PENRITH HIGH SCHOOL

2015
HSC TRIAL EXAMINATION

Mathematics Extension 1

General Instructions:
- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In questions 11 – 14, show relevant mathematical reasoning and/or calculations
- Answer each question on a new sheet of paper

Total marks–70

SECTION I **Pages 3–5**

10 marks
- Attempt Questions 1–10
- Allow about 15 minutes for this section

SECTION II **Pages 6–9**

60 marks
- Attempt Questions 11–14
- Allow about 1 hours 45 minutes for this section

Student Number:____________________ **Teacher Name:**____________________

This paper MUST NOT be removed from the examination room

Assessor: T Bales

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the provided multiple–choice answer sheet for Questions 1–10

1 For $x > 1,\ e^x - \ln x$ is:

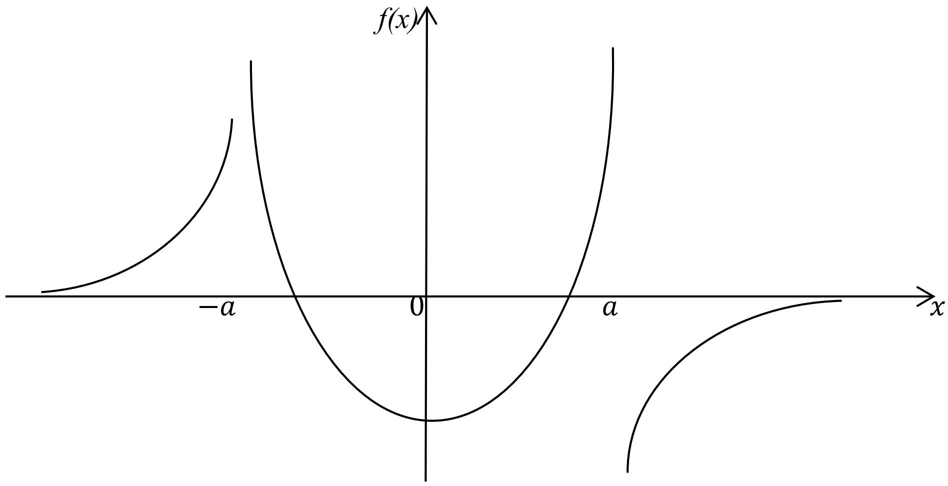
(A) $= 0$

(B) > 0

(C) < 0

(D) $= e$

2 Consider the function $f(x)$ given in the graph below:



Which domain of the function f(x) above is valid for the inverse function to exist?:

(A) $x > 0$

(B) $-a < x < a$

(C) $0 < x < a$

(D) $x < 0$

3 What is the acute angle between the lines $2x - y - 7 = 0$ and $3x - 5y - 2 = 0$?

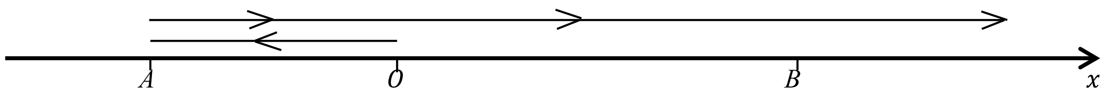
(A) $4°24'$

(B) $32°28'$

(C) $57°32'$

(D) $85°36'$

4 A particle is moving in a straight line with velocity $v\ m/s$ and acceleration $a\ m/s^2$. Initially the particle started moving to the left of a fixed point O. The particle is noticed to be slowing down during the course of the motion from 0 to A. It turns around at A, keeps speeding up for the rest of the course of motion, passing $O$and B and continues. The particle never comes back. Take left to be the negative direction.



During the course of the particle's motion from O to A, which statement of the following is correct?

(A) $v > 0$ and $a > 0$

(B) $v > 0$ and $a < 0$

(C) $v < 0$ and $a > 0$

(D) $v < 0$ and $a < 0$

5 A particle is moving in a straight line with velocity, $v = \frac{1}{1+x}\ m/s$ where x is the displacement of the particle from a fixed point O. If the particle was observed to have reached the position $x = -2\ m$ at a certain moment of time, then this particle:

(A) will definitely reach the position $x = 1m$

(B) may reach the position $x = 1m$

(C) will never reach the position $x = 1m$

(D) will come to rest before reaching $x = 1m$

6 If the rate of change of a function $y = f(x)$ at any point is proportional to the value of the function at that point then the function $y = f(x)$ is a:

