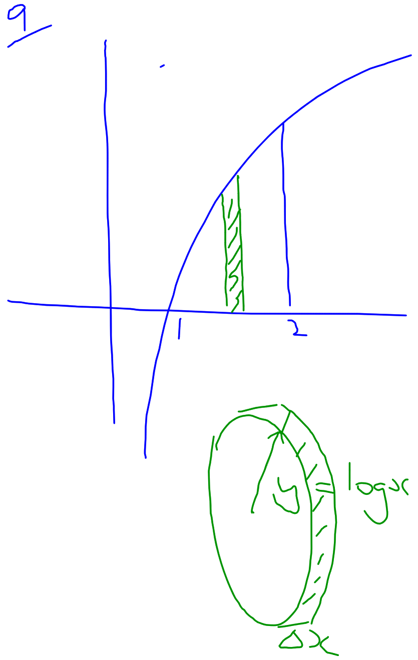
9



$$A(x) = \pi(\log x)^2$$

$$\Delta V = \pi(\log x)^2 \Delta x$$

$$V = \lim_{\Delta x \to 0} \sum_{x=1}^{2} \pi(\log x)^2 \Delta x$$

$$= \pi \int_1^2 (\log x)^2 \, dx \qquad u = (\log x)^2 \qquad v = x$$

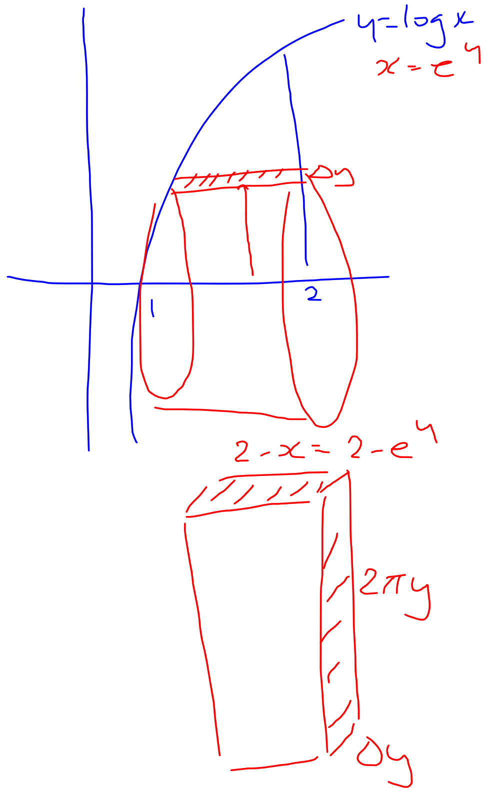
$$du = \frac{2\log x}{x} dx \qquad dv = dx$$

