3b(iv)

$${}^{12}C_4 = {}^{12}C_n$$

$$\underline{n = 8}$$

5b(vi)

$${}^{n}C_3 + {}^{n}C_2 = 8\,{}^{n}C_1$$

$$\frac{n!}{3!(n-3)!} + \frac{n!}{2!(n-2)!} = 8n$$

$$\frac{n(n-1)(n-2)}{6} + \frac{n(n-1)}{2} = 8n$$

$$(n-1)(n-2) + 3(n-1) = 48$$

$$n^2 - 49 = 0$$

$$\underline{n = 7}$$

5a)

(i) $^nC_0 = \frac{n!}{n!\,0!}$

$= 1$

(ii) $^nC_1 = \frac{n!}{(n-1)!\,1!}$

$= n$

(iv) $^nC_3$

$= \frac{n!}{3!(n-3)!}$

$= \frac{n(n-1)(n-2)}{6}$

5b(ii) $^9C_2 - {}^nC_1 = {}^6C_3$

$$\frac{9!}{2!7!} - \frac{n!}{1!(n-1)!} = \frac{6!}{3!3!}$$

$$\frac{9\times 8}{2\times 1} - n = \frac{6\times 5\times 4}{3\times 2\times 1}$$

$$36 - n = 20$$

$$\underline{n = 16}$$

5b)(iv) $\quad {}^nC_3 + {}^nC_2 = 8\,{}^nC_1$

$${}^{n+1}C_3 = 8\,{}^nC_1$$

$$\frac{(n+1)!}{3!\,(n-2)!} = 8n$$

$$\frac{(n+1)n(n-1)}{6} = 8n$$

$$n^3 - n = 48n$$

$$n^3 = 49n$$

$$n(n^2-49) = 0$$

$n=0$ not a solution $\quad n = \pm 7\ ,\ n>0$

$$\underline{\underline{n=7}}$$

7 a) $T_{i+1} = {}^9C_i (x^2)^{9-i} \left(\frac{1}{x}\right)^i$

$= {}^9C_i \, x^{18-2i} \cdot x^{-i}$

$= {}^9C_i \, x^{18-3i}$

b) $\underline{\underline{x^3}}$

$18 - 3i = 3$

$3i = 15$

$i = 5$

$\therefore T_6 = {}^9C_5 \, x^3$

(ii) $\underline{\underline{x^{-3}}}$

$18 - 3i = -3$

$3i = 21$

$i = 7$

$\therefore T_8 = {}^9C_7 \, x^{-3}$

9a) $\left(\frac{x}{2} - \frac{5}{x}\right)^{15}$

$$T_{j+1} = {}^{15}C_j \left(\frac{x}{2}\right)^{15-j}\left(-\frac{5}{x}\right)^{j}$$

