1 \leq \sin\alpha \leq 1)$

$\sin^2\alpha = 1$

$\alpha = 90^\circ$ $\quad$ (α must be acute)

9c (ii)

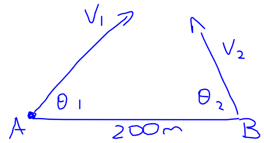
(i) $R = \dfrac{2V^2 \sin\alpha \cos\alpha}{g}$ $\quad T = \dfrac{2V\sin\alpha}{g}$

$\sin 2\alpha = \frac{1}{2}$

$2\alpha = \frac{\pi}{3}$

$\alpha = \frac{\pi}{6}$.

13/



$x_A = V_1 t \cos\theta_1$ $\qquad$ $x_B = V_2 t \cos\theta_2$

$y_A = -\frac{1}{2}gt^2 + V_1 t \sin\theta_1$ $\qquad$ $y_B = -\frac{1}{2}gt^2 + V_2 t \sin\theta_2$

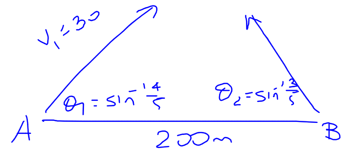
a) when collide

$$y_A = y_B$$

$$-\frac{1}{2}gt^2 + V_1 t \sin\theta_1 = -\frac{1}{2}gt^2 + V_2 t \sin\theta_2$$

$$V_1 t \sin\theta_1 = V_2 t \sin\theta_2$$

$$\underline{V_1 \sin\theta_1 = V_2 \sin\theta_2}$$



(i) $V_1 \sin\theta_1 = V_2 \sin\theta_2$

$30\left(\frac{4}{5}\right) = V_2\left(\frac{3}{5}\right)$

$V_2 = 30 \times \frac{4}{5} \times \frac{5}{3}$

$= \underline{40}$

Education

$x_A + x_B = 200$

$t(V_1\cos\theta_1 + V_2\cos\theta_2) = 200$

$t\left[30\left(\frac{3}{5}\right) + 40\left(\frac{4}{5}\right)\right] = 200$

$t(50) = 200$

$\underline{\underline{t = 4}}$

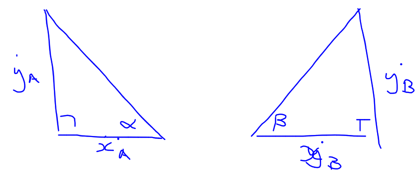
(iii) $t = 4$

$$y_A = -\frac{1}{2}(10)(4)^2 + (30)(4)\left(\frac{4}{5}\right)$$

$$= -80 + 96$$

$$= \underline{16}$$

A

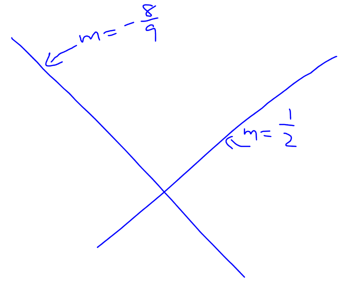


$$\tan\alpha = -\frac{8}{9}$$

$$\tan\beta = -\frac{1}{2}$$

$$\dot{x}_A = V_1\cos\theta_1 = (30)\left(\tfrac{3}{5}\right) = 18 \qquad \dot{y}_A = -gt + V_1\sin\theta_1 = -10(4) + (30)\left(\tfrac{4}{5}\right) = -16$$

$$\dot{x}_B = V_2\cos\theta_2 = (40)\left(\tfrac{4}{5}\right) = 32 \qquad \dot{y}_B = -gt + V_2\sin\theta_2 = -10(4) + (40)\left(\tfrac{3}{5}\right) = -16$$



$$\tan\gamma = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{\frac{1}{2} + \frac{8}{9}}{1 + \left(\frac{1}{2}\right)\left(-\frac{8}{9}\right)}\right|$$

$$= \left|\frac{\frac{25}{18}}{\frac{10}{18}}\right|$$

$$= \frac{5}{2}$$

$\therefore$ obtuse $\angle = \underline{112^\circ}$ $\quad \gamma = 68.12^\circ$