(A) Polynomial function

(B) Trigonometric function

(C) Exponential function

(D) Quadratic function

7 Let α and β be any two acute angles such that $\alpha < \beta$. Which of the following statements is correct ?

(A) $sin\alpha < sin\beta$

(B) $cos\alpha < cos\beta$

(C) $cosec\alpha < cosec\beta$

(D) $cot\alpha < cot\beta$

8 Which of the following is a primitive function of $sin^2x + x^2$?

(A) $x - \frac{1}{2}sin2x + \frac{x^3}{3} + c$

(B) $\frac{1}{2}x - \frac{1}{4}sin2x + \frac{x^3}{3} + c$

(C) $x - \frac{1}{2}sin2x + 2x + c$

(D) $\frac{1}{2}x - \frac{1}{4}sin2x + 2x + c$

9 Consider the binomial expansion $(1+x)^n = 1 + n_{C_1}x + n_{C_2}x^2 + \cdots + n_{C_n}x^n$.
Which of the following expressions is correct?

(A) ${}^nc_1 + 2\,{}^nc_2 + \cdots + n\,{}^nc_n = n2^{n-1}$

(B) ${}^nc_1 + 2\,{}^nc_2 + \cdots + n\,{}^nc_n = n2^{n+1}$

(C) ${}^nc_1 + {}^nc_2 + \cdots + {}^nc_n = 2^{n-1}$

(D) ${}^nc_1 + {}^nc_2 + \cdots + {}^nc_n = 2^{n+1}$

10 Using the substitution, $u = 1 + \sqrt{x}$, find the value of $\int_1^4 \frac{1}{(1+\sqrt{x})^2}\frac{1}{\sqrt{x}}dx$ is:

(A) $\frac{6}{5}$

(B) $\frac{1}{3}$

(C) $\frac{2}{3}$

(D) $\frac{3}{2}$

Section II

60 marks
Attempt Questions 11–14
Allow about 1 hour and 45 minutes for this section

Begin each question on a new sheet of paper. Extra sheets of paper are available.

In questions 11–14, your responses should include relevant mathematical reasoning and/or calculations

---

Question 11 (15 marks) Start a new sheet of paper.

(a) $(x+1)$ and $(x-2)$ are factors of $A(x) = x^3 - 4x^2 + x + 6$. Find the third factor. **2**

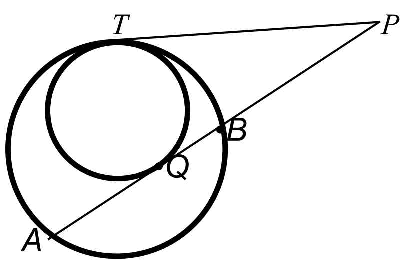
(b) Find the coordinates of the point P which divides the interval AB internally in the ratio $2:3$ with $A(-3, 7)$ and $B(15, -6)$ **3**

(c) Solve the inequality $\frac{1}{4x-1} < 2$, graphing your solution on a number line. **3**

(d) Use the method of mathematical induction to prove that, for all positive integers n: **3**

$$1^2 + 3^2 + 5^2 + 7^2 + \cdots + (2n-1)^2 = \frac{n}{3}(2n-1)(2n+1)$$

(e)



PT is the common tangent to the two circles which touch at T.
PA is the tangent to the smaller circle at Q, intersecting the larger circle at points B and A as shown.

i) State the property which would be used to explain why $PT^2 = PA \times PB$ **1**

ii) If $PT = m, QA = n \; and \; QB = r$, prove that $m = \dfrac{nr}{n-r}$ **3**

Proceed to next page for question (12)

Question 12 (15 marks) Start a new sheet of paper.

(a) The equation $sinx = 1 - 2x$ has a root near $x = 0.3$. Use one application of Newton's methods to find a better approximation, giving your answer correct to 2 decimal places. **3**

(b) Five couples sit at a round table. How many different seating arrangements are possible if:

i) there are no restrictions? **1**

ii) each person sits next to their partner? **2**

(c) In the expansion of $(4 + 2x - 3x^2)\left(2 - \frac{x}{5}\right)^6$, find the coefficient of x^5 **3**

(d) i) Write the binomial expansion for $(1 + x)^n$ **2**

ii) Using part (i), show that $\int_0^3 (1+x)^n \, dx = \sum_{k=0}^{n} \frac{1}{k+1} {}^nC_k 3^{k+1}$ **2**

iii) Hence show that $\sum_{k=0}^{n} \frac{1}{k+1} {}^nC_k 3^{k+1} = \frac{1}{n+1}\left[4^{n+1} - 1\right]$ **2**

Proceed to next page for question (13)

Question 13 (15 marks) Start a new sheet of paper.