$$= {}^{15}C_j \left(2^{-1}\right)^{15-j}(-5)^{j}(x)^{15-j}(x^{-1})^{j}$$

$$= {}^{15}C_j (-1)^{j}\, 2^{j-15}\, 5^{j}\, x^{15-2j}$$

10a)
$$\left(x+\frac{3}{x}\right)^8$$

$$T_{k+1} = {}^8C_k\,(x)^{8-k}\left(\frac{3}{x}\right)^k$$

$$(x)^{8-k}(x)^{-k} = x^0$$
$$8-2k=0$$
$$k=4$$

$$T_5 = {}^8C_4\,x^4\left(\frac{3}{x}\right)^4$$
$$= \underline{{}^8C_4\,3^4}$$

10b) $\left(2x^3 - \frac{1}{x}\right)^{12}$

$$T_{k+1} = {}^{12}C_k \left(2x^3\right)^{12-k}\left(-\frac{1}{x}\right)^k$$

$$\left(x^3\right)^{12-k}\left(x^{-1}\right)^k = x^0$$

$$x^{36-3k-k} = x^0$$

$$36-4k=0$$

$$k=9$$

12 a) $(3+x)(1-x)^{15}$

$$T_{k+1} = {}^{15}C_k (1)^{15-k}(-x)^k$$

$$= {}^{15}C_k (-x)^k$$

(i) x^4

$$(3)\left({}^{15}C_4 x^4\right) + (x)\left({}^{15}C_3 (-x)^3\right)$$

$$= 4095x^4 - 455x^4$$

$$= 3640x^4$$

12 c) $(x-3)(x+2)^{15}$

$T_{k+1} = {}^{15}C_k (x)^{15-k} 2^k$

x^7

$(1)\,{}^{15}C_9 2^9 - 3\,{}^{15}C_8 2^8$

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a) $(5+2x)^{15}$

$T_{k+1} = {}^{15}C_k (5)^{15-k} (2x)^k$

${}^{15}C_{10}\, 5^5\, 2^{10}\, x^{10} = {}^{15}C_{11}\, 5^4\, 2^{11}\, x^{11}$

$x = \dfrac{{}^{15}C_{10}\, 5^5\, 2^{10}}{{}^{15}C_{11}\, 5^4\, 2^{11}}$

$= \dfrac{15!}{10!\,5!} \times \dfrac{11!\,4!}{15!} \times \dfrac{5}{2}$

$= \dfrac{11}{5} \times \dfrac{5}{2}$

$= \dfrac{11}{2}$

15. a) $\left(x+\frac{1}{x}\right)^5\left(x-\frac{1}{x}\right)^4$

$=\left(x^2-\frac{1}{x^2}\right)^4\left(x+\frac{1}{x}\right)$

$T_{k+1} = {}^4C_k\,(x^2)^{4-k}\left(-\frac{1}{x^2}\right)^k$

$= {}^4C_k(-1)^k\,x^{8-2k}\cdot x^{-2k}$

$= {}^4C_k(-1)^k\,x^{8-4k}$

<u>coefficient of x</u>.

${}^4C_2(-1)^2 = {}^4C_2$

$= \underline{\underline{6}}$

15c)

$$\left(y+\frac{1}{y}\right)^{10}\left(y-\frac{1}{y}\right)^{7}$$

$$=\left(y^2-\frac{1}{y^2}\right)^{7}\left(y+\frac{1}{y}\right)^{3}$$

$$T_{k+1}={}^{7}C_{k}(-1)^{k}(y^2)^{7-k}\left(\frac{1}{y^2}\right)^{k}$$

$$={}^{7}C_{k}(-1)^{k}y^{14-4k}$$

$$\left(y^3+3y+\frac{3}{y}+\frac{1}{y^3}\right)$$

coefficient y^{-3}

$$=-{}^{7}C_{5}+3\,{}^{7}C_{4}$$

$=$

$14-4k=-6$

$4k=20$

$k=5$

$14-4k=-4$

$4k=18$

$14-4k=-2$

$4k=16$

$k=4$

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$$(5+\sqrt{11})^n + (5-\sqrt{11})^n$$

$$= {}^nC_0 5^n + {}^nC_1 5^{n-1}\sqrt{11} + {}^nC_2 5^{n-2}(\sqrt{11})^2 + {}^nC_3 5^{n-3}(\sqrt{11})^3 + {}^nC_4 5^{n-4}(\sqrt{11})^4 + \dots$$

$$+ {}^nC_0 5^n - {}^nC_1 5^{n-1}\sqrt{11} + {}^nC_2 5^{n-2}(\sqrt{11})^2 - {}^nC_3 5^{n-3}(\sqrt{11})^3 + {}^nC_4 5^{n-4}(\sqrt{11})^4 - \dots$$

$$= 2\,{}^nC_0 5^n \qquad + 2\,{}^nC_2 5^{n-2}(\sqrt{11})^2 \qquad + 2\,{}^nC_4 5^{n-4}(\sqrt{11})^4 + \dots$$

$$= 2\,{}^nC_0 5^n + 2\,{}^nC_2 5^{n-2}\,11 + 2\,{}^nC_4 5^{n-4}\,11^2 + \dots$$

$= \underline{\text{integer}}$

$$\sum_{k=0}^{n} {}^nC_k 5^{n-k}(\sqrt{11})^k + \sum_{k=0}^{n} {}^nC_k (5^{n-k})(-\sqrt{11})^k$$

$$= \sum_{k=0}^{n} {}^nC_k 5^{n-k}(\sqrt{11})^k(1+(-1)^k)$$

$$T_{k+1} = {}^nC_k 5^{n-k}(\sqrt{11})^k(1+(-1)^k)$$

<u>If k is odd</u>

$$T_{k+1} = 0$$

<u>If k is even</u>

$$T_{k+1} = 2\,{}^nC_k 5^{n-k}(\sqrt{11})^k$$

$$= 2\,{}^nC_k 5^{n-k}(11)^{\frac{k}{2}}$$

25a)

$$7^n + 2$$

$$= (6+1)^n + 2$$

$$= \underbrace{{}^nC_0 6^n + {}^nC_1 6^{n-1} + {}^nC_2 6^{n-2} + \dots + {}^nC_{n-1} 6}\ + {}^nC_n + 2$$

$$= 6\left\{{}^nC_0 6^{n-1} + {}^nC_1 6^{n-2} + {}^nC_2 6^{n-3} + \dots + {}^nC_{n-1}\right) + 3$$

6 ad 3 are duisible by 3

$\therefore$ $7^n + 2$ is divisible by 3

25c)

$$a^n - b^{n-1}(b+cn)$$

$$= (b+c)^n - b^{n-1}(b+cn)$$

$$= b^n + {}^nC_1 b^{n-1}c + {}^nC_2 b^{n-2}c^2 + {}^nC_3 b^{n-3}c^3 + \dots + {}^nC_n c^n - b^n - b^{n-1}cn$$

$$= {}^nC_2 b^{n-2}c^2 + {}^nC_3 b^{n-3}c^3 + \dots + {}^nC_n c^n$$