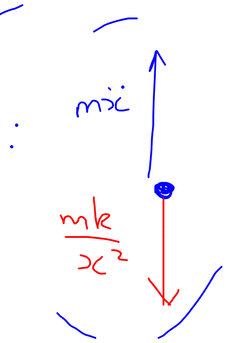
3/



$t=0$

$v=U \qquad \ddot{x}=-g$

$x=R$

$$\ddot{x} = -\frac{k}{x^2}$$

$$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = -\frac{k}{x^2}$$

$$\frac{1}{2}v^2 = \frac{k}{x} + c$$

$x=R, \; v=U$

$$\frac{1}{2}U^2 = \frac{gR^2}{R} + c$$

$$c = \frac{1}{2}U^2 - gR$$

$\ddot{x} = -g, \quad x = R$

$$-g = -\frac{k}{R^2}$$

$$k = gR^2$$

$$v^2 = \frac{2gR^2}{x} + U^2 - 2gR$$
$$= U^2 - 2gR^2\left(\frac{1}{R} - \frac{1}{x}\right)$$

$R = 6400km \qquad g = 10m/s^2$

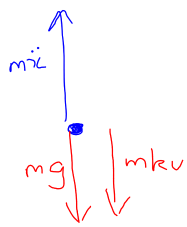
$$\lim_{x\to\infty} v^2 = U^2 - 2gR$$

but $v^2 \geq 0$

$$U^2 - 2gR \geq 0$$
$$U^2 \geq 2gR$$
$$= 2(10)(6400000$$

escape velocity $U = \sqrt{128000000}$
$$= 11313.7 \text{ m/s}$$
$$= 11.3137 \text{ km/s}$$

4



$$m\ddot{x} = -mg - mkv$$

$$\ddot{x} = -g - kv$$

$t=0, \quad v = U = \frac{g}{k}, \quad x = 0$

$$\frac{dv}{dt} = -g - kv$$

$$\frac{dt}{dv} = \frac{-1}{g+kv}$$

$$t = -\frac{1}{k}\log(g+kv) + c$$

When $t=0, \quad v = U$

$$0 = -\frac{1}{k}\log(g+kU) + c$$

$$c = -\frac{1}{k}\log(g+kU)$$

$$t = \frac{1}{k}\log\left(\frac{g+kU}{g+kv}\right)$$

$$kt = \log\left(\frac{g+kU}{g+kv}\right)$$

$$e^{kt} = \frac{g+kU}{g+kV}$$

$$e^{-kt} = \frac{g+kv}{g+kU}$$

$$g+kv = e^{-kt}(g+kU)$$

$$kv = e^{-kt}(g+kU) - g$$

$$v = \frac{1}{k}\left(e^{-kt}(g+kU) - g\right)$$

$$v = \frac{1}{k}\left(e^{-kt}(g+kU) - g\right)$$

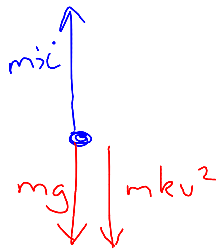
$$= \frac{1}{k}\left(e^{-kt}(2g) - g\right) \qquad U = \frac{g}{k}$$

$$= \frac{g}{k}\left(2e^{-kt} - 1\right)$$

$$= U\left(2e^{-kt} - 1\right)$$

5 (ii)

$t=0,\ x=0,\ v=U$



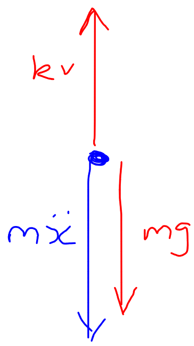
$$x = \frac{1}{2k}\log\left[\frac{g+kU^2}{g+kv^2}\right]$$

when $x = H$, $v = 0$

$$H = \frac{1}{2k}\log\left[\frac{g+kU^2}{g}\right]$$

$$= \frac{1}{2k}\log\left[1+\frac{kU^2}{g}\right]$$

6/



$t=0,\ v=0,\ x=0$

$$m\ddot{x} = mg - kv$$

$$\ddot{x} = g - \frac{k}{m}v$$

$$v\frac{dv}{dx} = \frac{mg-kv}{m}$$

$$\frac{dx}{dv} = \frac{mv}{mg-kv}$$

$$\begin{array}{r l}
 & -\frac{m}{k} \\
-kv+mg \big) & mv + 0 \\
 & mv - \frac{m^2g}{k} \\
\hline
 & \frac{m^2g}{k}
\end{array}$$

