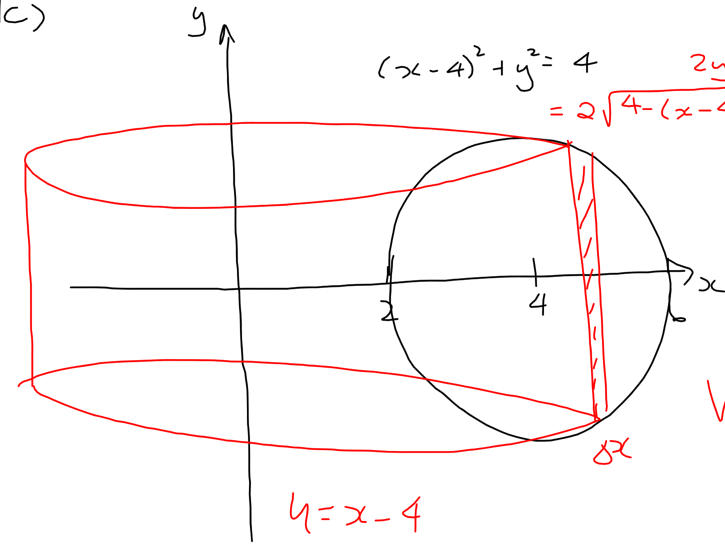
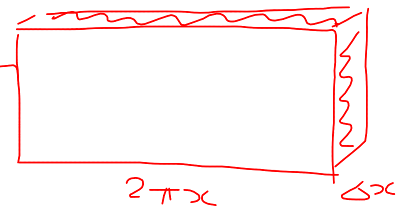
1c)

$(x-4)^2 + y^2 = 4$



$2y = 2\sqrt{4-(x-4)^2}$



$A(x) = 4\pi x\sqrt{4-(x-4)^2}$

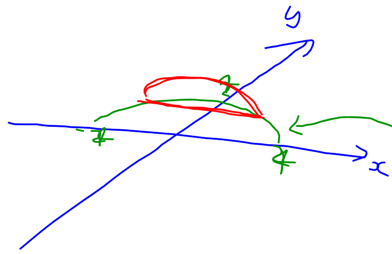
$$V = 4\pi\int_2^6 4\pi x\sqrt{4-(x-4)^2}\,dx$$

$u = x-4$

$du = dx$

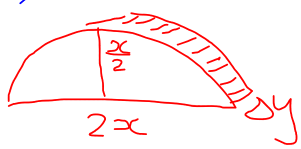
$$= 4\pi\int_{-2}^{2}(u+4)\sqrt{4-u^2}\,du$$

2/



$y = \sqrt{16 - x^2}$

$x = \sqrt{16 - y^2}$



$A(y) = \frac{1}{2}\pi ab$

$= \frac{1}{2}\pi(x)\left(\frac{x}{2}\right)$

$= \frac{\pi}{4}x^2$

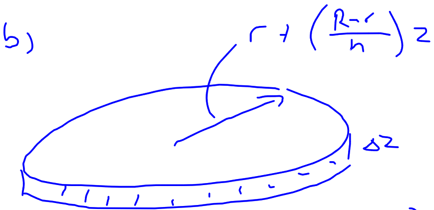
$\Delta V = \frac{\pi}{4}(16 - y^2)\Delta y$

$V = \lim_{\Delta y \to 0} \sum_{y=0}^{4} \frac{\pi}{4}(16 - y^2)\Delta y$

$V = \frac{\pi}{4}\int_0^4 (16 - y^2)\,dy$

$= \frac{\pi}{4}\int_0^4 (16 - x^2)\,dx$

7b)



$$A(z) = \pi\left(r + \left(\frac{R-r}{h}\right)z\right)^2$$

$$\Delta V = \frac{\pi}{h^2}\left[hr + (R-r)z\right]^2 \Delta z$$

$$V = \lim_{\Delta z \to 0} \sum_{z=0}^{h} \frac{\pi}{h^2}\left[hr + (R-r)z\right]^2 \Delta z$$