$$= \pi\Big[x(\log x)^2\Big]_1^2 - 2\pi \int_1^2 \log x \, dx$$

$$= \pi\left(2(\log 2)^2\right) - 2\pi\Big[x\log x - x\Big]_1^2$$

$$= 2\pi(\log 2)^2 - 4\pi \log 2 + 4\pi - 2\pi$$

$$= \underline{2\pi(\log 2)^2 - 4\pi\log 2 + 2\pi} \ \text{unit}^3$$



$$A(x) = 2\pi y(2-e^y)$$

$$\Delta V = 2\pi(2y - ye^y)\,\Delta y$$

$$V = \lim_{\Delta y \to 0} \sum_{y=0}^{\log 2} 2\pi(2y - ye^y)\,\Delta y$$

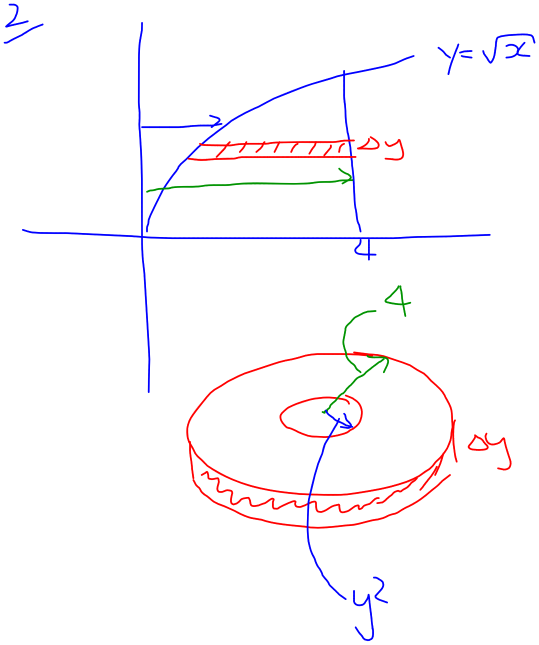
$$V = 2\pi \int_0^{\log 2} (2y - ye^y)\,dy \qquad \begin{matrix} u = y & v = e^y \\ du = dy & dv = e^y dy \end{matrix}$$

$$= 2\pi \Big[y^2 - ye^y \Big]_0^{\log 2} + 2\pi \int_0^{\log 2} e^y\,dy$$

$$= 2\pi \Big[y^2 - ye^y + e^y \Big]_0^{\log 2}$$

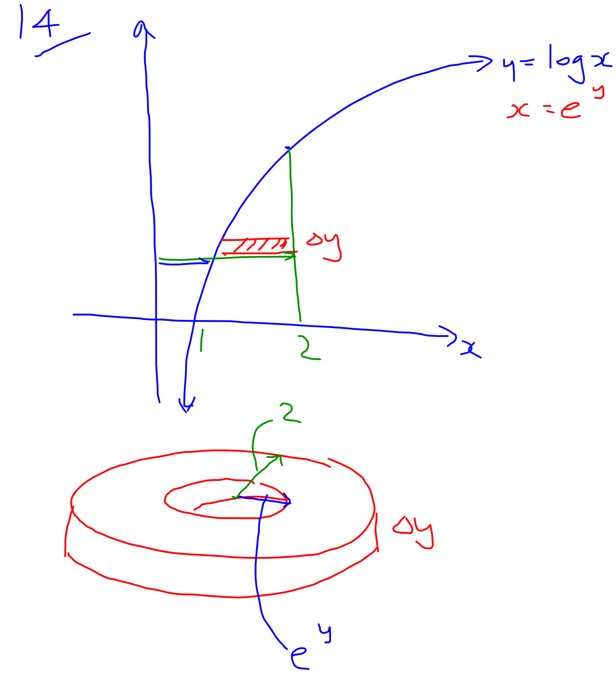
$$= 2\pi(\log 2)^2 - 4\pi \log 2 + 4\pi - 2\pi$$

12



$$A(y) = \pi\left\{4^2 - (y^2)^2\right\}$$

14



$$A(y) = \pi\left(2^2 - (e^y)^2\right)$$

$$\Delta V = \pi\left(4 - e^{2y}\right)\Delta y$$

$$V = \lim_{\Delta y \to 0} \sum_{y=0}^{\log 2} \pi\left(4 - e^{2y}\right)\Delta y$$

$$= \pi \int_0^{\log 2} \left(4 - e^{2y}\right) dy$$

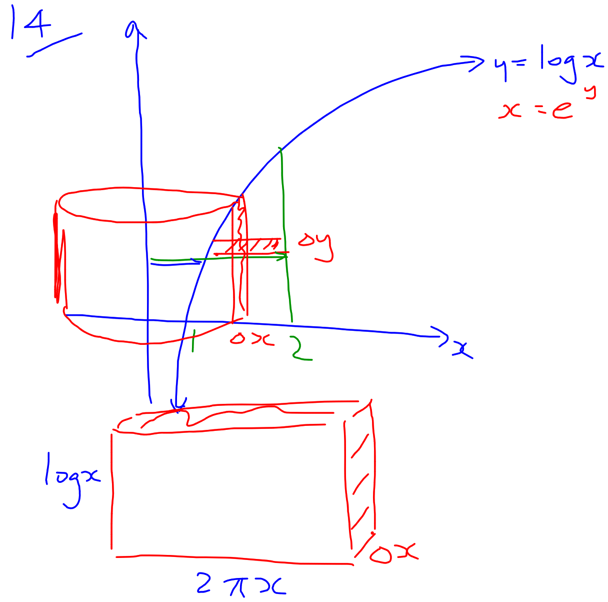
$$= \pi\left[4y - \tfrac{1}{2}e^{2y}\right]_0^{\log 2}$$

