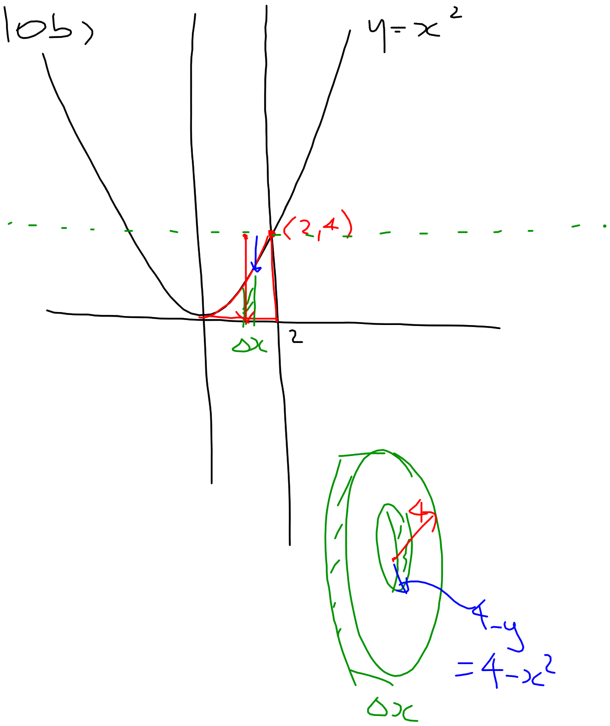
10b)



$$A(x) = \pi\left[4^2 - (4 - x^2)^2\right]$$

$$\Delta V = \pi(8x^2 - x^4)\,\Delta x$$

$$V = \lim_{\Delta x \to 0} \sum_{x=0}^{2} \pi(8x^2 - x^4)\,\Delta x$$

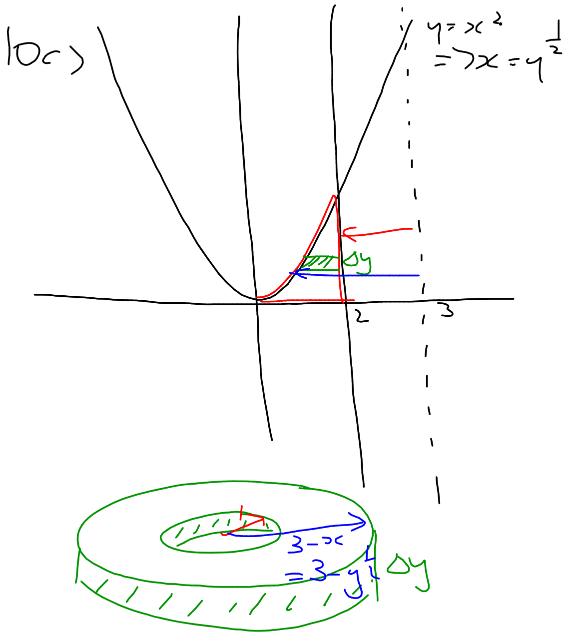
$$= \pi \int_0^2 (8x^2 - x^4)\,dx$$

$$= \pi\left[\frac{32}{3}x^3 - \frac{1}{5}x^5\right]_0^2$$

$$= \pi\left(\frac{256}{3} - \frac{32}{5}\right)$$

$$= \frac{224\pi}{15} \text{ units}^3$$

10c)



$$A(y) = \pi\left[(3 - y^{\frac{1}{2}})^2 - 1^2\right]$$

$$\Delta V = \pi\left(8 - 6y^{\frac{1}{2}} + y\right)\Delta y$$

$$V = \lim_{\Delta y \to 0} \sum_{y=0}^{4} \pi\left(8 - 6y^{\frac{1}{2}} + y\right)\Delta y$$