$$\int_0^x dx = \int_0^v \left[-\frac{m}{k} + \frac{m^2g}{k(mg-kv)}\right] dv$$

$$x = \left[-\frac{mv}{k} - \frac{m^2g}{k^2}\log(mg-kv)\right]_0^v$$

$$x = -\frac{mv}{k} + \frac{m^2 g}{k^2}\log\left(\frac{mg}{mg-kv}\right)$$

$$\ddot{x} = g - \frac{k}{m}v$$

terminal velocity occurs when $\ddot{x} = 0$

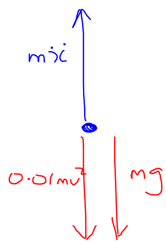
$$0 = 10 - \frac{\cdot 14}{70}v$$

$$v = 50\ m/s$$

$$= \frac{50 \times 60 \times 60}{1000}$$

$$= 180\ km/h$$

8 a)



$t=0,\ x=0,\ v=60$

$$m\ddot{x} = -mg - 0.01mv^2$$

$$\ddot{x} = -g - \frac{v^2}{100}$$

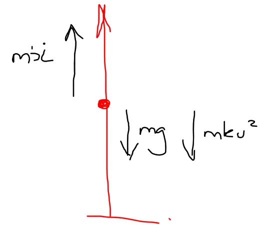
$$\frac{dv}{dt} = \frac{-100g - v^2}{100}$$

$$\int_0^T dt = -100\int_{60}^{0} \frac{dv}{100g + v^2}$$

$$T = \left[-100 \times \frac{1}{10\sqrt{g}} \tan^{-1} \frac{v}{10\sqrt{g}}\right]_{60}^{0}$$

$$= \underline{\frac{10}{\sqrt{g}} \tan^{-1} \frac{6}{\sqrt{g}}}$$

9 $\quad x = h, \quad v = v_T\left[\sqrt{1-e^{-2kh}}\right]$



$$m\ddot{x} = -mg - mkv^2$$

$$\ddot{x} = -g - kv^2$$

$$v\frac{dv}{dx} = -g - kv^2$$

$$\frac{dx}{dv} = \frac{-v}{g+kv^2}$$

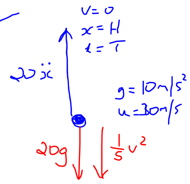
$$\int_0^h dx = -\frac{1}{2k}\int_{V_I}^{0} \frac{2kv}{g+kv^2}$$

$$\Big[x\Big]_0^h = -\frac{1}{2k}\Big[\log(g+kv^2)\Big]_{V_I}^{0}$$

$$h = -\frac{1}{2k}\log\left(\frac{g}{g+kV_I^2}\right)$$

$$e^{-2kh} = \frac{g}{g+kV_I^2}$$

10



$$20\ddot{x} = -20g - \frac{1}{5}v^2$$

$$\ddot{x} = -g - \frac{v^2}{100}$$

$$= \frac{-100g - v^2}{100}$$

$$\frac{dv}{dt} = \frac{-100g - v^2}{100}$$

$$\int_0^T dt = \int_{30}^{0} \frac{-100}{100g + v^2}\,dv$$

$$T = \frac{10}{\sqrt{g}}\left[\tan^{-1}\frac{v}{10\sqrt{g}}\right]_0^{30}$$

$$= \frac{10}{\sqrt{10}}\tan^{-1}\frac{3}{\sqrt{10}}$$

$$= \sqrt{10}\tan^{-1}\frac{3}{\sqrt{10}}$$

$$\ddot{x} = \frac{-100g - v^2}{100}$$

$$v\frac{dv}{dx} = \frac{-100g - v^2}{100}$$

$$\frac{dv}{dx} = \frac{-100g - v^2}{100v}$$

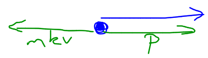
$$\int_0^H dx = -50\int_{30}^{0} \frac{2v}{100g + v^2}$$