(a) Use the substitution $u = \frac{x}{\sqrt{1-x^2}}$ to show that $\frac{d}{dx}\left[tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)\right] = \frac{1}{\sqrt{1-x^2}}$ **3**

(You can use the result that $\int \frac{1}{a^2+x^2} dx = \frac{1}{a} tan^{-1}\frac{x}{a} + c$. You do not need to prove this).

(b) Give the exact value for $\int_{-\sqrt{3}}^{3} \frac{dx}{3+x^2}$ **3**

(c) The distinct points P, Q have parameters $t = t_1$ and $t = t_2$ respectively on the parabola $x = 2t, y = t^2$. The equations of the tangents to the parabola at P and Q respectively are given by:
$y - t_1x + t_1{}^2 = 0$ and $y - t_2x + t_2{}^2 = 0$ (You do not need to prove these)

i) Show that the equation of the chord $PQ\ is\ 2y - (t_1 + t_2)x + 2t_1t_2 = 0$ **2**

ii) Show that M, the point of intersection of the tangents to the parabola at P and Q, has coordinates $(t_1 + t_2\ ,\ t_1t_2)$. **2**

iii) $\propto$) Prove that for any value of t_1, except $t_1 = 0$, there are exactly two values of t_2 for which M lies on the parabola $x^2 = -4y$. **3**

β) Find these two values of t_2 in terms of t_1. **2**

Proceed to next page for question (14)

Question 14 (15 marks) Start a new sheet of paper.

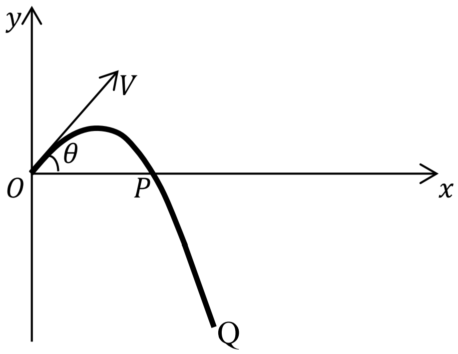
(a) A particle P is moving in simple harmonic motion on the x axis, according to the law $x = 4sin3t$ where x is the displacement of P in centimetres from O at time t seconds.

i) State the period and amplitude of the motion. **2**

ii) Find the first time when the particle is 2cm to the positive side of the origin and it's velocity at this time. **2**

iii) Find the greatest speed and greatest acceleration of P **3**

(b)



A projectile is fired from O, with speed Vms^{-1}, at an angle of elevation of θ to the horizontal. After t seconds, its horizontal and vertical displacements from O (as shown) are x metres and y metres, repectively.

i) Prove that $x = Vtcos\theta$ and $y = -\frac{1}{2}gt^2 + Vtsin\theta$ **3**

ii) Show that the time taken to reach P is given by $t = \dfrac{2Vsin\theta}{g}$ **2**

iii) The projectile falls to Q, where its angle of depression from O is θ. Prove that, in its flight from O to Q, P is the half-way point in terms of time. **3**

End of paper

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1;\ x \neq 0,\ if\ n < 0$$

$$\int \frac{1}{x}\,dx = \ln x,\ x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\ a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\ a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\ a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\ a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax,\ a \neq 0$$

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a},\ a \neq 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \ln(x + \sqrt{x^2 - a^2}),\ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}\,dx = \ln(x + \sqrt{x^2 + a^2})$$

NOTE: $\ln x = \log_e x,\ x > 0$

Student Number:_____________ Teacher Name:________________

Section I - Multiple Choice

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ~~●~~ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.

correct

A ~~●~~ B ~~●~~ C ○ D ○

Start Here ➔

1. A ○ B ○ C ○ D ○

2. A ○ B ○ C ○ D ○

3. A ○ B ○ C ○ D ○

4. A ○ B ○ C ○ D ○

5. A ○ B ○ C ○ D ○

6. A ○ B ○ C ○ D ○

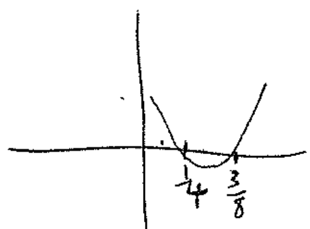
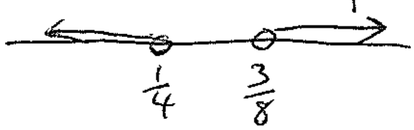
7. A ○ B ○ C ○ D ○