$$= \frac{\pi}{h^2}\int_0^h \left[hr + (R-r)z\right]^2 dz$$

$$= \frac{\pi}{3h^2(R-r)}\left[\left(hr + (R-r)z\right)^3\right]_0^h$$

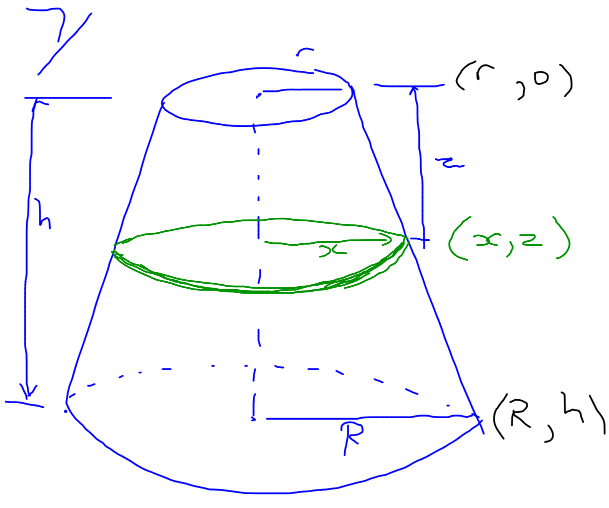
$$= \frac{\pi}{3h^2(R-r)}\left[h^3R^3 - h^3r^3\right]$$

$$= \frac{\pi h}{3(R-r)}\left(R^3 - r^3\right)$$

$$= \frac{\pi h}{3(R-r)}(R-r)\left(R^2 + rR + r^2\right)$$

$$= \underline{\frac{\pi h}{3}\left(R^2 + rR + r^2\right)}$$

7)



$$m = \frac{h - 0}{R - r}$$

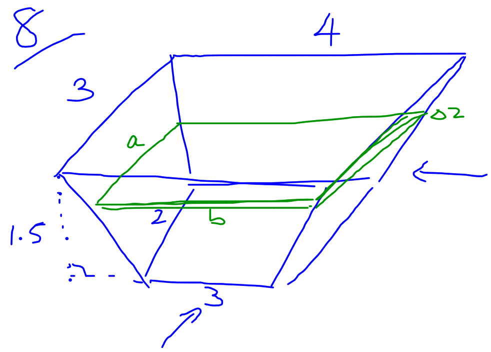
$$= \frac{h}{R - r}$$

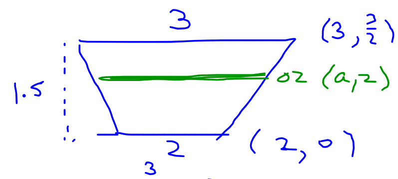
$$z - 0 = \frac{h}{R - r}(x - r)$$

$$\frac{R - r}{h} z = x - r$$

$$\underline{x = r + \frac{R - r}{h} z}$$

8/





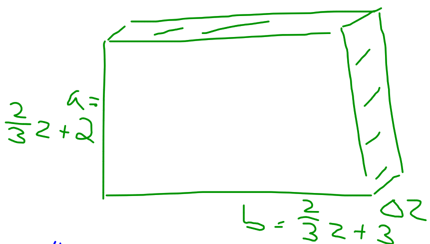
$$m = \frac{\frac{3}{2} - 0}{3 - 2} = \frac{3}{2}$$

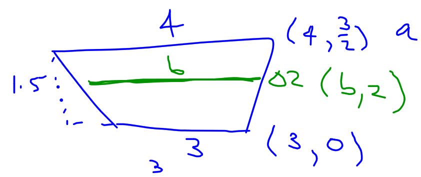
$$z - 0 = \frac{3}{2}(a - 2)$$

$$2z = 3a - 6$$

$$3a = 2z + 6$$

$$a = \frac{2}{3}z + 2$$





$$m = \frac{\frac{3}{2} - 0}{4 - 3} = \frac{3}{2}$$

$$z - 0 = \frac{3}{2}(b - 3)$$

$$2z = 3b - 9$$

$$3b = 2z + 9$$

$$b = \frac{2}{3}z + 3$$

$$A(z) = \frac{2}{9}(z + 3)(2z + 9)$$

$$\Delta V = \frac{2}{9}\left(2z^2 + 15z + 27\right)\Delta z$$

$$V = \lim_{\Delta z \to 0} \sum_{z=0}^{\frac{3}{2}} \frac{2}{9}\left(2z^2 + 15z + 27\right)\Delta z$$