16b)

a) P_1 max height $= \dfrac{V^2 \sin^2\theta}{2g}$

b) P_2 max height $= \dfrac{\left(\frac{3}{2}V\right)^2 \sin^2\frac{\theta}{2}}{2g}$

$$V^2\sin^2\theta = \left(\tfrac{3}{2}V\right)^2 \sin^2\tfrac{\theta}{2}$$

$$\sin^2\theta = \tfrac{9}{4}\sin^2\tfrac{\theta}{2}$$

$$1-\cos^2\theta = \tfrac{9}{8}\left(1-\cos\theta\right)$$

$$8-8\cos^2\theta = 9-9\cos\theta$$

$$8\cos^2\theta - 9\cos\theta + 1 = 0$$

$$(8\cos\theta - 1)(\cos\theta - 1) = 0$$

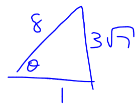
$$\cos\theta = \tfrac{1}{8} \quad \text{or} \quad \cos\theta = 1$$

$$\underline{\theta = \cos^{-1}\tfrac{1}{8}} \qquad \theta = 0^\circ$$

not possible

P_1 $\dot{y} = -gt + V\sin\theta$

max height $\dot{y} = 0$



$$t = \frac{V\sin\theta}{g}$$

$$= \frac{3\sqrt{7}V}{8g}$$

$\therefore$ Yes, take same time.

P_2: $\dot{y} = -gt + \frac{3}{2}V\sin\frac{\theta}{2}$

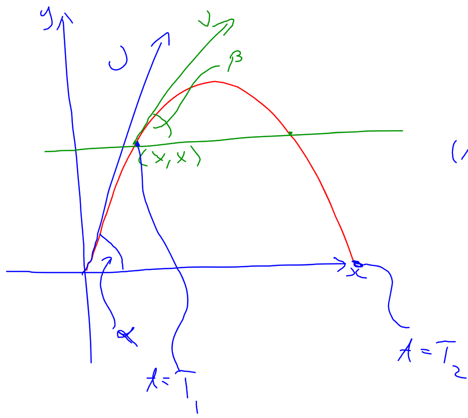
$$t = \frac{\frac{3}{2}V\sin\frac{\theta}{2}}{g}$$

$$= \frac{\frac{3}{2}V\sqrt{\frac{1}{2}(1-\cos\theta)}}{g}$$

$$= \frac{3V\sqrt{\frac{7}{16}}}{2g}$$

$$= \frac{3\sqrt{7}V}{8g}$$

17



(i) $T_1 = \frac{2v}{g}(\sin\alpha - \cos\alpha)$

(ii) $T_2 = \frac{T_1}{1 - \cot\alpha}$

$$y = -\frac{1}{2}gt^2 + Ut\sin\alpha$$

$$y = 0$$

$$-\frac{1}{2}gt^2 + Ut\sin\alpha = 0$$

$$t\left(U\sin\alpha - \frac{1}{2}gt\right) = 0$$

$$t = 0, \text{ or } t = \frac{2U\sin\alpha}{g}$$

$$\frac{2U\sin\alpha}{g} = \frac{2U(\sin\alpha - \cos\alpha)}{g(1-\cot\alpha)}$$

$$\sin\alpha = \frac{\sin\alpha - \cos\alpha}{1 - \frac{\sin\alpha}{\cos\alpha}}$$

$$\sin\alpha - \frac{\sin^2\alpha}{\cos\alpha} = \sin\alpha - \cos\alpha$$

$$\sin^2\alpha = \cos^2\alpha$$

$$\sin\alpha = \cos\alpha$$

$$\tan\alpha = 1$$

$$\alpha = \frac{5\pi}{4}$$

$$1 - \cot\alpha > 0$$

$$\cot\alpha < 1$$

$$\frac{\pi}{4} < \alpha < \frac{\pi}{2}$$

b) $V\cos\beta = U\cos\alpha$

$V\cos\beta = U\cos 2\beta$

$V\cos\beta = U(2\cos^2\beta - 1)$

18a (iii) $y = 2\sin(x-\theta)\cos x$ ..... (1)

(ii) $\frac{dy}{dx} = 2\cos(2x-\theta)$

(iii) $2\sin(x-\theta)\cos x = \sin(2x-\theta) - \sin\theta$

$y = 2\int \cos(2x-\theta)\,dx$

$= \sin(2x-\theta) + c$ .... (2)

in ① $x=0$, $y = 2\sin(-\theta) \times (1)$

$= -2\sin\theta$

in ② where $x=0$, $y = -2\sin\theta$

$-2\sin\theta = \sin(-\theta) + c$

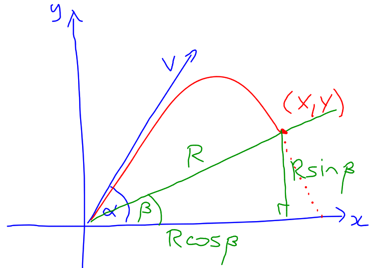
$= -\sin\theta + c$

$c = -\sin\theta$

$\therefore y = \sin(2x-\theta) - \sin\theta$

Thus, $\underline{2\sin(x-\theta)\cos x = \sin(2x-\theta) - \sin\theta}$

b)