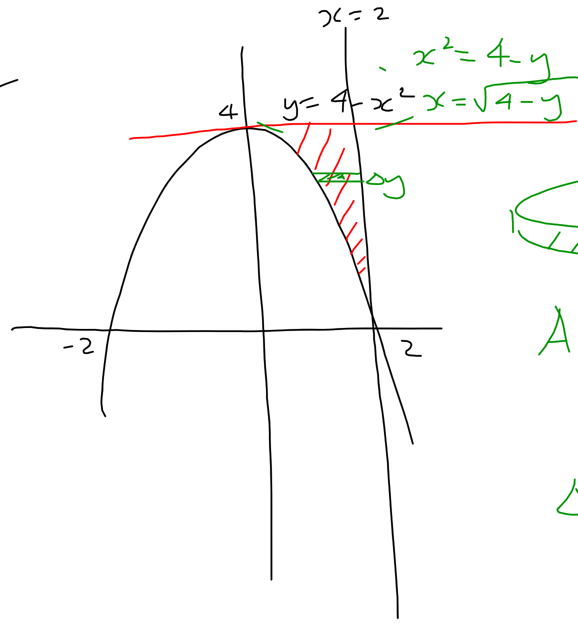
$$= \pi \int_0^4 \left(8 - 6y^{\frac{1}{2}} + y\right) dy$$

$$= \pi\left[8y - 4y^{\frac{3}{2}} + \frac{1}{2}y^2\right]_0^4$$

$$= \pi(32 - 32 + 8)$$

$$= 8\pi \text{ units}^3$$

12



$2 - x = 2 - \sqrt{4-y}$

$$A(y) = \pi\left(4 - 4\sqrt{4-y} + 4 - y\right)$$

$$\Delta V = \pi\left(8 - 4\sqrt{4-y} - y\right)$$

$$V = \lim_{\Delta y \to 0} \sum_{y=0}^{4} \pi\left(8 - 4\sqrt{4-y} - y\right)\Delta y$$

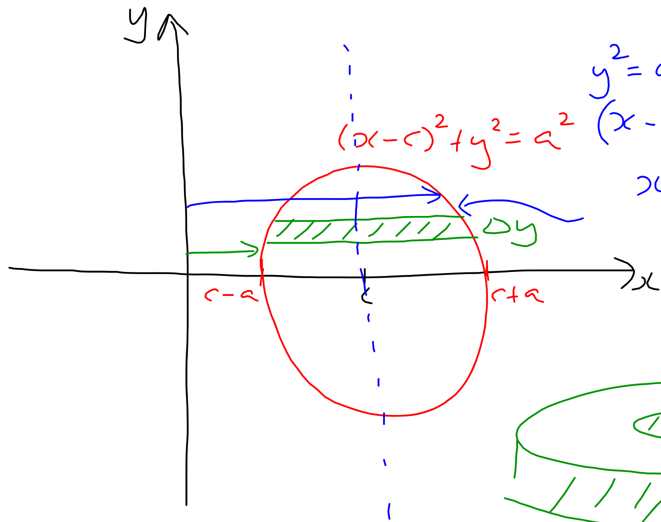
$$= \pi \int_0^4 \left(8 - 4\sqrt{4-y} - y\right) dy$$

$$= \pi \left[8y - 4 \times \frac{2}{-3}(4-y)^{\frac{3}{2}} - \frac{1}{2}y^2 \right]_0^4$$

$$= \pi\left(32 + 0 - 8 - \frac{8}{3} \times 8\right)$$

$$= \underline{\underline{\frac{8}{3}\pi}}$$

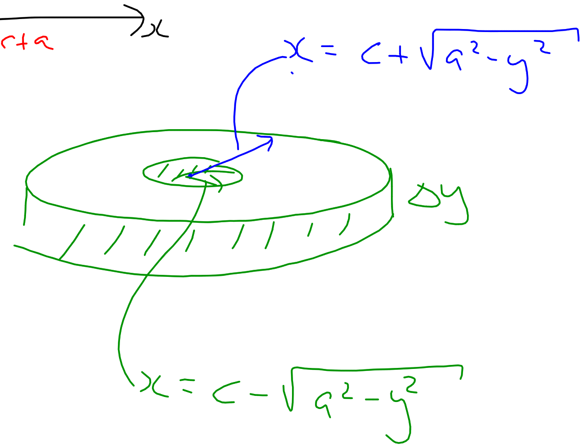
18



$$y^2 = a^2-(x-c)^2$$

$$(x-c)^2 = a^2-y^2$$

$$x = c \pm \sqrt{a^2-y^2}$$



$$A(y) = \pi\left[\left(c+\sqrt{a^2-y^2}\right)^2 - \left(c^2-\sqrt{a^2-y^2}\right)^2\right]$$

