5a)

$$2\sin 15 \cos 15$$

$$= \sin 30^\circ$$

$$= \frac{1}{2}$$

5e)

$$\cos\frac{\pi}{12}\sin\frac{\pi}{12}$$

$$= \frac{1}{2} \times 2\sin\frac{\pi}{12}\cos\frac{\pi}{12}$$

$$= \frac{1}{2}\sin\frac{\pi}{6}$$

$$= \frac{1}{2} \times \frac{1}{2}$$

$$= \underline{\frac{1}{4}}$$

5j)

$$\frac{2\cos^2\frac{2\pi}{5}-1}{1-2\sin^2\frac{\pi}{10}}$$

$$=\frac{\cos\frac{4\pi}{5}}{\cos\frac{\pi}{5}}$$

$$=\frac{-\cos\frac{\pi}{5}}{\cos\frac{\pi}{5}}$$

$$=\underline{-1}$$

7a) θ is acute

$\cos\theta = \frac{4}{5}$

$$\cos^2\theta = \tfrac{1}{2}(1+\cos 2\theta)$$

$$\cos^2\frac{\theta}{2} = \frac{1}{2}(1+\cos\theta)$$

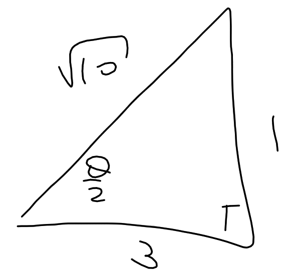
$$= \frac{1}{2}\left(1+\frac{4}{5}\right)$$

$$= \frac{9}{10}$$

$$\cos\frac{\theta}{2} = \pm\frac{3}{\sqrt{10}}$$

but $\frac{\theta}{2}$ is acute

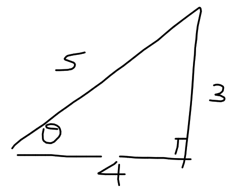
$$\therefore \cos\frac{\theta}{2} = \frac{3}{\sqrt{10}}$$



c) $\tan\frac{\theta}{2} = \frac{1}{3}$

7(c) θ is acute

$\cos\theta = \frac{4}{5}$



$\tan\theta = \frac{3}{4}$

$$\tan\theta = \frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}$$

$$\frac{3}{4} = \frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}$$

$$3 - 3\tan^2\frac{\theta}{2} = 8\tan\frac{\theta}{2}$$

$$3\tan^2\frac{\theta}{2} + 8\tan\frac{\theta}{2} - 3 = 0$$

$$\left(3\tan\frac{\theta}{2} - 1\right)\left(\tan\frac{\theta}{2} + 3\right) = 0$$

$$\tan\frac{\theta}{2} = \frac{1}{3} \text{ or } \tan\frac{\theta}{2} = -3$$

but $\frac{\theta}{2}$ is acute

$$\therefore \underline{\tan\frac{\theta}{2} = \frac{1}{3}}$$

8g)

$$\frac{1-\tan^2\theta}{1+\tan^2\theta} = \cos 2\theta$$

$$\frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{1-\tan^2\theta}{\sec^2\theta}$$

$$= \cos^2\theta\left(1-\frac{\sin^2\theta}{\cos^2\theta}\right)$$

$$= \cos^2\theta - \sin^2\theta$$

$$= \underline{\cos 2\theta}$$

10b) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$$8x^3 - 6x + 1 = 0$$

let $x = \cos\theta$

$$8\cos^3\theta - 6\cos\theta + 1 = 0$$

$$2\cos 3\theta + 1 = 0$$

$$\cos 3\theta = -\frac{1}{2}$$

Q 2, 3

$$\cos\alpha = \frac{1}{2}$$

$$\alpha = 60^\circ$$

$$3\theta = 120^\circ, 240^\circ, 480^\circ$$

$$\theta = 40^\circ, 80^\circ, 160^\circ$$

$$x = \underline{\cos 40^\circ, \cos 80^\circ, \cos 160^\circ}$$

11/

$$\sin 3x = \sin(2x + x)$$

$$= \sin 2x \cos x + \cos 2x \sin x$$

$$= 2\sin x \cos x \cos x + (1 - 2\sin^2 x)\sin x$$

$$= 2\sin x(1 - \sin^2 x) + \sin x - 2\sin^3 x$$

$$= 2\sin x - 2\sin^3 x + \sin x - 2\sin^3 x$$

$$= \underline{3\sin x - 4\sin^3 x}$$

11b)

$$\sin 3x = 3\sin x - 4\sin^3 x$$
$$\cos 3x = 4\cos^3 x - 3\cos x$$

$$\tan 3x = \frac{\sin 3x}{\cos 3x}$$
$$= \frac{3\sin x - 4\sin^3 x}{4\cos^3 x - 3\cos x}$$
$$= \frac{\frac{3\sin x}{\cos^3 x} - 4\frac{\sin^3 x}{\cos^3 x}}{\frac{4\cos^3 x}{\cos^3 x} - \frac{3\cos x}{\cos^3 x}}$$
$$= \frac{3\tan x \sec^2 x - 4\tan^3 x}{4 - 3\sec^2 x}$$
$$= \frac{3\tan x(1+\tan^2 x) - 4\tan^3 x}{4 - 3(1+\tan^2 x)}$$
$$= \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$$