8. A ○ B ○ C ○ D ○

9. A ○ B ○ C ○ D ○

10. A ○ B ○ C ○ D ○

Exam Trial Mathematics Ext MATHEMATICS : Question 11

Suggested Solutions	Marker's Comments
a) $P(3) = 27 - 4\times 9 + 3 + 6$ $= 0$ $\therefore (x-3)$ is third factor.	A lot of students found by polynomial division
b) $A(-3,7)$ $B(15,-6)$ $M:N$ $2:3$ $x = \frac{3(-3)+2(15)}{5} = \frac{21}{5}$ $y = \frac{3(7)+2(-6)}{5} = \frac{9}{5}$ $P\left(\frac{21}{5}, \frac{9}{5}\right)$	Extremely well done. problems occurred with getting signs wrong
c) $\frac{1}{4x-1} < 2$ $4x-1 < 2(4x-1)^2$ $2(4x-1)^2 - (4x-1) > 0$ $(4x-1)[2(4x-1)-1] > 0$ $(4x-1)(8x-3) > 0$  $\therefore x < \frac{1}{4}$ ~~and~~ or $x > \frac{3}{8}$. 	students need to ensure a 'number line' is represented at the end and not shaded regions on a number plane
Mult. Choice 1) B 2) C 3) B 4) C 5) C 6) C 7) A 8) B 9) A 10) B	

Exam	MATHEMATICS : Question......	
	Suggested Solutions	Marker's Comments

a) Test for $n=1$.

$L.H.S = [2(1)-1]^2$
$= 1$

$R.H.S = \frac{1}{3}(1)(3)$
$= 1$

$\therefore$ True for $n=1$

Assume true for $n=k$

$1^2+3^2+5^2+\cdots+(2k-1)^2 = \frac{k}{3}(2k-1)(2k+1)$

Test for $n=k+1$

that is we are required to prove

$\underbrace{1^2+3^2+5^2+\cdots+(2k-1)^2}+(2k+1)^2 = \frac{k+1}{3}(2k+1)(2k+3)$

$L.H.S = \frac{k}{3}(2k-1)(2k+1)+(2k+1)^2$ by induction assumption

$= (2k+1)\left[\frac{k}{3}(2k-1)+(2k+1)\right]$

$= (2k+1)\left[\frac{2k^2-k}{3}+(2k+1)\right]$

$= (2k+1)\left[\frac{2k^2-k+6k+3}{3}\right]$

$= (2k+1)(\frac{1}{3})(2k^2+5k+3)$

$= \frac{1}{3}(2k+1)(k+1)(2k+3)$

$= \frac{k+1}{3}(2k+1)(2k+3)$

$= R.H.S$ $\therefore$ proved by mathematical induction

well done, some students didn't realise $S_{k+1} = S_k + T_{k+1}$ and tried to prove $S_{k+1} = T_{k+1}$

e(i) tangent-secant theorem.

Only brief answer was required.

(ii) $PT=m$ $QA=n$ $QB=r$. $PT^2 = PA \times PB$.

$PT=PQ$ (equal tangents from an external point)