$$\Delta V = 4\pi c\sqrt{a^2-y^2}\,\Delta y$$

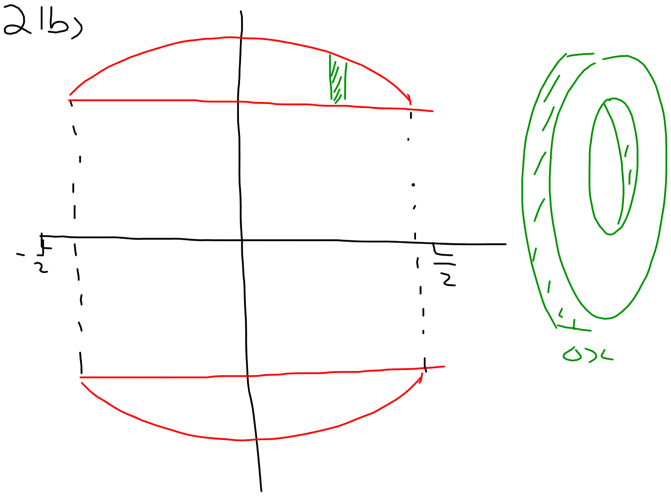
$$V = \lim_{\Delta y \to 0} \sum_{y=-a}^{a} 4\pi c\sqrt{a^2-y^2}\,\Delta y$$

$$= 4\pi c \int_{-a}^{a} \sqrt{a^2-y^2}\,dy$$

$$= 4\pi c \times \frac{1}{2}\pi a^2$$

$$= \underline{2\pi^2 c a^2}$$

21b)



$V_{\text{sphere, diameter}=L}$

$= \frac{4}{3}\pi\left(\frac{L}{2}\right)^3$

$= \frac{\pi L^3}{6}$

$A(x) = \frac{\pi}{4}\left(L^2 - 4x^2\right)$

$\Delta V = \frac{\pi}{4}\left(L^2 - 4x^2\right)\Delta x$

$V = \lim_{\Delta x \to 0} \sum_{x=-\frac{L}{2}}^{\frac{L}{2}} \frac{\pi}{4}\left(L^2 - 4x^2\right)\Delta x$

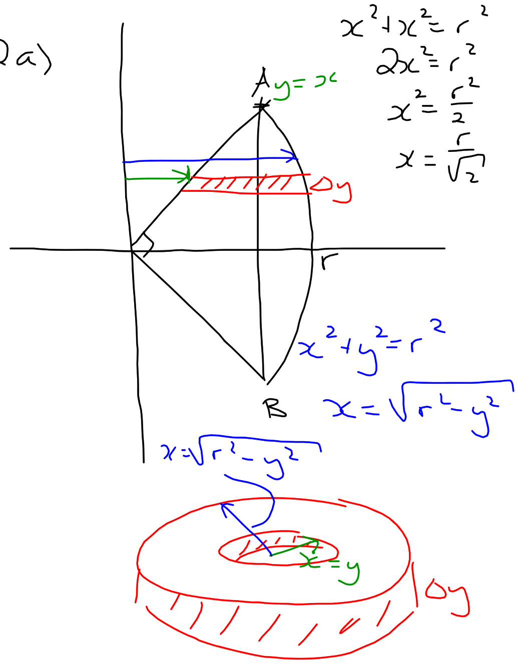
$= \frac{\pi}{2}\int_0^{\frac{L}{2}} \left(L^2 - 4x^2\right)dx$

$= \frac{\pi}{2}\left[L^2 x - \frac{4}{3}x^3\right]_0^{\frac{L}{2}}$

$= \frac{\pi}{2}\left(\frac{L^3}{2} - \frac{L^3}{6}\right)$

$= \frac{\pi L^3}{6}$ units^3

22a)



$$x^2+x^2=r^2$$
$$2x^2=r^2$$
$$x^2=\frac{r^2}{2}$$
$$x=\frac{r}{\sqrt{2}}$$

$$A(y)=\pi\left[\left(\sqrt{r^2-y^2}\right)^2-y^2\right]$$

$$\Delta V=\pi\left(r^2-2y^2\right)\Delta y$$

$$V=\lim_{\Delta y\to 0}\sum_{y=-\frac{r}{\sqrt{2}}}^{\frac{r}{\sqrt{2}}}\pi\left(r^2-2y^2\right)\Delta y$$