13c)

$$\boxed{\frac{2\sin^3\theta + 2\cos^3\theta}{\sin\theta + \cos\theta} = 2 - \sin 2\theta}$$

$$\frac{2\sin^3\theta + 2\cos^3\theta}{\sin\theta + \cos\theta} = \frac{2(\sin\theta + \cos\theta)(\sin^2\theta - \sin\theta\cos\theta + \cos^2\theta)}{(\sin\theta + \cos\theta)}$$

$$= 2(1 - \sin\theta\cos\theta)$$

$$= 2 - 2\sin\theta\cos\theta$$

$$= \underline{2 - \sin 2\theta}$$

13k) $$\boxed{\cos^2\alpha\cos^2\beta - \sin^2\alpha\sin^2\beta = \frac{1}{2}(\cos 2\alpha + \cos 2\beta)}$$

$$\cos^2\alpha\cos^2\beta - \sin^2\alpha\sin^2\beta = \cos^2\alpha(1-\sin^2\beta) - (1-\cos^2\alpha)\sin^2\beta$$

$$= \cos^2\alpha - \cos^2\alpha\sin^2\beta - \sin^2\beta + \cos^2\alpha\sin^2\beta$$

$$= \cos^2\alpha - \sin^2\beta$$

$$\frac{1}{2}\left(\cos 2\alpha + \cos 2\beta\right)$$

$$= \frac{1}{2}\left[2\cos^2\alpha - 1 + 1 - 2\sin^2\beta\right]$$

$$= \cos^2\alpha - \sin^2\beta$$

$$\therefore \frac{1}{2}\left(\cos 2\alpha + \cos 2\beta\right) = \cos^2\alpha\cos^2\beta - \sin^2\alpha\sin^2\beta$$

$$\cos^2\alpha \cos^2\beta - \sin^2\alpha \sin^2\beta$$

$$= \left[\frac{1}{2}(1+\cos 2\alpha)\frac{1}{2}(1+\cos 2\beta)\right] - \left[\frac{1}{2}(1-\cos 2\alpha)\frac{1}{2}(1-\cos 2\beta)\right]$$

$$= \frac{1}{4}\left[1 + \cos 2\alpha + \cos 2\beta + \cos 2\alpha \cos 2\beta - 1 + \cos 2\alpha + \cos 2\beta - \cos 2\alpha \cos 2\beta\right]$$

$$= \frac{1}{4}\left[2\cos 2\alpha + 2\cos 2\beta\right]$$

$$= \frac{1}{2}(\cos 2\alpha + \cos 2\beta)$$

13m) $$\frac{\sin 2\alpha + \cos 2\alpha}{2\cos\alpha + \sin\alpha - 2(\cos^3\alpha + \sin^3\alpha)} = \operatorname{cosec}\alpha$$

$$= \frac{2\sin\alpha\cos\alpha + 2\cos^2\alpha - 1}{2(\sin\alpha + \cos\alpha)(1 - \cos^2\alpha + \sin\alpha\cos\alpha - \sin^2\alpha) - \sin\alpha}$$

$$= \frac{2\sin\alpha\cos\alpha + 2\cos^2\alpha - 1}{2(\sin\alpha + \cos\alpha)\sin\alpha\cos\alpha - \sin\alpha}$$

$$= \frac{2\sin\alpha\cos\alpha + 2\cos^2\alpha - 1}{\sin\alpha\left[2\sin\alpha\cos\alpha + 2\cos^2\alpha - 1\right]}$$

$$= \frac{1}{\sin\alpha}$$

$$= \underline{\operatorname{cosec}\alpha}$$

13n)

$$(\tan\theta + \tan 2\theta)(\cot\theta + \cot 3\theta)$$

$$= \left(\tan\theta + \frac{2\tan\theta}{1-\tan^2\theta}\right)\left(\frac{1}{\tan\theta} + \frac{1-3\tan^2\theta}{3\tan\theta - \tan^3\theta}\right)$$

$$= \frac{\tan\theta - \tan^3\theta + 2\tan\theta}{1-\tan^2\theta} \times \frac{3\tan\theta - \tan^3\theta + \tan\theta - 3\tan^3\theta}{\tan\theta(3\tan\theta - \tan^3\theta)}$$

$$= \frac{\tan\theta(3-\tan^2\theta)}{(1-\tan^2\theta)} \times \frac{\tan\theta(4-4\tan^2\theta)}{\tan\theta(3\tan\theta - \tan^3\theta)}$$

$$=$$

16(b)(i) a) $\cos 45° = \frac{1}{\sqrt{2}} = \underline{\underline{\frac{1}{2}\sqrt{2}}}$