$\therefore PQ = m$

$PA = PQ+PA$ $PB = PQ - QB$
$= m+n$ $= m-r$

$\therefore m^2 = (m+n)(m-r) \Rightarrow \cancel{m^2} = \cancel{m^2}+mn-rm-rn$

$m(n-r) = rn$ $m = \frac{nr}{n-r}$

students needed to state reason why $PT=PQ$

Exam Trial Mathematics Ext 1 MATHEMATICS : Question 12

Suggested Solutions	Marker's Comments
a) $\sin x = 1-2x$ $f(x): \sin x + 2x - 1 = 0$ $f'(x): \cos x + 2$ $f(a_1) = a - \frac{f(a)}{f'(a)}$ $= 0.3 - \frac{\sin 0.3 + 0.6 - 1}{\cos 0.3 + 2}$ $= 0.33535$ $= 0.34$ to 2 decimal places.	- some students forgot to rearrange to $=0$. - some student put equation around the wrong way so that the numerator and denominator gave a negative overall value.
b)(i) $9!$ (ii) $4! \times 2^5$	- students forgot that all 5 pairs could be swapped between the pair.
c) $(4+2x-3x^2)(2-\frac{x}{5})^6$ General term for $(2-\frac{x}{5})^6$: ${}^6C_k\, 2^{6-k}(-1)^k(\frac{x}{5})^5$ ${}^6C_k\, 2^{6-k} \cdot 5^{-k}(-1)^k x^k$ when $k=5$. multiply by 4 (other brackets) $-{}^6C_5\, 2 \cdot 5^{-5}(4) = \boxed{\frac{-48}{3125}}$ when $k=4$ multiply by 2 (other brackets) ${}^6C_4\, 2^2 5^{-4}(2) = \boxed{\frac{120}{625}}$ when $k=3$ multiply by (-3) (other brackets) $-{}^6C_3\, 2^3 5^{-3}(-3) = \boxed{\frac{480}{125}}$ $\frac{-48}{3125} + \frac{120}{625} + \frac{480}{125} = \frac{12552}{3125}$ or 4.01664	General problems with signs.

Exam Trial Mathematics Ext 1 MATHEMATICS : Question 12

Suggested Solutions	Marker's Comments
d(i) $(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \cdots \binom{n}{n}x^n.$	well done
(ii) $\int_0^3 (1+x)^n\,dx = \int_0^3 \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \cdots \binom{n}{n}x^n\,dx$ $= \left[\binom{n}{0}x + \binom{n}{1}\frac{x^2}{2} + \binom{n}{3}\frac{x^3}{3} + \cdots \binom{n}{n}\frac{x^{n+1}}{n+1}\right]_0^3$ $= \left(3\binom{n}{0} + \binom{n}{1}\frac{3^2}{2} + \binom{n}{2}\frac{3^3}{3} + \cdots \binom{n}{n}\frac{3^{n+1}}{n+1}\right) - 0$ $= \sum_{k=0}^{n} \frac{1}{k+1}\,{}^nC_k\,3^{k+1}$	students needed to demonstrate integration
(iii) $\int_0^3 (1+x)^n\,dx = \left[\frac{(1+x)^{n+1}}{n+1}\right]_0^3$ $= \frac{4^{n+1}}{n+1} - \frac{1}{n+1} = \frac{1}{n+1}\left[4^{n+1} - 1\right]$ $\therefore$ from (ii)(iii) $\sum_{k=0}^{n} \frac{1}{k+1}\,{}^nC_k\,3^{k+1} = \frac{1}{n+1}\left[4^{n+1} - 1\right]$	if students realised method they were successful.

Exam Trial Maths Ext 1 MATHEMATICS : Question 13.	
Suggested Solutions	Marker's Comments

a) Let $u = \dfrac{x}{\sqrt{1-x^2}}$ Let $y = \tan^{-1}\left(\dfrac{x}{\sqrt{1-x^2}}\right)$

$$\boxed{\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}}$$

$$\frac{dy}{du} = \frac{d}{du}(\tan^{-1} u) = \frac{1}{1+u^2}$$

$$\frac{dy}{du} = \frac{1}{1+\frac{x^2}{1-x^2}} = \frac{1}{\frac{1-x^2+x^2}{1-x^2}} = 1-x^2$$

$$u = x(1-x^2)^{-\frac{1}{2}}$$

$$\frac{du}{dx} = x \times \frac{1}{2}(1-x^2)^{-\frac{3}{2}} \times -2x + 1(1-x^2)^{-\frac{1}{2}}$$

$$= \frac{x^2}{(\sqrt{1-x^2})^3} + \frac{1}{\sqrt{1-x^2}} = \frac{x^2+1-x^2}{(\sqrt{1-x^2})^3}$$

$$\frac{dy}{dx} = (1-x^2) \times \frac{1}{(\sqrt{1-x^2})^3} = \boxed{\frac{1}{\sqrt{1-x^2}}}$$

Marker's comments:
- Some students did not use the given substitution which was required.
- Some students did not seem familiar with $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
- Note: If you use Product rule $u=x, v=(1-x^2)^{-\frac{1}{2}}$. For quotient rule $u=x, v=(1-x^2)^{\frac{1}{2}}$. Students were mixing these up.