$$= \pi\left(4\log 2 - \tfrac{1}{2}e^{2\log 2} - 0 + \tfrac{1}{2}\right)$$

$$= \pi\left(4\log 2 - 2 + \tfrac{1}{2}\right)$$

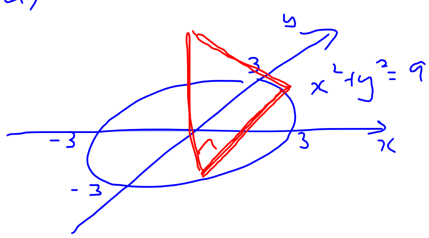
$$= \pi\left(4\log 2 - \tfrac{3}{2}\right) \text{ unit}^3$$

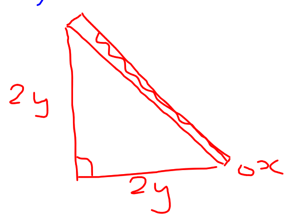
14



$A(x) = 2\pi x \log x$

16d)





$$A(x) = 2y^2$$

$$= 2(9-x^2)$$

$$\Delta V = 2(9-x^2)\Delta x$$

$$V = \lim_{\Delta x \to 0} \sum_{x=-3}^{3} 2(9-x^2)\Delta x$$

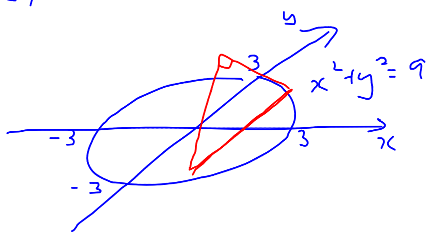
$$= 4\int_0^3 (9-x^2)\,dx$$

$$= 4\left[9x - \frac{1}{3}x^3\right]_0^3$$

$$= 4(27-9)$$

$$= \underline{72 \text{ units}^3}$$

16e)