$x = Vt\cos\alpha$

$y = -\frac{1}{2}gt^2 + Vt\sin\alpha$

when $t = T$ $\quad X = R\cos\beta \qquad Y = R\sin\beta$

$$Y = -\frac{1}{2}gT^2 + VT\sin\alpha$$

$$X = VT\cos\alpha$$

$$T = \frac{X}{V\cos\alpha}$$

$$Y = -\frac{1}{2}g\frac{X^2}{V^2\cos^2\alpha} + X\tan\alpha$$

$$Y = -\frac{g}{2V^2}X^2\sec^2\alpha + X\tan\alpha$$

$$R\sin\beta = R\cos\beta\left(-\frac{g}{2V^2}X\sec^2\alpha + \tan\alpha\right)$$

$$\tan\beta - \tan\alpha = -\frac{g}{2V^2}X\sec^2\alpha$$

$$X = \frac{2V^2\cos^2\alpha(\tan\alpha - \tan\beta)}{g}$$

(iii) $$R\cos\beta = \frac{2V^2\cos^2\alpha(\tan\alpha - \tan\beta)}{g}$$

$$= \frac{2V^2\cos^2\alpha(\tan\alpha - \tan\beta)}{g\cos\beta}$$

$$= \frac{2V^2\cos\alpha\left(\frac{\sin\alpha}{\cos\alpha} - \frac{\sin\beta}{\cos\beta}\right)\cos\alpha\cos\beta}{g\cos^2\beta}$$

$$= \frac{2V^2\cos\alpha(\sin\alpha\cos\beta - \cos\alpha\sin\beta)}{g\cos^2\beta}$$

$$= \frac{2V^2\cos\alpha\sin(\alpha-\beta)}{g\cos^2\beta}$$

(iii) $R = \dfrac{V^2\left[\sin(2\alpha-\beta) - \sin\beta\right]}{g\cos^2\beta}$

max $\sin(2\alpha-\beta)$

$= 1$

$\max R = \dfrac{V^2(1 - \sin\beta)}{g(1+\sin\beta)(1-\sin\beta)}$

$= \dfrac{V^2}{g(1+\sin\beta)}$

(iv) $\beta = 14^\circ$

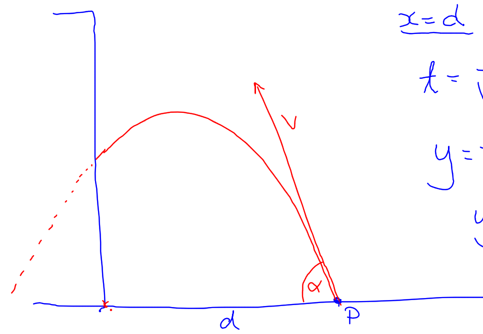
$\sin(2\alpha - \beta) = 1$

$2\alpha - 14 = 90$

$2\alpha = 104$

$\underline{\alpha = 52^\circ}$

19b)



$x = Vt\cos\alpha \quad y = -\frac{1}{2}gt^2 + Vt\sin\alpha$

$\underline{x = d}$

$t = \frac{d}{V\cos\alpha}$

$y = -\frac{1}{2}g\frac{d^2}{V^2\cos^2\alpha} + \frac{d\sin\alpha}{\cos\alpha}$

$\underline{y > 0}$.