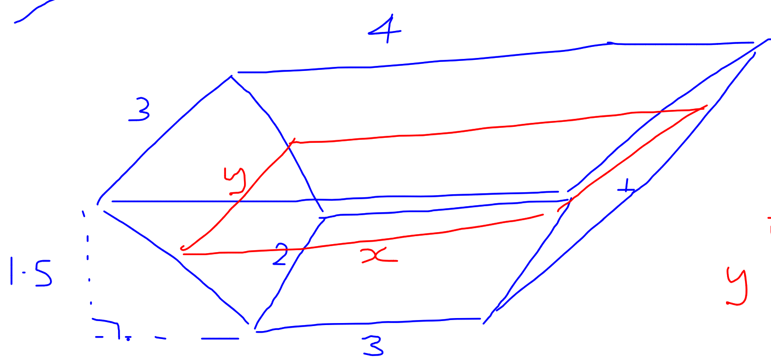
$$= \frac{2}{9}\int_0^{\frac{3}{2}} \left(2z^2 + 15z + 27\right) dz$$

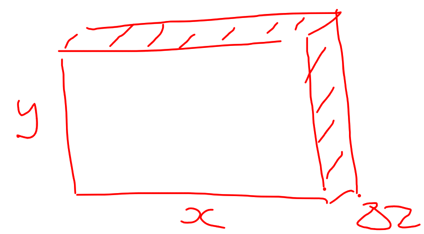
$$= \frac{2}{9}\left[\frac{2}{3}z^3 + \frac{15}{2}z^2 + 27z\right]_0^{\frac{3}{2}}$$

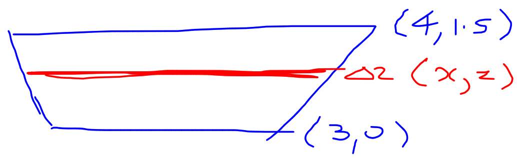
$$= \frac{2}{9}\left(\frac{9}{4} + \frac{135}{8} + \frac{81}{2}\right)$$

$$= \underline{\frac{53}{4}\ \text{units}^3}$$

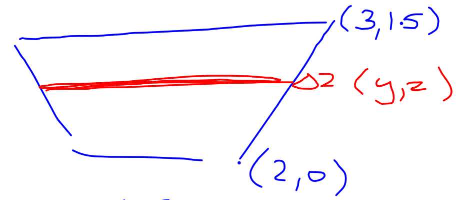
8





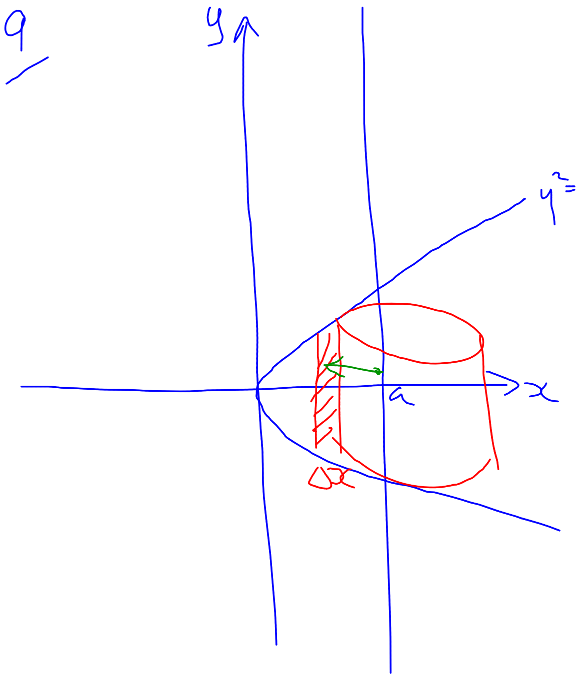


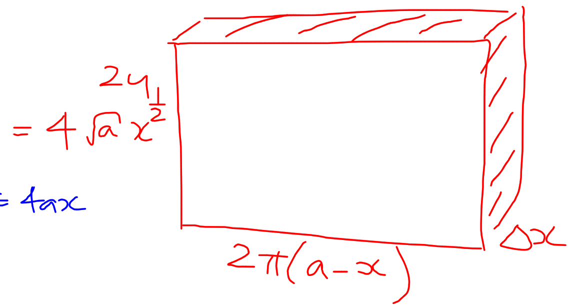
$$m = \frac{1.5}{1}$$
$$= \frac{3}{2}$$
$$z - 0 = \frac{3}{2}(x - 3)$$
$$\frac{2}{3}z = x - 3$$
$$x = 3 + \frac{2}{3}z$$



$$m = \frac{1.5}{1}$$
$$= \frac{3}{2}$$
$$z - 0 = \frac{3}{2}(y - 2)$$
$$\frac{2}{3}z = y - 2$$
$$y = 2 + \frac{2}{3}z$$