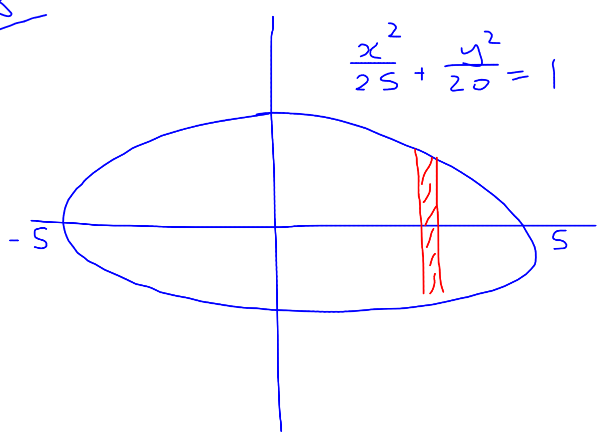


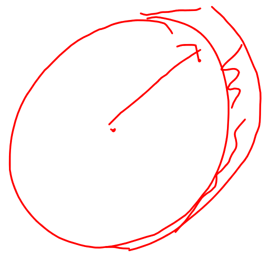
$$2a^2 = 4y^2$$
$$a^2 = 2y^2$$

$$A(x) = \frac{1}{2} a^2$$
$$= y^2$$
$$= 9 - x^2$$

18

$$\frac{x^2}{25} + \frac{y^2}{20} = 1$$





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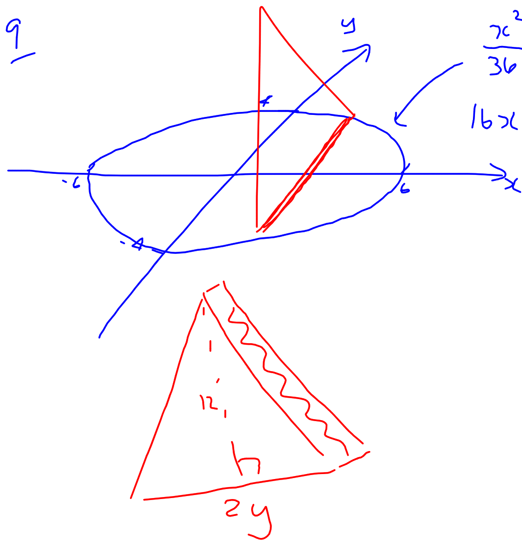


$$\frac{x^2}{36} + \frac{y^2}{16} = 1$$

$$16x^2 + 36y^2 = 576$$

$$36y^2 = 576 - 16x^2$$

$$y^2 = 16 - \frac{4}{9}x^2$$

$$A(x) = 12y$$

$$= 12\sqrt{16 - \frac{4}{9}x^2}$$

$$= 8\sqrt{36 - x^2}$$

$$\Delta V = 8\sqrt{36 - x^2}\,\Delta x$$

$$V = \lim_{\Delta x \to 0} \sum_{x=-6}^{6} 8\sqrt{36 - x^2}\,\Delta x$$

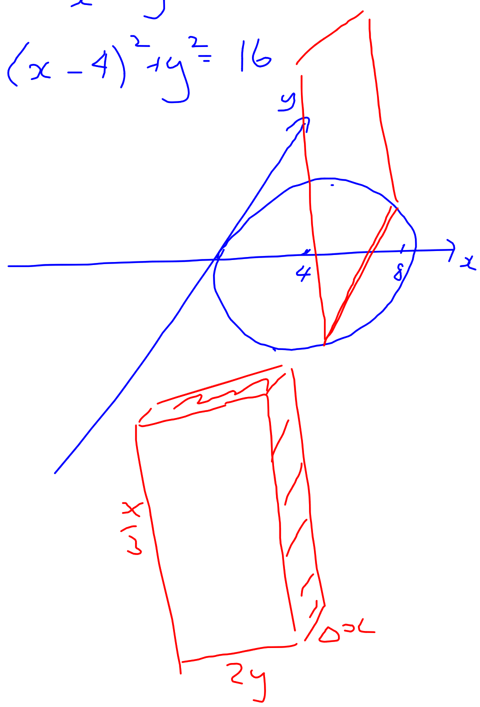
$$= 8\int_{-6}^{6} \sqrt{36 - x^2}\,dx$$

$$= 8 \times \frac{1}{2}\pi(6)^2$$

$$= 144\pi \text{ units}^3$$

20/ $x^2+y^2=8x$

$(x-4)^2+y^2=16$



$A(x) = \frac{2}{3}xy$

$= \frac{2}{3}x\sqrt{8x-x^2}$

$\Delta V = \frac{2}{3}x\sqrt{8x-x^2}\,\Delta x$

$V = \lim_{\Delta x \to 0} \sum_{x=0}^{8} \frac{2}{3}x\sqrt{8x-x^2}\,\Delta x$

$= \frac{2}{3}\int_0^8 x\sqrt{8x-x^2}\,dx$

$= -\frac{1}{3}\int_0^8 (8-2x)\sqrt{8x-x^2}\,dx + \frac{8}{3}\int_0^8 \sqrt{16-(x-4)^2}\,dx$

$= -\frac{4}{9}\left[(8x-x^2)^{\frac{3}{2}}\right]_0^8 + \frac{8}{3}\times\frac{1}{2}\pi(4)^2$

$= \frac{64\pi}{3}$ units^3