2 c) (ii)

$$y = -\frac{1}{10}x^2 + x$$

$x = 8$, $y = 1.6$

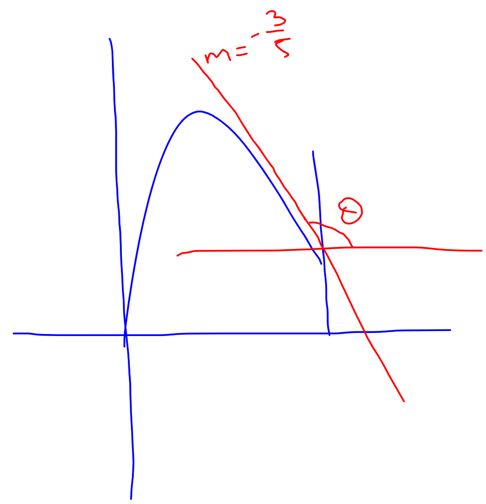
$$\frac{dy}{dx} = -\frac{1}{5}x + 1$$

when $x = 8$, $\frac{dy}{dx} = \frac{-3}{5}$

$$\tan\theta = \frac{-3}{5}$$

$$\theta = \tan^{-1}\frac{-3}{5}$$

$$= -\tan^{-1}\frac{3}{5}$$



2d) $y = 2.1\text{m}$

$$y = -\frac{1}{10}x^2 + x$$

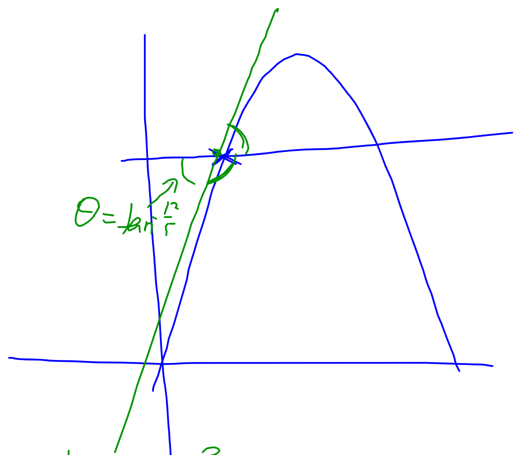
$$2.1 = -\frac{1}{10}x^2 + x$$

$$x^2 - 10x + 21 = 0$$

$$(x-3)(x-7) = 0$$

$$x = 3 \text{ or } 7$$

$$\underline{\underline{\text{hits } 3\text{m}}}$$

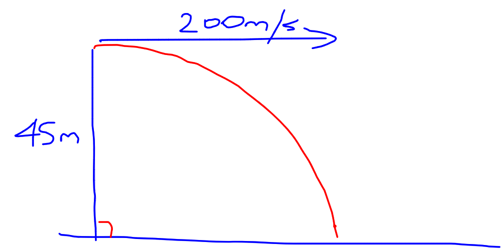


when $x = 3$

$$\frac{dy}{dx} = -\frac{3}{5} + 1$$

$$= \frac{2}{5}$$

4



a) Intial $\dot{x} = 200$ $\dot{y} = 0$

b) $\ddot{x} = 0$ $\ddot{y} = -10$

$\dot{x} = c_1$ $\dot{y} = -10t + c_2$

when $t = 0$, $\dot{x} = 200$, $\dot{y} = 0$

$200 = c_1$ $0 = 0 + c_2$

$c_2 = 0$

$\therefore \dot{x} = 200$ $\dot{y} = -10t$

$x = 200t + c_3$ $y = -5t^2 + c_4$

when $t = 0$, $x = 0$, $y = 0$

$0 = 0 + c_3$

$c_3 = 0$

$x = 200t$

$0 = 0 + c_4$

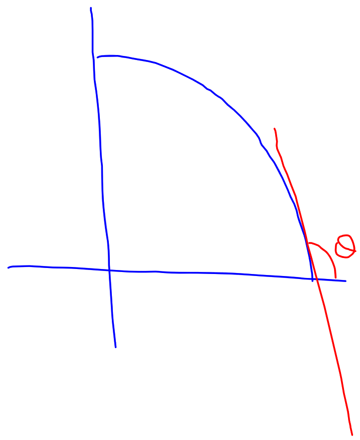
$c_4 = 0$

$y = -5t^2$

$$t = \frac{x}{200}$$

$$y = -5\left(\frac{x^2}{40000}\right)$$

$$y = \frac{-x^2}{8000}$$



6 a) $x = Vt\cos\alpha$ $\qquad$ $y = -\frac{1}{2}gt^2 + Vt\sin\alpha$

$$y = x\tan\alpha - \frac{gx^2}{2V^2}\sec^2\alpha$$

$$2V^2 y = 2V^2 x\tan\alpha - gx^2(1+\tan^2\alpha)$$

6b)

