1)

$$(2+3x)^{12} = \sum_{k=0}^{12} t_k x^k$$

$$t_k = {}^{12}C_k \, 2^{12-k} \, 3^k$$

$$t_{k+1} = {}^{12}C_{k+1} \, 2^{11-k} \, 3^{k+1}$$

$$\frac{A_{k+1}}{A_k} = \frac{12!}{(k+1)!(11-k)!} \times \frac{k!(12-k)!}{12!} \times \frac{2^{11-k}\,3^{k+1}}{2^{12-k}\,3^{k}}$$

$$= \frac{12-k}{k+1} \times \frac{3}{2}$$

$$= \frac{36-3k}{2k+2}$$

b) If $t_{k+1} > t_k$ then t_{k+1} is greatest

$$t_{k+1} > t_k$$

$$\frac{t_{k+1}}{t_k} > 1$$

$$\frac{36-3k}{2k+2} > 1$$

$$36-3k > 2k+2$$

$$5k < 34$$

$$k < \frac{34}{5}$$

$$k = 6$$

$\therefore t_7$ is greatest

${}^{12}C_7\, 2^5\, 3^7$

2/ $(7+3x)^{25} = \sum_{k=0}^{25} c_k x^k$

$T_{k+1} = c_k x^k$

$= {}^{25}C_k (7)^{25-k} (3x)^k$

$c_k = {}^{25}C_k 7^{25-k} 3^k$

3/ $T_k = {}^{13}C_k \, 3^{13-k} (4x)^k$

$T_{k+1} = {}^{13}C_{k+1} \, 3^{12-k} (4x)^{k+1}$

$$\frac{T_{k+1}}{T_k} = \frac{4x(13-k)}{3(k+1)}$$

If $T_{k+1} > T_k$ then T_{k+1} is greatest

$$\frac{T_{k+1}}{T_k} > 1$$

$\underline{x = \frac{1}{2}}$

$$\frac{2(13-k)}{3(k+1)} > 1$$

$$26-2k > 3k+3$$

$$-5k > -23$$

$$k < \frac{23}{5}$$

$$k = 4$$

$\therefore$ $T_5 = {}^{13}C_5\, 3^7 2^5$ is greatest term.

7b) $(7-2x)^{14}$

$$T_{k+1} = {}^{14}C_k (7)^{14-k} (-2x)^k$$

$$T_k = {}^{14}C_{k-1} (7)^{15-k} (-2x)^{k-1}$$

if $T_{k+1} > T_k$ then T_{k+1} is greatest term.

$$ {}^{14}C_k\, 7^{14-k}\, 2^k > {}^{14}C_{k-1}\, 7^{15-k}\, 2^{k-1}$$

$$\frac{14!}{k!(14-k)!} \times \frac{(k-1)!(15-k)!}{14!} \times \frac{2}{7} > 1$$

$$\frac{2(15-k)}{7k} > 1$$

$$30 - 2k > 7k$$

$$-9k > -30$$

$$k < \frac{30}{9}$$

$$k = 3$$

$$\therefore \underline{T_4 = {}^{14}C_3\, 7^{11}\, 2^3}$$

d (iv) $(2x-y)^{15}$

$x = \frac{3}{2}$, $y = \frac{4}{9}$

$$T_{k+1} = {}^{15}C_k (3)^{15-k} \left(-\frac{4}{9}\right)^k$$

$$T_k = {}^{15}C_{k-1} (3)^{16-k} \left(-\frac{4}{9}\right)^{k-1}$$

If $T_{k+1} > T_k$ then T_{k+1} is greatest

$$ {}^{15}C_k \, 3^{15-k} \left(\frac{4}{9}\right)^k > {}^{15}C_{k-1} \, 3^{16-k} \left(\frac{4}{9}\right)^{k-1}$$

$$\frac{15!}{k!(15-k)!} \times \frac{(k-1)!(16-k)!}{15!} \times \frac{\frac{4}{9}}{3} > 1$$

$$\frac{4(16-k)}{27k} > 1$$

$$64 - 4k > 27k$$

$$-31k > -64$$

$$k < \frac{64}{31}$$

$$k = 2$$

$$\therefore T_3 = {}^{15}C_2\, 3^{13} \left(\frac{4}{9}\right)^2$$

$$= {}^{15}C_2\, 3^{9}\, 4^{2}$$

11) $(1+x)^n = \sum_{r=0}^{n} t_r x^r$

$^nC_{r+1} = 5\,^nC_r \qquad ^nC_{r+4} = 2\,^nC_{r+3}$

$$\frac{n!}{(r+1)!\,(n-r-1)!} = \frac{5n!}{r!\,(n-r)!}$$

$$\frac{n!\,r!\,(n-r)!}{n!\,(r+1)!\,(n-r-1)!} = 5$$

$$\frac{n-r}{r+1} = 5$$