9





$$A(x) = \delta\sqrt{a}\,\pi\,(a-x)\,x^{\frac{1}{2}}$$

$$\Delta V = \delta\sqrt{a}\,\pi\left(a x^{\frac{1}{2}} - x^{\frac{3}{2}}\right)\Delta x$$

$$V = \lim_{\Delta x \to 0} \sum_{x=0}^{a} \delta\sqrt{a}\,\pi\left(a x^{\frac{1}{2}} - x^{\frac{3}{2}}\right)\Delta x$$

$$= \delta\sqrt{a}\,\pi \int^{a} \left(a x^{\frac{1}{2}} - x^{\frac{3}{2}}\right)dx$$

$$= \delta\sqrt{a}\,\pi\left[\frac{2}{3}a x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}}\right]_0^a$$

$$A(z) = \left(3 + \frac{2}{3}z\right)\left(2 + \frac{2}{3}z\right)$$

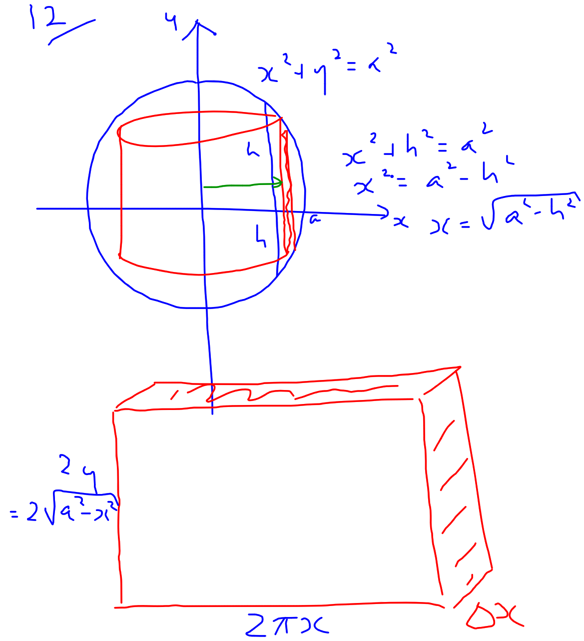
$$\Delta V = \left(6 + \frac{10}{3}z + \frac{4}{9}z^2\right)\Delta z$$

$$V = \lim_{\Delta z \to 0} \sum_{z=0}^{1.5} \left(6 + \frac{10}{3}z + \frac{4}{9}z^2\right)\Delta z$$

$$= \int_0^{1.5} \left(6 + \frac{10}{3}z + \frac{4}{9}z^2\right) dz$$

$$= \left[6z + \frac{5}{3}z^2 + \frac{4}{27}z^3\right]_0^{1.5}$$

12.



$$A(x)=2\pi x\cdot 2\sqrt{a^2-x^2}$$

$$\Delta V=4\pi x\sqrt{a^2-x^2}\,\Delta x$$

$$V=\lim_{\Delta x\to 0}\sum_{x=\sqrt{a^2-h^2}}^{a}4\pi x\sqrt{a^2-x^2}\,\Delta x$$

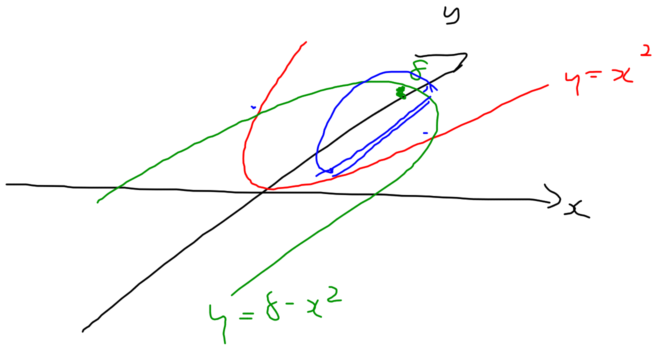
$$=2\pi\int_{\sqrt{a^2-h^2}}^{a}2x\sqrt{a^2-x^2}\,dx$$