$$gx^2\tan^2\alpha - 2xV^2\tan\alpha + (2yV^2 + gx^2) = 0$$

$V = 200 \quad g = 10 \quad x = 3000 \quad y = 500$

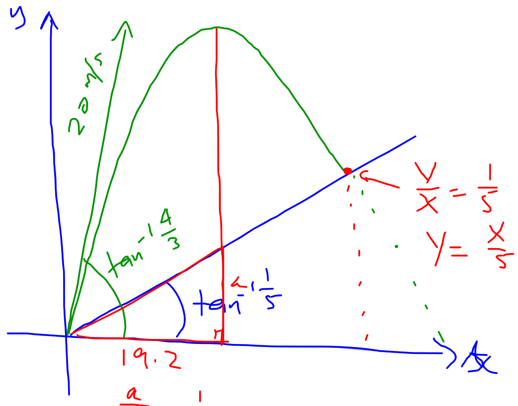
$$90\,000\,000\tan^2\alpha - 240\,000\,000\tan\alpha + 130\,000\,000 = 0.$$

$$9\tan^2\alpha - 24\tan\alpha + 13 = 0$$

$$\tan\alpha = \frac{24 \pm \sqrt{108}}{18}$$

$$= \frac{4 \pm \sqrt{3}}{3}$$

7



$\frac{a}{19.2} = \frac{1}{5}$

$a = \frac{19.2}{5} = 3.84$

$\therefore$ greatest height is 8.95 m above road.

$x = 12t$ $\qquad$ $y = 16t - 5t^2$

$R = 38.4$ m $\qquad$ greatest $y = 12.79$ m

$X = 12t$ $\qquad$ $\frac{X}{5} = 16t - 5t^2$

$\frac{12t}{5} = 16t - 5t^2$

$25t^2 - 80t + 12t = 0$

$25t^2 - 68t = 0$

$t(25t - 68) = 0$

$t = 0$ or $t = \frac{68}{25}$

$= \underline{2.72 s}$

when $t = 2.72$.

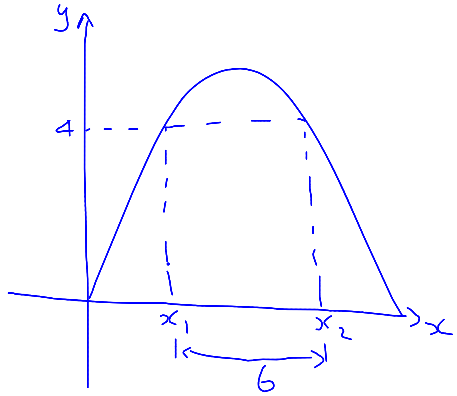
$x = 12(2.72)$
$= 32.64$

$y = 16(2.72) - 5(2.72)^2$
$= 6.528$

$R^2 = 32.64^2 + 6.528^2$

$R = \underline{\underline{33.28}}$

10b)



$$R = \frac{V^2 \sin 2\alpha}{g}$$

c) $\alpha = 45^\circ$, $y = x\left(1 - \frac{x}{R}\right)$

when $x = x_1$ and x_2, $y = 4$

$$4 = x\left(1 - \frac{x}{R}\right)$$

$$x^2 - Rx + 4R = 0$$

$$x = Vt\cos\alpha \qquad y = Vt\sin\alpha - \tfrac{1}{2}gt^2$$

$$t = \frac{x}{V\cos\alpha}$$

$$y = \frac{x\sin\alpha}{\cos\alpha} - \frac{gx^2}{2V^2\cos^2\alpha}$$

$$= x\left(\tan\alpha - \frac{gx}{2V^2\cos^2\alpha}\right)$$

$$= x\tan\alpha\left(1 - \frac{gx}{2V^2\sin\alpha\cos\alpha}\right)$$

$$= x\tan\alpha\left(1 - \frac{gx}{V^2\sin 2\alpha}\right)$$

$$= x\tan\alpha\left(1 - \frac{x}{R}\right)$$

$$(x_2 - x_1)^2 = (x_2 + x_1)^2 - 4x_1x_2$$

$$6^2 = R^2 - 16R$$

$$R^2 - 16R - 36 = 0$$

$$(R+2)(R-18) = 0$$

$$R = -2 \text{ or } R = 18$$

$$\therefore \underline{R = 18} \quad (\because R > 0)$$

11)

