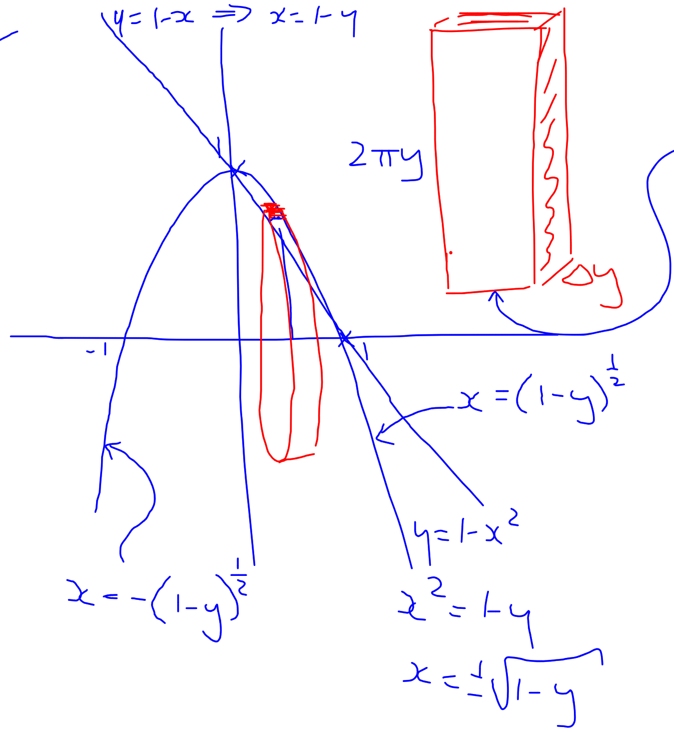
3



$\left(\sqrt{1-y} - (1-y)\right)$

$A(y) = 2\pi y\left(\sqrt{1-y} - 1 + y\right)$

$\Delta V = 2\pi y\left(\sqrt{1-y} - 1 + y\right)\Delta y$

$V = \lim_{\Delta y \to 0} \sum_{y=0}^{1} \left(2\pi\left(y\sqrt{1-y} - y + y^2\right)\Delta y\right)$

$$V = 2\pi \int_0^1 y(1-y)^{\frac{1}{2}}\,dy - 2\pi \int_0^1 (y - y^2)\,dy$$

$$= 2\pi \int_0^1 (1-y)y^{\frac{1}{2}}\,dy - 2\pi \left[\tfrac{1}{2}y^2 - \tfrac{1}{3}y^3\right]_0^1$$

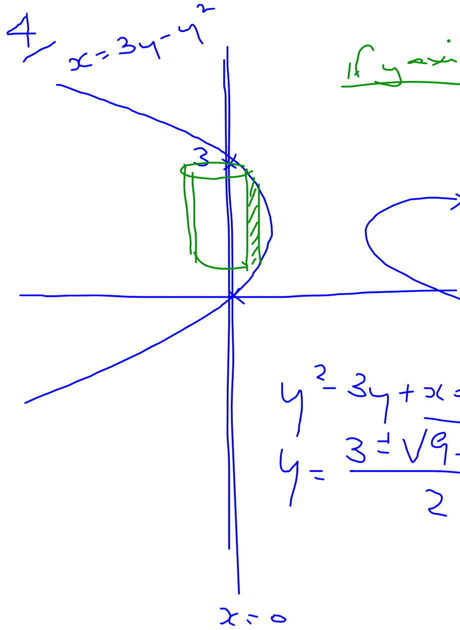
$$= 2\pi \int_0^1 \left(y^{\frac{1}{2}} - y^{\frac{3}{2}}\right)dy - \frac{\pi}{3}$$

$$= 2\pi \left[\frac{2}{3}y^{\frac{3}{2}} - \frac{2}{5}y^{\frac{5}{2}}\right]_0^1 - \frac{\pi}{3}$$

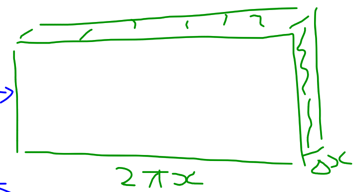
$$= \frac{8\pi}{15} - \frac{\pi}{3}$$

$$= \underline{\underline{\frac{\pi}{5}\ \text{units}^3}}$$

4



If y-axis:



