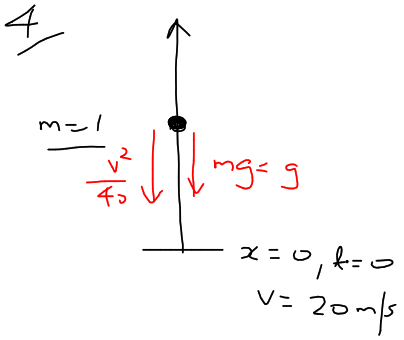
4



$$\ddot{x} = -g - \frac{v^2}{40}$$

$$= -\frac{40g + v^2}{40}$$

(a)(ii) $$v\frac{dv}{dx} = -\frac{40g + v^2}{40}$$

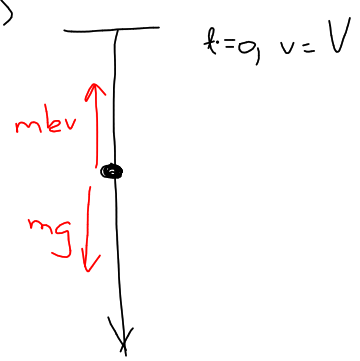
$$-20\int_{20}^{0} \frac{2v}{40g + v^2} = \int_0^H dx$$

$$20\left[\ln(40g + v^2)\right]_0^{20} = H$$

$$H = 20\ln\left(\frac{400 + 400}{400}\right)$$

$$= 20\ln 2$$

5d)



$m\ddot{x} = mg - mkv$

$\ddot{x} = g - kv$

terminal $v = 20$

$\frac{g}{k} = 20$

$k = \frac{9}{20}$

$k = \frac{1}{2}$

$x = 20t + (2V - 40)\left(1 - e^{-\frac{t}{2}}\right)$

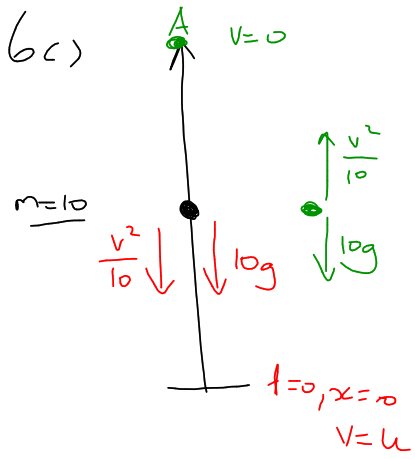
$x_2 - x_1 = \left[0 + 40\right]\left(1 - e^{-\frac{t}{2}}\right)$

$= 40\left(1 - e^{-\frac{t}{2}}\right)$

limiting distance $= \lim_{A \to \infty} 40\left(1 - e^{-\frac{t}{2}}\right)$

$\therefore \quad = 40$

6c)



$$10\ddot{x} = 100 - \frac{v^2}{10}$$

$$\ddot{x} = 10 - \frac{v^2}{100}$$

$$v\frac{dv}{dx} = \frac{1000 - v^2}{100}$$

$$-50\int_u^w \frac{-2v\,dv}{1000 - v^2} = \int_0^{50\ln\frac{1000+u^2}{1000}} dx$$

$$-50\left[\ln(1000 - v^2)\right]_u^w = 50\ln\left(\frac{1000+u^2}{1000}\right)$$

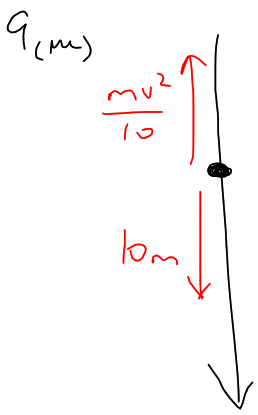
$$\ln\left(\frac{1000}{1000 - w^2}\right) = \ln\left(\frac{1000+u^2}{1000}\right)$$

$$\frac{1000 - w^2}{1000} = \frac{1000}{1000+u^2}$$

$$1000 - w^2 = \frac{1000000}{1000+u^2}$$

$$w^2 = 1000 - \frac{1000000}{1000+u^2}$$

$$= \frac{1000u^2}{1000+u^2}$$



$$\ddot{x} = 10 - \frac{v^2}{10} = \frac{100 - v^2}{10}$$

$$\frac{dv}{dt} = \frac{100 - v^2}{10}$$

$$10 \int_V^v \frac{dv}{100 - v^2} = \int_0^t dt$$

$$t = 10 \left[\frac{1}{20} \ln \left(\frac{10 + v}{10 - v} \right) \right]_V^v$$

$$= \frac{1}{2} \ln \left(\frac{10 + v}{10 - v} \times \frac{10 - V}{10 + V} \right)^v$$

$$= \frac{1}{2} \ln \left(\frac{(10 + v)(10 - V)}{(10 - v)(10 + V)} \right)$$

When $\ddot{x} = 0$

$$\frac{100 - v^2}{10} = 0$$

$$v = 10$$

$v = 10.5$, $V = 25.7$

$$t = \frac{1}{2} \ln \left[\frac{20.5 \times 15.7}{0.5 \times 35.7} \right]$$