10a)

$$(3+\sqrt{5})^6 + (3-\sqrt{5})^6$$

$$= 2\left\{ {}^6C_0\, 3^6 + {}^6C_2\, 3^4(\sqrt{5})^2 + {}^6C_4\, 3^2(\sqrt{5})^4 + {}^6C_6(\sqrt{5})^6 \right\}$$

ii

$$\frac{1}{(\sqrt{3}-1)^4} + \frac{1}{(\sqrt{3}+1)^4}$$

$$= \frac{(\sqrt{3}+1)^4}{(\sqrt{3}-1)^4(\sqrt{3}+1)^4} + \frac{(\sqrt{3}-1)^4}{(\sqrt{3}+1)^4(\sqrt{3}-1)^4}$$

$$= \frac{(\sqrt{3}+1)^4 + (\sqrt{3}-1)^4}{(\sqrt{3}-1)^4(\sqrt{3}+1)^4}$$

$$\frac{2\left({}^4C_0(\sqrt{3})^4 + {}^4C_2(\sqrt{3})^2(1)^2 + {}^4C_4(1)^4\right)}{(3-1)^4}.$$

$$= \frac{9 + 18 + 1}{2^3}$$

$$= \frac{28}{8}$$

$$= \frac{7}{2}$$

13/ $(x+2y)^5 = x^5 + 10x^4y + 40x^3y^2 + 80x^2y^3 + 80xy^4 + 32y^5$

$(1.02)^5 = (1 + 2(0.01))^5$

$= 1 + 10(0.01) + 40(0.0001) + 80(0.000001) + \dots$

$= \underline{1.10408}$

14

$$\left(x+\frac{1}{x}\right)^3 = x^3 + 3x + \frac{3}{x} + \frac{1}{x^3}$$

$$\underline{x+\frac{1}{x} = 2}$$

$$x^3 + \frac{1}{x^3} = \left(x+\frac{1}{x}\right)^3 - 3\left(x+\frac{1}{x}\right)$$

$$= 2^3 - 6$$

$$= \underline{2}$$

17b) $\left(x+\frac{1}{x}\right)^6$

$= x^6 + 6x^4 + 15x^2 + 20 + \frac{15}{x^2} + \frac{6}{x^4} + \frac{1}{x^6}$

If $U = x + \frac{1}{x}$

$\left(x+\frac{1}{x}\right)^4 = x^4 + 4x^2 + 6 + \frac{4}{x^2} + \frac{1}{x^4}$

$\left(x+\frac{1}{x}\right)^2 = x^2 + 2 + \frac{1}{x^2}$

$x^2 + \frac{1}{x^2} = U^2 - 2$

$x^4 + \frac{1}{x^4} = U^4 - 4\left(x^2 + \frac{1}{x^2}\right) - 6$

$$x^6+\frac{1}{x^6}=u^6-6\left(x^4+\frac{1}{x^4}\right)-15\left(x^2+\frac{1}{x^2}\right)-20$$

$$=u^6-6\left[u^4-4\left(x^2+\frac{1}{x^2}\right)-6\right]-15\left(x^2+\frac{1}{x^2}\right)-20$$

$$=u^6-6u^4+9\left(x^2+\frac{1}{x^2}\right)+16$$

$$=u^6-6u^4+9\left[u^2-2\right]+16$$

$$=u^6-6u^4+9u^2-2$$