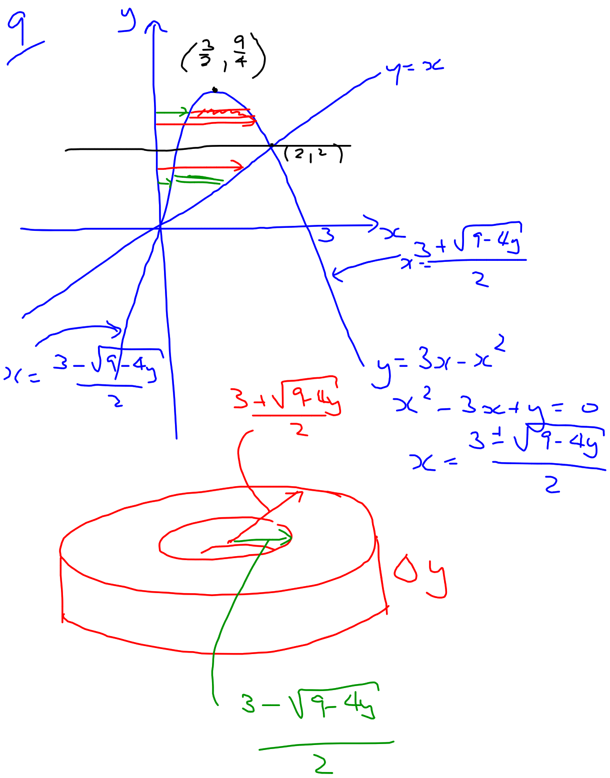
$$y^2 - 3y + x = 0$$

$$y = \frac{3 \pm \sqrt{9-4x}}{2}$$

$$\frac{3+\sqrt{9-4x}}{2} - \left(\frac{3-\sqrt{9-4x}}{2}\right)$$

$$= \underline{\sqrt{9-4x}}$$

9



$$A_1(y) = \pi\left[\frac{\left(3+\sqrt{9-4y}\right)^2}{4} - \frac{\left(3-\sqrt{9-4y}\right)^2}{4}\right]$$

$$\Delta V_1 = 3\pi\sqrt{9-4y}\ \Delta y$$

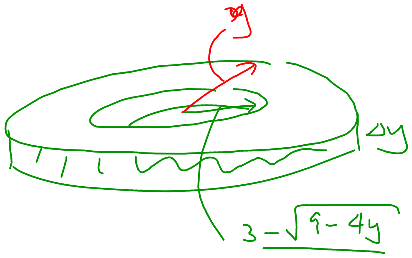
$$V_1 = \lim_{\Delta y \to 0} \sum_{y=2}^{\frac{9}{4}} 3\pi\sqrt{9-4y}\ \Delta y$$

$$= 3\pi \int_2^{\frac{9}{4}} \sqrt{9-4y}\ dy$$

$$= 3\pi \times \frac{2}{3} \times -\frac{1}{4}\left[(9-4y)^{\frac{3}{2}}\right]_2^{\frac{9}{4}}$$

$$= -\frac{\pi}{2}(-1)$$

$$= \frac{\pi}{2}$$



$$A_2(y) = \pi\left[y^2 - \frac{\left(3-\sqrt{9-4y}\right)^2}{4}\right]$$

$$= \frac{\pi}{4}\left(4y^2 - 9 + 6\sqrt{9-4y} - 9 + 4y\right)$$

$$\Delta V_2 = \frac{\pi}{4}\left(4y^2 + 4y - 18 + 6\sqrt{9-4y}\right)\Delta y$$

$$V_2 = \lim_{\Delta y \to 0} \sum_{y=0}^{2} \frac{\pi}{2}\left(2y^2 + 2y - 9 + 3\sqrt{9-4y}\right)\Delta y$$

$$= \frac{\pi}{2}\int_0^2 \left(2y^2 + 2y - 9 + 3\sqrt{9-4y}\right)dy$$

$$= \frac{\pi}{2}\left[\frac{2}{3}y^3 + y^2 - 9y - \frac{1}{2}(9-4y)^{\frac{3}{2}}\right]_0^2$$

