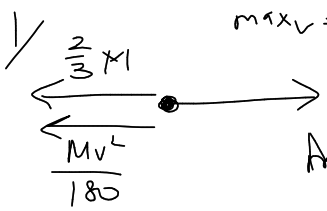
1/



$$M\ddot{x} = -\frac{2}{3}M1 - \frac{Mv^2}{180}$$

$$\ddot{x} = -\frac{2}{3} - \frac{v^2}{180}$$

$$v\frac{dv}{dx} = -\frac{120+v^2}{180}$$

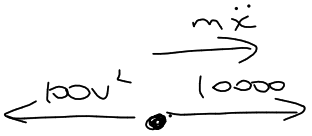
$$\int_{80}^{v} \frac{2v\,dv}{120+v^2} = -\int_{0}^{x} \frac{1}{90}\,dx$$

$$\left[\ln(120+v^2)\right]_{80}^{v} = -\frac{1}{90}x$$

$$\ln\left(\frac{120+v^2}{6520}\right) = -\frac{1}{90}x$$

$$x = 90\ln\left(\frac{6520}{120+v^2}\right)$$

2/



$$10000\ddot{x} = 10000 - 100v^2$$

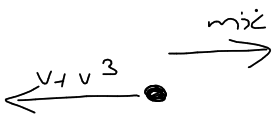
$$\ddot{x} = 1 - \frac{v^2}{100}$$

When $\ddot{x} = 0$

$$\frac{v^2}{100} = 1$$

$$v = 10\,m/s$$

3/



$t=0,\ v=Q$
$x=0$

$\frac{1}{v+v^3} = \frac{A}{v} + \frac{Bv+C}{1+v^2}$

$A(1+v^2)+(Bv+C)v = 1$

$A=1 \quad C=0$

$B=-1$

$\dot{x} = -(v+v^3)$

$v\frac{dv}{dx} = -(v+v^3)$

$-\int_Q^v \frac{v\,dv}{v+v^3} = \int_0^x dx$

$-\int_Q^v \frac{dv}{1+v^2} = x$

$x = -\left[\tan^{-1}v\right]_Q^v$

$= \tan^{-1}Q - \tan^{-1}v$

$= \tan^{-1}\left(\frac{Q-v}{1+Qv}\right)$

b) $\frac{dv}{dt} = -(v+v^3)$

$-\int_Q^v \frac{dv}{v+v^3} = \int_0^t dt$

$-\int_Q^v \left[\frac{1}{v} - \frac{v}{1+v^2}\right]dv = \int_0^t dt$

$t = \left[-\ln v + \frac{1}{2}\ln(1+v^2)\right]_Q^v$

$= \left[\frac{1}{2}\ln\left(\frac{1+v^2}{v^2}\right)\right]_Q^v$

$= \frac{1}{2}\ln\left[\frac{1+v^2}{v^2}\times\frac{Q^2}{1+Q^2}\right]$

$= \frac{1}{2}\ln\left(\frac{Q^2(1+v^2)}{v^2(1+Q^2)}\right)$

b) $t = \frac{1}{2} \ln \frac{Q^2(1+v^2)}{v^2(1+Q^2)}$

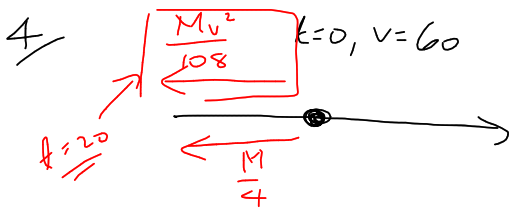
c) $2t = \ln \frac{Q^2(1+v^2)}{v^2(1+Q^2)}$

$v^2(1+Q^2)e^{2t} = Q^2(1+v^2)$

$v^2\left((1+Q^2)e^{2t} - Q^2\right) = Q^2$

$v^2 = \frac{Q^2}{(1+Q^2)e^{2t} - Q^2}$

4/



$$M\ddot{x} = -\frac{M}{4}$$

$$\ddot{x} = -\frac{1}{4}$$

$$\frac{dv}{dt} = -\frac{1}{4}$$

$$-4\int_{60}^{v} dv = \int_{0}^{20} dt$$

$$-4\left[v\right]_{60}^{v} = \left[t\right]_{0}^{20}$$

$$-4v + 240 = 20 - 0$$

$$4v = 220$$

$$v = 55\ m/s$$

$$v\frac{dv}{dx} = -\frac{1}{4}$$

$$-4\int_{60}^{55} v\,dv = \int_{0}^{x} dx$$

$$x = -2\left[v^2\right]_{60}^{55}$$

$$= 2\left(60^2 - 55^2\right)$$

$$= 1150\ m$$

b)

$$M\ddot{x} = -\frac{M}{4} - \frac{Mv^2}{108}$$

$$\ddot{x} = -\frac{27+v^2}{108}$$

$$v\frac{dv}{dx} = -\frac{27+v^2}{108}$$

$$-54\int_{55}^{v}\frac{2v\,dv}{27+v^2} = \int_{1150}^{x} dx$$

$$-54\left[\ln(27+v^2)\right]_{55}^{v} = x - 1150$$

$$x = 1150 + 54\left(\ln(27+55^2) - \ln(27+v^2)\right)$$

c) $V = 0$

$$x = 1150 + 54 \ln\left(\frac{27 + 55^2}{27}\right)$$

$$= 1405 \text{ m} \quad (\text{to nearest m})$$