$\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$

$\cos^2 22\frac{1}{2}° = \frac{1}{2}(1 + \cos 45°)$

$= \frac{1}{2}\left(1 + \frac{1}{2}\sqrt{2}\right)$

$\cos 22\frac{1}{2}° = \frac{1}{\sqrt{2}}\sqrt{\left(1 + \frac{1}{2}\sqrt{2}\right)}$

$= \sqrt{\frac{1}{2} + \frac{1}{4}\sqrt{2}}$

$= \frac{1}{2}\sqrt{2 + \sqrt{2}}$

$$\cos^2 11\tfrac{1}{4} = \tfrac{1}{2}\left(1 + \cos 22\tfrac{1}{2}^\circ\right)$$

$$= \tfrac{1}{2}\left(1 + \tfrac{1}{2}\sqrt{2+\sqrt{2}}\right)$$

$$= \tfrac{1}{4}\left(2 + \sqrt{2+\sqrt{2}}\right)$$

$$\cos 11\tfrac{1}{4}^\circ = \tfrac{1}{2}\sqrt{2+\sqrt{2+\sqrt{2}}}$$

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a) $\sin 54° = \cos 36°$

b) $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

c) $4\sin^3 18° - 2\sin^2 18° - 3\sin 18° + 1$

$= -\sin(3\times 18) - 2\sin^2 18° + 1$

$= -\sin 54° + \cos(2\times 18)$

$= -\sin 54° + \cos 36°$

$= \underline{\underline{0}}$

d) $x = \sin 18°$ is a solution to

$$4x^3 - 2x^2 - 3x + 1 = 0$$

$$(4x^2 + 2x - 1)(x - 1) = 0$$

$$4x^2 + 2x - 1 = 0 \quad \text{or} \quad x = 1$$

$$\sin 18° \neq 1$$

$\therefore$ $x = \sin 18°$ is a solution

to $4x^2 + 2x - 1 = 0$

$$x = \frac{-2 \pm \sqrt{20}}{8}$$

$$= \frac{-1 \pm \sqrt{5}}{4}$$

$\sin 18° > 0$

$$\therefore \sin 18° = \frac{-1 + \sqrt{5}}{4}$$

$$\sin 54^\circ = \sin(3 \times 18)$$

$$= 3\sin 18^\circ - 4\sin^3 18^\circ$$

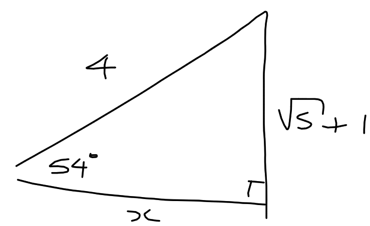
$$= 3\left(\frac{\sqrt{5}-1}{4}\right) - 4\left(\frac{\sqrt{5}-1}{4}\right)^3$$

$$= \frac{3\sqrt{5}-3}{4} - \frac{5\sqrt{5}-15+3\sqrt{5}-1}{16}$$

$$= \frac{4\sqrt{5}+4}{16}$$

$$= \frac{\sqrt{5}+1}{4}$$

$$\cos 54° = \frac{\sqrt{10-2\sqrt{5}}}{4}$$



$$x^2 = 16 - (\sqrt{5}+1)^2$$
$$= 16 - 6 - 2\sqrt{5}$$
$$= 10 - 2\sqrt{5}$$

f) $\sqrt{8+2\sqrt{10-2\sqrt{5}}} = \sqrt{5+\sqrt{5}} + \sqrt{3-\sqrt{5}}$

$$\left(\sqrt{5+\sqrt{5}} + \sqrt{3-\sqrt{5}}\right)^2 = 5+\sqrt{5} + 2\sqrt{(5+\sqrt{5})(3-\sqrt{5})} + 3-\sqrt{5}$$

$$= 8 + 2\sqrt{10-2\sqrt{5}}$$

$$\therefore \sqrt{8+2\sqrt{10-2\sqrt{5}}} = \underline{\sqrt{5+\sqrt{5}} + \sqrt{3-\sqrt{5}}}$$

g) $\cos^2 27 = \frac{1}{2}(1+\cos 54^\circ)$

$$= \frac{1}{2}\left(1+\frac{\sqrt{10-2\sqrt{5}}}{4}\right)$$

$$= \frac{4+\sqrt{10-2\sqrt{5}}}{8}$$

$$= \frac{8+2\sqrt{10-2\sqrt{5}}}{16}$$

$$\cos 27^\circ = \frac{1}{4}\left(\sqrt{8+2\sqrt{10-2\sqrt{5}}}\right)$$

$$= \frac{1}{4}\left(\sqrt{5+\sqrt{5}}+\sqrt{3-\sqrt{5}}\right)$$