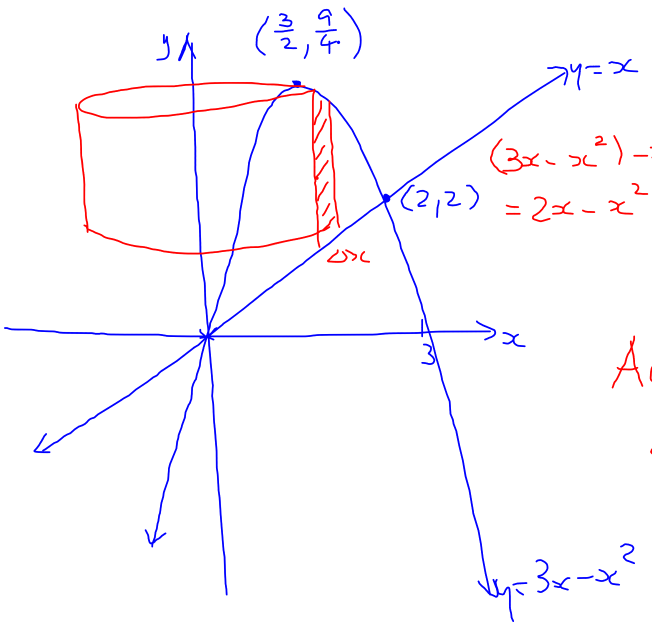
$$= \frac{\pi}{2}\left(\frac{16}{3} + 4 - 18 - \frac{1}{2} + \frac{1}{2}(27)\right)$$

$$= \frac{13\pi}{6}$$

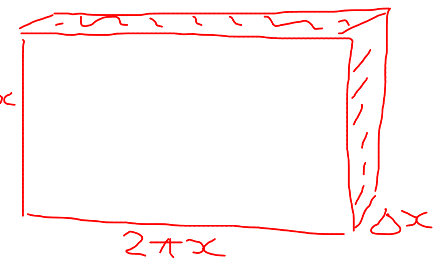
$$V = V_1 + V_2$$

$$= \frac{\pi}{2} + \frac{13\pi}{6}$$

$$= \frac{8\pi}{3} \text{ units}^3$$



$(3x - x^2) - x$
$= 2x - x^2$



$$A(x) = 2\pi x \cdot (2x - x^2)$$

$$\Delta V = 2\pi(2x^2 - x^3)\,\Delta x$$

$$V = \lim_{\Delta x \to 0} \sum_{x=0}^{2} 2\pi(2x^2 - x^3)\,\Delta x$$

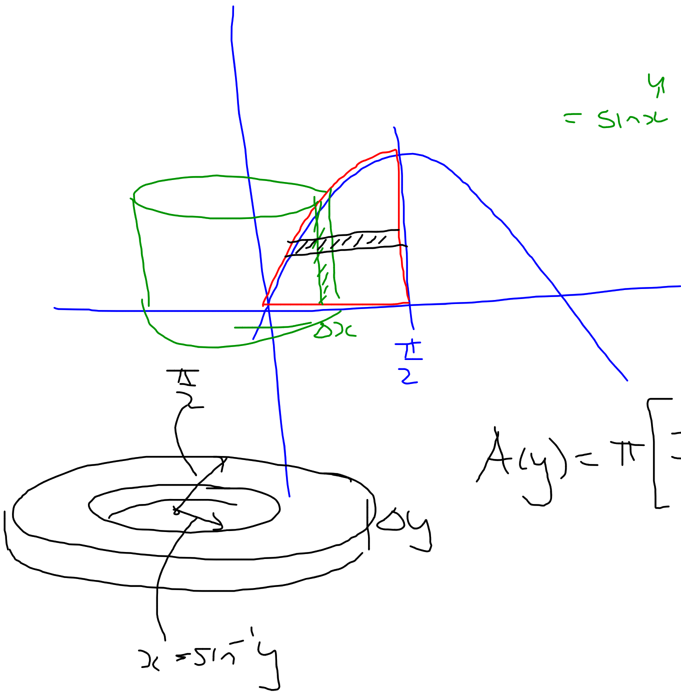
$$= 2\pi \int_0^2 (2x^2 - x^3)\,dx$$

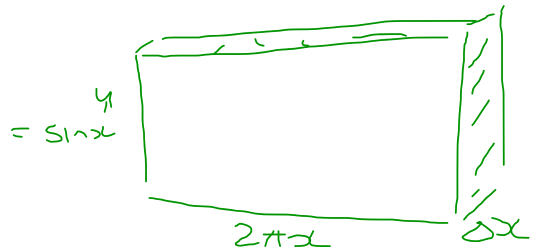
$$= 2\pi \left[\frac{2}{3}x^3 - \frac{1}{4}x^4\right]_0^2$$

$$= 2\pi\left(\frac{16}{3} - \frac{16}{4}\right)$$

$$= \frac{8\pi}{3}$$

b)