$$H = 50\left[\log(100g + v^2)\right]_0^{30}$$

$$= 50\log\left(\frac{100g + 900}{100g}\right)$$

$$= 50\log\frac{19}{10}\ \text{m}$$

13/



$$m\ddot{x} = P - mkv$$

$$\ddot{x} = \frac{P - mkv}{m}$$

$$\frac{dv}{dt} = \frac{P - mkv}{m}$$

$$\int_0^s dt = -\frac{1}{k}\int_2^4 \frac{-mk}{P - mkv}$$

$$S = -\frac{1}{k}\Big[\log(P - mkv)\Big]_2^4$$

$$= \frac{1}{k}\left(\log \frac{P-2mk}{P-4mk}\right)$$

$$e^{sk} = \frac{P - 2mk}{P - 4mk}$$

$$(P - 4mk)e^{sk} = P - 2mk$$

$$P(e^{sk} - 1) = 2mk(2e^{sk} - 1)$$

$$P = \frac{2mk(2e^{sk} - 1)}{e^{sk - 1}}$$

$$v\frac{dv}{dx} = \frac{P - mkv}{m}$$

$$\frac{dv}{dx} = \frac{P - mkv}{mv}$$

$$\int_0^x dx = -\frac{1}{k}\int_2^4 \left[\frac{P - mkv}{P - mkv} - \frac{P}{P - mkv}\right] dv$$

$$X = -\frac{1}{k}\left[v + \frac{P}{mk}\log(P - mkv)\right]_2^4$$

$$= -\frac{1}{k}\left(2 + \frac{P}{mk}\log\left(\frac{P - 4mk}{P - 2mk}\right)\right)$$

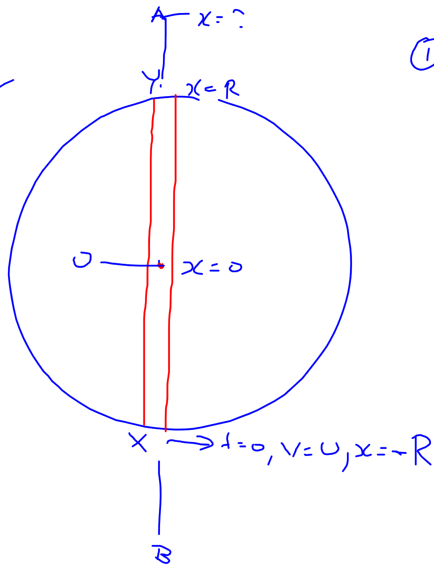
c) $P = 2mk\left[\dfrac{2e^{5k}-1}{e^{5k}-1}\right]$

$k = 0.5$

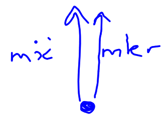
$P = m\left[\dfrac{2e^{2.5}-1}{e^{2.5}-1}\right]$.