$d\tan\alpha > \frac{gd^2}{2 \cdot V^2 \cos^2\alpha}$

$2V^2\cos^2\alpha > \frac{gd^2}{d\tan\alpha}$

$V^2 > \frac{gd}{2\sin\theta\cos\theta}$

$V^2 > \frac{gd}{\sin 2\theta}$

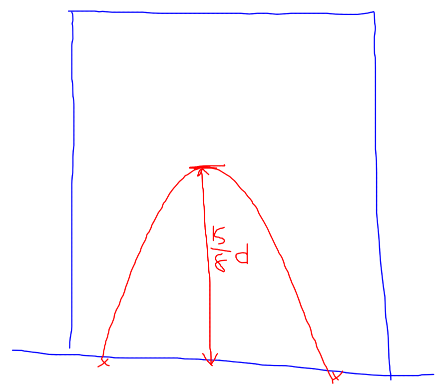
$V^2 > gd \quad (\sin 2\theta \leq 1)$

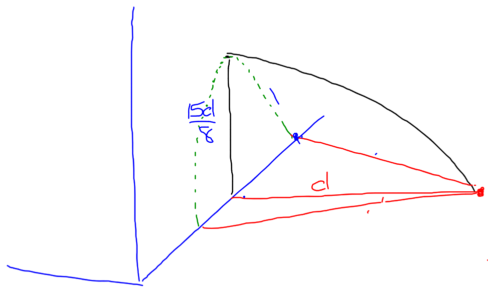
$\underline{V > \sqrt{gd}}$

b) $V = 2\sqrt{gd}$

$$y = -\frac{1}{2} g \frac{d^2}{4gd\cos^2\alpha} + d\tan\alpha$$

$$= -\frac{d}{8}\sec^2\alpha + d\tan\alpha$$





$$x = Vt\cos\alpha$$
$$d = 2\sqrt{gd}\, t\cos\alpha$$
$$t = \frac{\sqrt{d}}{2\sqrt{g}\cos\alpha}$$

$$\dot{y} = 0$$
$$-gt + V\sin\alpha = 0$$
$$\frac{\sqrt{dg}}{2\cos\alpha} = 2\sqrt{gd}\sin\alpha$$
$$4\sin\alpha\cos\alpha = 1$$
$$\sin 2\alpha = \frac{1}{2}$$
$$\underline{\alpha = 15^\circ},\ 75^\circ$$

$$y = -\frac{1}{2}gt^2 + Vt\sin\alpha$$
$$= -\frac{1}{2}g\left(\frac{d}{4g\cos^2\alpha}\right) + 2\sqrt{gd}\left(\frac{\sqrt{d}}{2\sqrt{g}\cos\alpha}\right)\sin\alpha$$
$$= \frac{-d}{8\cos^2\alpha} + \frac{d}{\cos\alpha}\cdot\sin\alpha$$
$$= \frac{-d(2\sqrt{2})^2}{8(\sqrt{3}+1)^2} + \frac{d(\sqrt{3}-1)}{(\sqrt{3}+1)}$$
$$=$$

$$y = -\frac{1}{2} g \frac{d^2}{V^2 \cos^2\alpha} + \frac{d \sin\alpha}{\cos\alpha}$$

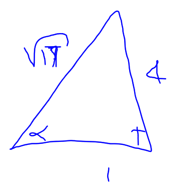
$$\frac{dy}{d\alpha} = \frac{-gd^2}{2V^2}\left(2\sec^2\alpha \tan\alpha\right) + d\sec^2\alpha$$

$$0 = \sec^2\alpha \left(d - \frac{gd^2}{V^2} \tan\alpha \right)$$

$$\tan\alpha = \frac{V^2}{gd}$$

$$= 4$$

$$\alpha =$$



$$y = -\frac{1}{2} g \frac{d^2}{V^2 \cos^2\alpha} + \frac{d \sin\alpha}{\cos\alpha}$$

$$= -\frac{1}{2} g \left(\frac{d^2}{4gd \times \frac{1}{17}} \right) + 4d$$

$$= \frac{-17d}{8} + \frac{32d}{8}$$

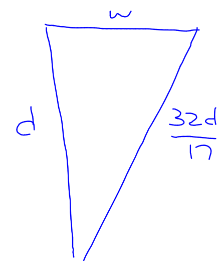
$$= \underline{\frac{15d}{8}}$$

$y = 0, \quad V = 2\sqrt{gd}, \quad \tan\alpha = 4$

$$R = \frac{V^2}{g}\sin 2\alpha$$

$$= \frac{4\cancel{g}d}{\cancel{g}} \times 2 \times \frac{4}{17}$$

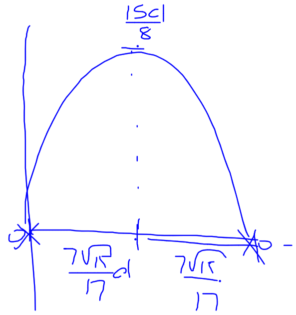
$$= \frac{32d}{17}$$



$$w^2 = \left[\left(\frac{32}{17}\right)^2 - 1^2\right] d$$

$$= \frac{735d}{289}$$

$$= \frac{7\sqrt{15}}{17} d$$



$$A = \frac{7\sqrt{15}d}{51}\left(0 + 4\left(\frac{15d}{8}\right) + 0\right)$$

$$= \frac{7\sqrt{15}d}{51} \times \frac{15d}{2}$$

$$y = x\left(x - \frac{14\sqrt{15}}{17}\right)$$