$A(x) = 2\pi x \sin x$

$A(y) = \pi\left[\frac{\pi^2}{4} - \left(\sin^{-1} y\right)^2\right]$

$$\Delta V = 2\pi x \sin x \, \Delta x$$

$$V = \lim_{\Delta x \to 0} \sum_{x=0}^{\frac{\pi}{2}} 2\pi x \sin x \, \Delta x$$

$u = x$ $\qquad$ $v = -\cos x$

$du = dx$ $\qquad$ $dv = \sin x \, dx$

$$= 2\pi \int_0^{\frac{\pi}{2}} x \sin x \, dx$$

$$= 2\pi \Big[-x \cos x \Big]_0^{\frac{\pi}{2}} + 2\pi \int_0^{\frac{\pi}{2}} \cos x \, dx$$

$$= 2\pi \Big[\sin x \Big]_0^{\frac{\pi}{2}}$$

$$= \underline{\underline{2\pi}} \text{ units}^3$$

$$\Delta V = \pi\left(\frac{\pi^2}{4} - (\sin^{-1}y)^2\right)\Delta y$$

$$V = \lim_{\Delta y \to 0} \sum_{y=0}^{1} \pi\left(\frac{\pi^2}{4} - (\sin^{-1}y)^2\right)\Delta y$$

$$= \frac{\pi^3}{4}\int_0^1 dy \quad - \pi\int_0^1 (\sin^{-1}y)^2 dy \qquad \begin{aligned} y &= \sin u \\ dy &= \cos u\, du \end{aligned}$$

$$= \frac{\pi^3}{4}\Big[y\Big]_0^1 \quad - \pi\int_0^{\frac{\pi}{2}} u^2 \cos u\, du \qquad U = u^2 \quad v = \sin u$$

$$= \frac{\pi^3}{4} - \pi\Big[u^2 \sin u\Big]_0^{\frac{\pi}{2}} + 2\pi\int_0^{\frac{\pi}{2}} u \sin u\, du \qquad dU = 2u\,du \quad dv = \cos u\, du$$

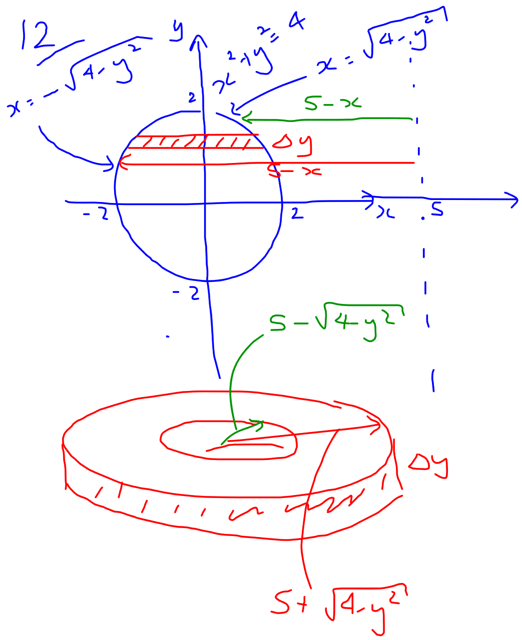
$$U = u \quad v = -\cos u$$

$$= \frac{\pi^3}{4} - \frac{\pi^3}{4} - 2\pi\Big[u\cos u\Big]_0^{\frac{\pi}{2}} + 2\pi\int_0^{\frac{\pi}{2}} \cos u\, du \qquad dU = du \quad dv = \sin u\, du$$

$$= 0 - 0 + 2\pi\Big[\sin u\Big]_0^{\frac{\pi}{2}}$$

$$= \underline{\underline{2\pi}} \text{ units}^3$$

12



$$A(y) = \pi\left[\left(5+\sqrt{4-y^2}\right)^2 - \left(5-\sqrt{4-y^2}\right)^2\right]$$

$$\Delta V = 20\pi\sqrt{4-y^2}\;\Delta y$$

$$V = \lim_{\Delta y \to 0} \sum_{y=-2}^{2} 20\pi\sqrt{4-y^2}\;\Delta y$$

$$= 20\pi \int_{-2}^{2} \sqrt{4-y^2}\;dy$$

$$= 20\pi \times \frac{1}{2}\pi(2)^2$$

$$= 40\pi^2 \text{ units}^3$$