$\doteqdot 2.1\,m$

14

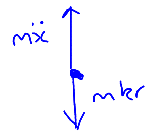


① X to O



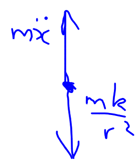
$$\ddot{x} = kr$$

② O to Y



$$\ddot{x} = -kr$$

③ Y to A



$$\ddot{x} = -\frac{k}{r^2}$$

④ A to Y

$$\ddot{x} = \frac{k}{r^2}$$

$$t = -\frac{1}{k}\log(g + kV_y) + c$$

when $t = 0$, $V_y = U\sin\theta$

$$0 = -\frac{1}{k}\log(g + kU\sin\theta) + c$$

$$c = \frac{1}{k}\log(g + kU\sin\theta)$$

$$t = \frac{1}{k}\log\left(\frac{g + kU\sin\theta}{g + kV_y}\right)$$

$$\frac{g + kU\sin\theta}{g + kV_y} = e^{kt}$$

$$(g + kV_y)e^{kt} = g + kU\sin\theta$$

$$g + kV_y = (g + kU\sin\theta)e^{-kt}$$

$$kV_y = (g + kU\sin\theta)e^{-kt} - g$$

$$V_y = \frac{1}{k}(g + kU\sin\theta)e^{-kt} - \frac{g}{k}.$$

$$t = -\frac{1}{k}\log V_x + c$$

$$V_x = U\cos\theta$$

$$0 = -\frac{1}{k}\log U\cos\theta + c$$

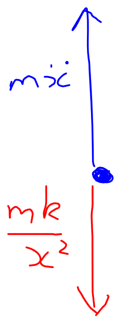
$$c = \frac{1}{k}\log U\cos\theta$$

$$t = \frac{1}{k}\log\left(\frac{U\cos\theta}{V_x}\right)$$

$$\frac{U\cos\theta}{V_x} = e^{kt}$$

$$V_x = U\cos\theta\, e^{-kt}$$

15



$t=0, \; v=U$

$$\ddot{x} = -\frac{k}{x^2}$$

$$v^2 = U^2 - 2gR^2\left(\frac{1}{R} - \frac{1}{x}\right)$$

$$\lim_{x \to \infty} v^2 = 0$$

$$U^2 - 2gR^2\left(\frac{1}{R}\right) = 0$$

$$U^2 = 2gR$$

$\therefore$ particle will escape if $\underline{U^2 \geq 2gR}$

b) $U^2 = 2gR$

$$v^2 = \frac{2gR^2}{x}$$

$$v = \sqrt{\frac{2gR^2}{x}}$$

$$\frac{dx}{dt} = \sqrt{\frac{2gR^2}{x}}$$

$$dt = \sqrt{\frac{x}{2gR^2}}\, dx$$

$$\int_0^T dt = \frac{1}{\sqrt{2g}\,R}\int_R^{2R} x^{\frac{1}{2}}\, dx$$

$$T = \frac{1}{\sqrt{2g}\,R}\left[\frac{2}{3}x\sqrt{x}\right]_R^{2R}$$

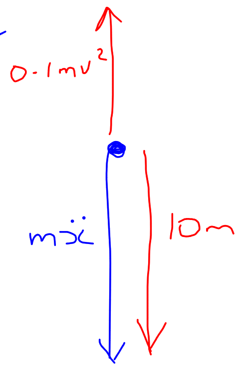
$$= \frac{2}{3\sqrt{2g}\,R}\left\{2R\sqrt{2R} - R\sqrt{R}\right\}$$

$$= \frac{1}{3\sqrt{g}}\left(2\sqrt{2}\sqrt{R} - \sqrt{R}\right)$$

$$= \frac{2\sqrt{2}-1}{3\sqrt{g}}\sqrt{R}.$$

$$\doteqdot 0.273\sqrt{R}$$

16



$$m\ddot{x} = 10m - 0.1mv^2$$

$$\ddot{x} = 10 - \frac{v^2}{10}$$

terminal velocity occurs when $\ddot{x} = 0$

$$0 = 10 - \frac{v^2}{10}$$

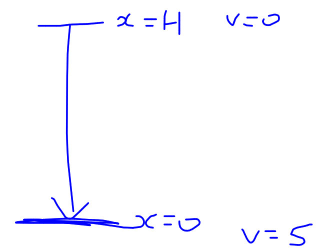
$$v^2 = 100$$

$$\underline{v = 10\ m/s}$$

(iii) $t = 0, \; x = H, \; v = 0$

$x = H, \; v = 0.5U$

$= 5$



$$\ddot{x} = 10 - \frac{v^2}{10}$$

$$v\frac{dv}{dx} = \frac{100 - v^2}{10}$$

$$\int_0^H dx = \int_5^0 \frac{10v}{100 - v^2}\,dv$$

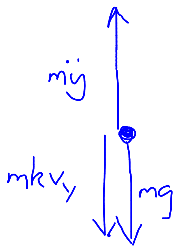
$$H = -5\left[\log(100 - v^2)\right]_5^0$$

$$H = -5\log\left(\frac{4}{3}\right)$$

$$= -5\log\frac{4}{3}\text{ m}$$

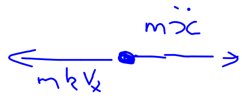
18/

vertical



$$m\ddot{y} = -mg - mkv_y$$
$$\ddot{y} = -g - kv_y$$
$$\frac{dv_y}{dt} = -g - kv_y$$
$$t = -\frac{1}{k}\int \frac{-k\,dv_y}{-g - kv_y}$$
$$= -\frac{1}{k}\int \frac{k\,dv_y}{g + kv_y}$$

horizontal



$$m\ddot{x} = -mkv_x$$
$$\ddot{x} = -kv_x$$
$$\frac{dV_x}{dt} = -kV_x$$
$$t = -\frac{1}{k}\int \frac{dV_x}{V_x}$$
$$= -\frac{1}{k}\log V_x + c$$