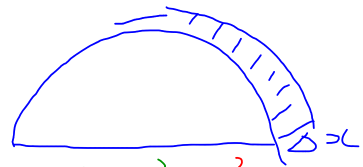
$$=-\frac{4\pi}{3}\Big[(a^2-x^2)\sqrt{a^2-x^2}\Big]_{\sqrt{a^2-h^2}}^{a}$$

$$=\frac{4\pi}{3}\left(h^2\sqrt{h^2}\right)$$

$$=\frac{4\pi h^3}{3}$$

16





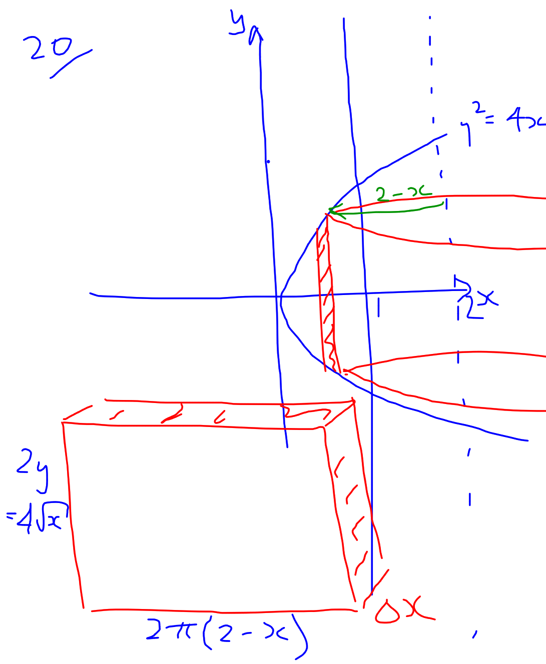
$$d = 8 - x^2 - x^2$$

$$d = 8 - 2x^2$$

$$r = 4 - x^2$$

$$A(x) = \pi(4 - x^2)^2$$

20/



$A(x) = 8\pi(2-x)\sqrt{x}$

$\Delta V = 8\pi(2-x)x^{\frac{1}{2}}\Delta x$

$$V = \lim_{\Delta x \to 0} \sum_{x=0}^{1} 8\pi(2-x)x^{\frac{1}{2}}\Delta x$$

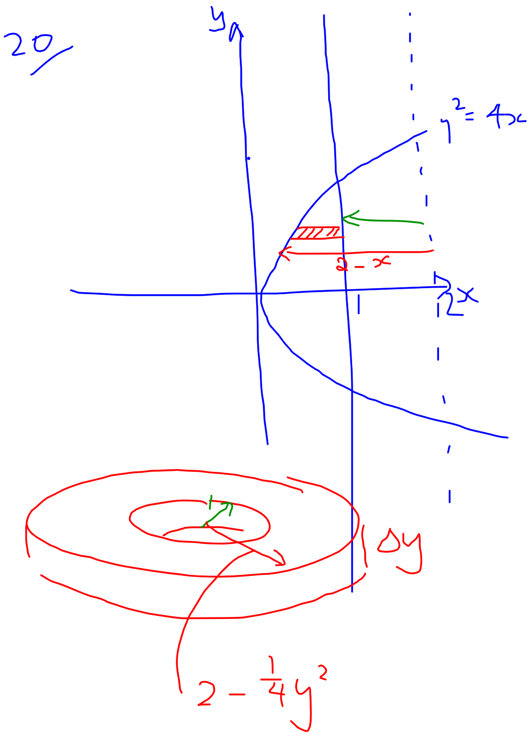
$$= 8\pi\int_0^1 \left(2x^{\frac{1}{2}} - x^{\frac{3}{2}}\right)dx$$

$$= 8\pi\left[\frac{4}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}}\right]_0^1$$

$$= 8\pi\left(\frac{4}{3} - \frac{2}{5}\right)$$

$$= \frac{112\pi}{15} \text{ units}^3$$

20/



$$A(y)=\pi\left[\left(2-\tfrac{1}{4}y^2\right)^2-1^2\right]$$

$$\Delta V=\pi\left(3-y^2+\tfrac{1}{16}y^4\right)\Delta y$$

$$V=\lim_{\Delta y\to 0}\sum_{y=-2}^{2}\pi\left(3-y^2+\tfrac{1}{16}y^4\right)\Delta y$$