$x = Vt\cos\alpha$

$t = \frac{x}{V\cos\alpha}$

$y = Vt\sin\alpha - \frac{1}{2}gt^2$

$y = \frac{x\sin\alpha}{\cos\alpha} - \frac{gx^2}{2V^2\cos^2\alpha}$

$y = x\tan\alpha - \frac{gx^2}{2v^2}(1+\tan^2\alpha)$

$2V^2y = 2V^2x\tan\alpha - gx^2 - gx^2\tan^2\alpha$

$x^2\tan^2\alpha - \frac{2V^2}{g}x\tan\alpha + \frac{2V^2}{g}y + x^2 = 0$

$k = \frac{V^2}{2g}$

$x^2\tan^2\alpha - 4kx\tan\alpha + 4ky + x^2 = 0$

2 solutions when $\Delta > 0$

$16k^2X^2 - 4X^2(4kY + X^2) > 0$

$4k^2 - (4kY + X^2) > 0$

$$4k^2 - 4kY - X^2 > 0$$

$$-X^2 > 4kY - 4k^2$$

$$\underline{X^2 < 4k^2 - 4kY}$$

c) $\tan\alpha_1 \tan\alpha_2 = \dfrac{4kY + X^2}{X^2}$

$$= 1 + \frac{4kY}{X^2}$$

$$> 1 \qquad (\because k>0,\ Y>0)$$

if $\alpha < 45°$

$\tan\alpha < 1$

if both α_1 and $\alpha_2 < 45°$

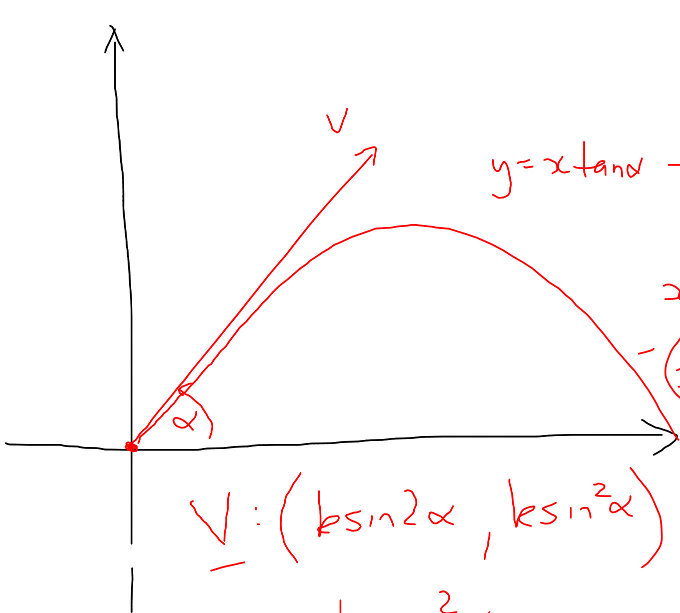
$\tan\alpha_1 < 1$ and $\tan\alpha_2 < 1$

$\tan\alpha_1 \tan\alpha_2 < 1$

but $\tan\alpha_1 \tan\alpha_2 > 1$

$\therefore$ they cannot both be $< 45°$

14.



$$y = x\tan\alpha - \frac{x^2}{4k\cos^2\alpha} \qquad k = \frac{V^2}{2g}$$

$$x^2 - 4kx\sin\alpha\cos\alpha + 4ky\cos^2\alpha = 0$$

$$(x - k\sin 2\alpha)^2 = 4ky\cos^2\alpha - k^2\sin^2 2\alpha$$

$$= 4k\cos^2\alpha\,(y - k\sin^2\alpha)$$

$\underline{V}$: $(k\sin 2\alpha,\ k\sin^2\alpha)$

$a = -k\cos^2\alpha$

$\underline{D}$: $k(\sin^2\alpha + \cos^2\alpha)$

$= \underline{\underline{k}}$

$\underline{F}$: $(k\sin 2\alpha,\ -k\cos 2\alpha)$

<u>max height</u>

$y = k\sin^2\alpha$

max $\sin^2\alpha = 1$

$\therefore$ max $\underline{\underline{y = k}}$

focus

$$\left(k\sin 2\alpha,\ -k\cos 2\alpha\right)$$

$$\sin^2 2\alpha + \cos^2 2\alpha = 1$$

$$x^2 + y^2 = k^2$$

vertex

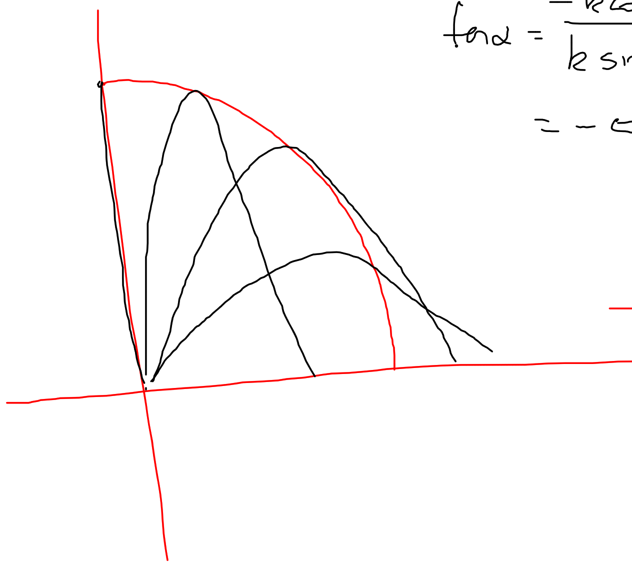
$$\left(k\sin 2\alpha,\ k\sin^2\alpha\right)$$

$$\sin 2\alpha = 2\sin\alpha\cos\alpha.$$

$$\sin^2 2\alpha = 4\sin^2\alpha\cos^2\alpha$$

$$= 4\sin^2\alpha\left(1-\sin^2\alpha\right)$$

$$\frac{x^2}{k^2} = \frac{4y}{k}\left(1-\frac{y}{k}\right)$$



$$\tan\alpha = \frac{-k\cos 2\alpha}{k\sin 2\alpha}$$

$$= -\cot 2\alpha$$