1

$$\ddot{x} = kr$$

$$\frac{d}{dr}\left(\frac{1}{2}v^2\right) = kr$$

$$\frac{1}{2}v^2 = \frac{1}{2}kr^2 + c$$

when $r = -R$, $v = U$

$$\frac{1}{2}U^2 = \frac{1}{2}kR^2 + c$$

$$c = \frac{1}{2}U^2 - \frac{1}{2}kR^2$$

$$\underline{V^2 = kr^2 + U^2 - kR^2}$$

when $r = 0$, $V^2 = U^2 - kR^2$

2

$$\ddot{x} = -kr$$

$$\frac{1}{2}v^2 = -\frac{1}{2}kr^2 + c$$

when $r = 0$, $v^2 = U^2 - kR^2$

$$\frac{1}{2}\left(U^2 - kR^2\right) = c$$

$$\underline{V^2 = -kr^2 + U^2 - kR^2}$$

$r = R$, $V^2 = -2kR^2 + U^2$

3 $\ddot{x} = -\dfrac{k}{r^2}$

$$\frac{d}{dr}\left(\frac{1}{2}v^2\right) = -\frac{k}{r^2}$$

$$\frac{1}{2}v^2 = \frac{k}{r} + c$$

when $r = R$, $v^2 = -2kR^2 + U^2$

$$-kR^2 + \frac{1}{2}U^2 = \frac{k}{R} + c$$

$$c = -kR^2 + \frac{1}{2}U^2 - \frac{k}{R}$$

$$v^2 = \frac{2k}{r} - 2kR^2 + U^2 - \frac{2k}{R}$$

when $v = 0$

$$0 = \frac{2k}{r} - 2kR^2 + U^2 - \frac{2k}{R}$$

$$\frac{2k}{r} = \frac{2kR^3 - U^2R + 2k}{R}$$

$$r = \frac{2Rk}{2kR^3 - U^2R + 2k}$$

4 $\ddot{x} = \frac{k}{r^2}$

$\frac{1}{2} v^2 = -\frac{k}{r} + c$

when $r = \frac{-2Rk}{2kR^3 - U^2R + 2k}$, $v = 0$

$0 = \frac{2kR^3 - U^2R + 2k}{2R} + c$

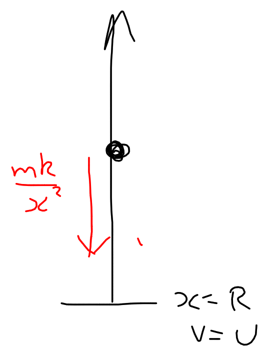
$c = \frac{2kR^3 - U^2R + 2k}{-2R}$

$V^2 = -\frac{2k}{r} - \frac{2kR^3 - U^2R + 2k}{R}$

$r = -R$

$V^2 =$

19/



$$m\ddot{x} = -\frac{mk}{x^2}$$

$$\ddot{x} = -\frac{k}{x^2}$$

$$x = R, \quad \ddot{x} = -g$$

$$-g = -\frac{k}{R^2}$$

$$k = gR^2$$

$$v\frac{dv}{dx} = \frac{-gR^2}{x^2}$$

$$\int_U^0 v\,dv = -gR^2\int_R^x \frac{dx}{x^2}$$

$$\left[v^2\right]_U^0 = \left[\frac{2gR^2}{x}\right]_R^x$$

$$-U^2 = \frac{2gR^2}{x} - 2gR$$

$$-gR = \frac{2gR^2}{x} - 2gR$$

$$\frac{2gR^2}{x} = gR$$

$$\frac{2R}{x} = 1$$

$$\underline{x = 2R}$$

$$\left[v^2 \right]_0^v = \left[\frac{2gR^2}{x} \right]_{2R}^{x}$$

$$v^2 = \frac{2gR^2}{x} - gR$$

$$= \frac{gR(2R-x)}{x}$$

$$\frac{dx}{dt} = \sqrt{\frac{gR(2R-x)}{x}}$$

(upwards motion so $v > 0$)

$$\int_0^t dt = \frac{1}{\sqrt{gR}} \int_R^{2R} \sqrt{\frac{x}{2R-x}}\, dx$$

$$t = \frac{1}{\sqrt{gR}} \int_R^{2R} \frac{x}{\sqrt{2Rx - x^2}}\, dx$$

$$t = \frac{1}{\sqrt{gR}}\left[-\frac{1}{2}\int_{R}^{2R}\frac{-2x+2R}{\sqrt{2Rx-x^2}}\,dx + R\int_{R}^{2R}\frac{dx}{\sqrt{R^2-(x-R)^2}}\right]$$

$$t = \frac{1}{\sqrt{gR}}\left[-\sqrt{2Rx-x^2} + R\sin^{-1}\left(\frac{x-R}{R}\right)\right]_{R}^{2R}$$

$$= \frac{1}{\sqrt{gR}}\left(\frac{R\pi}{2}+R\right)$$

$$= \sqrt{\frac{R}{g}}\left(\frac{\pi}{2}+1\right)$$