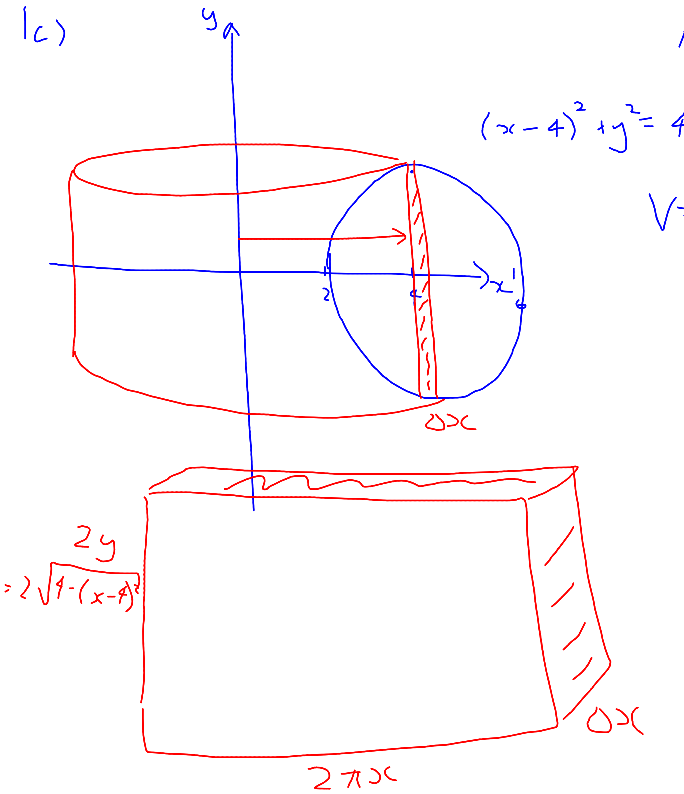
$$=2\pi\int_0^2\left(3-y^2+\tfrac{1}{16}y^4\right)dy$$

$$=2\pi\left[3y-\tfrac{1}{3}y^3+\tfrac{1}{80}y^5\right]_0^2$$

$$=2\pi\left(6-\frac{8}{3}+\frac{32}{80}\right)$$

$$=\frac{112\pi}{15}\text{ units}^3$$

1c)



$$A(x) = 4\pi x\sqrt{4-(x-4)^2}$$

$$\Delta V = 4\pi x\sqrt{4-(x-4)^2}\,\Delta x$$

$$V = \lim_{\Delta x \to 0} \sum_{x=2}^{6} 4\pi x\sqrt{4-(x-4)^2}\,\Delta x$$

$$= 4\pi \int_2^6 x\sqrt{4-(x-4)^2}\,dx$$

$u = x-4$

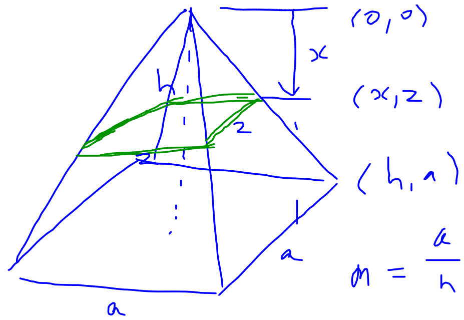
$du = dx$

$$= 4\pi \int_{-2}^{2} (u+4)\sqrt{4-u^2}\,du$$

$$= 4\pi \int_{-2}^{2} u\sqrt{4-u^2}\,du + 16\pi\int_{-2}^{2}\sqrt{4-u^2}\,du$$

$$= 0 + 16\pi \times \frac{1}{2}\pi(2)^2$$

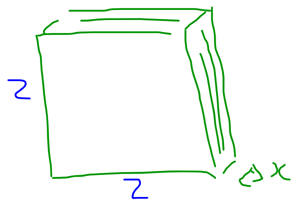
$$= \underline{32\pi^2 \text{ units}^3}$$



$$m = \frac{a}{h}$$

$$z - 0 = \frac{a}{h}(x - 0)$$

$$z = \frac{ax}{h}$$



$$A(z) = z^2$$

$$= \left(\frac{ax}{h}\right)^2$$