$$=2\pi\int_0^{\frac{r}{\sqrt{2}}}\left(r^2-2y^2\right)dy$$

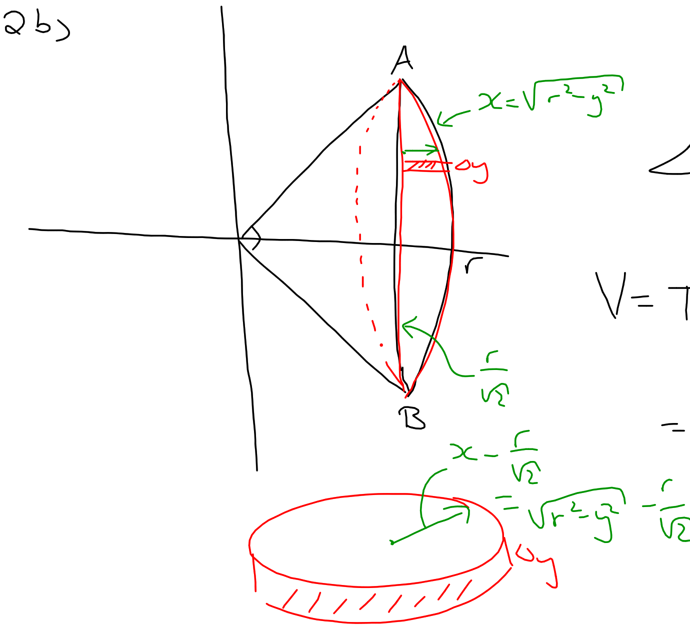
$$=2\pi\left[r^2y-\frac{2}{3}y^3\right]_0^{\frac{r}{\sqrt{2}}}$$

$$=2\pi\left(\frac{r^3}{\sqrt{2}}-\frac{r^3}{3\sqrt{2}}\right)$$

$$=\frac{4\pi r^3}{3\sqrt{2}}\text{ unit}^3$$

$$=\frac{2\sqrt{2}\pi r^3}{3}$$

22b)



$$A(y) = \pi\left(\sqrt{r^2-y^2} - \frac{r}{\sqrt{2}}\right)^2$$

$$\Delta V = \pi\left(\frac{3r^2}{2} - y^2 - \sqrt{2}\,r\sqrt{r^2-y^2}\right)\Delta y$$

$$V = \pi \lim_{\Delta y \to 0} \sum_{y=-\frac{r}{\sqrt{2}}}^{\frac{r}{\sqrt{2}}} \left(\frac{3r^2}{2} - y^2 - \sqrt{2}\,r\sqrt{r^2-y^2}\right)\Delta y$$

$$= 2\pi \int_0^{\frac{r}{\sqrt{2}}} \left(\frac{3r^2}{2} - y^2 - \sqrt{2}\,r\sqrt{r^2-y^2}\right)dy$$

$$V = 2\pi \int_0^{\frac{r}{\sqrt{2}}} \left(\frac{3r^2}{2} - y^2 \right) dy \quad - 2\sqrt{2}\pi r \int_0^{\frac{r}{\sqrt{2}}} \sqrt{r^2 - y^2}\, dy$$

$y = r\sin\theta$
$dy = r\cos\theta\, d\theta$

$$= 2\pi \left[\frac{3r^2}{2} y - \frac{1}{3} y^3 \right]_0^{\frac{r}{\sqrt{2}}} \quad - 2\sqrt{2}\pi r \int_0^{\frac{\pi}{4}} r^2 \cos^2\theta\, d\theta$$

$$= 2\pi \left(\frac{3r^3}{2\sqrt{2}} - \frac{r^3}{6\sqrt{2}} \right) \quad - \sqrt{2}\pi r^3 \int_0^{\frac{\pi}{4}} (1 + \cos 2\theta)\, d\theta$$

$$= \frac{8\pi r^3}{3\sqrt{2}} \quad - \sqrt{2}\pi r^3 \left[\theta + \frac{1}{2} \sin 2\theta \right]_0^{\frac{\pi}{4}}$$

$$= \frac{8\pi r^3}{3\sqrt{2}} \quad - \frac{\sqrt{2}\pi^2 r^3}{4} \quad - \sqrt{2}\pi r^3$$