b) $$\int_{-\sqrt{3}}^{3} \frac{1}{3+x^2}\,dx = \frac{1}{\sqrt{3}}\left[\tan^{-1}\frac{x}{\sqrt{3}}\right]_{-\sqrt{3}}^{3}$$

$$= \frac{1}{\sqrt{3}}\left(\tan^{-1}\frac{3}{\sqrt{3}} - \tan^{-1}\frac{-\sqrt{3}}{3}\right)$$

$$= \frac{1}{\sqrt{3}}\left(\tan^{-1}\sqrt{3} - \tan^{-1}(-1)\right)$$

$$= \frac{1}{\sqrt{3}}\left(\frac{\pi}{3} - -\frac{\pi}{4}\right)$$

$$= \frac{7\pi}{12\sqrt{3}} \left(\text{or } \frac{7\pi\sqrt{3}}{36}\right)$$

Marker's comments:
- This question was well answered.
- Some students did not know the range of $\tan^{-1}$ to be $-\frac{\pi}{2}$ to $\frac{\pi}{2}$

Exam Trial Maths Ext1	MATHEMATICS	: Question....13c

Suggested Solutions	Marker's Comments
13c i. $m = \dfrac{t_2^2 - t_1^2}{2t_2 - 2t_1} = \dfrac{(t_2+t_1)(t_2-t_1)}{2(t_2-t_1)} = \dfrac{t_2+t_1}{2}$ $y - t_1^2 = \dfrac{(t_1+t_2)}{2}(x - 2t_1)$ $2y - \cancel{2t_1^2} = (t_1+t_2)x - \cancel{2t_1^2} - 2t_1t_2$ $2y - (t_1+t_2)x + 2t_1t_2 = 0$	• keep (t_1+t_2) in brackets to avoid complicated algebraic manipulation
ii. $y = t_1x - t_1^2$ $y = t_2x - t_2^2$ $t_1x - t_1^2 = t_2x - t_2^2$ $t_1x - t_2x = t_1^2 - t_2^2$ $x\cancel{(t_1 - t_2)} = (t_1+t_2)\cancel{(t_1-t_2)}$ $x = t_1 + t_2$ sub into $y = t_1x - t_1^2$ $y = t_1(t_1+t_2) - t_1^2$ $= \cancel{t_1^2} + t_1t_2 - \cancel{t_1^2}$ $y = t_1t_2$ $M(t_1+t_2, t_1t_2)$	
iii. $x^2 = -4y$ $(t_1+t_2)^2 = -4(t_1t_2)$ $t_1^2 + 2t_1t_2 + t_2^2 = -4t_1t_2$ $t_1^2 + 6t_1t_2 + t_2^2 = 0$ $\Delta = 36t_1^2 - 4.1.t_1^2$ $= 32t_1^2$ $\Delta > 0$ for all t_1 $\therefore$ 2 values for all t_2	• Some students didn't know where to start. • sub M into $x^2 = -4y$ • Use discriminant to show 2 values ie $\Delta > 0$

Exam Ext 1 Maths	MATHEMATICS	: Question...... 13 c iii β

Suggested Solutions	Marker's Comments
$t_2 = \frac{-6t_1 \pm \sqrt{32t_1^2}}{2}$ $= \frac{-6t_1 \pm 4\sqrt{2}\,t_1}{2}$ $= -3t_1 \pm 2\sqrt{2}\,t_1$ $= (-3 \pm 2\sqrt{2})t_1$	Some students found t_1 in terms of t_2

Exam Trial Maths Ext1 MATHEMATICS : Question 14a

Suggested Solutions	Marker's Comments

Suggested Solutions

a i $x = 4\sin 3t$

amplitude $= 4$

period $= \frac{2\pi}{n} = \frac{2\pi}{3}$

ii $2 = 4\sin 3t$

$\frac{1}{2} = \sin 3t$

$3t = \frac{\pi}{6}$

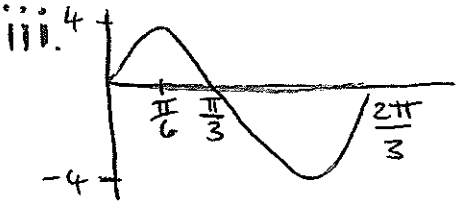
$t = \frac{\pi}{18}$

$\dot{x} = 12\cos 3t$

$= 12\cos 3\left(\frac{\pi}{18}\right)$

$= 6\sqrt{3}$

iii.



max velocity when $t = 0$

$\dot{x} = 12\cos 3(0)$

$= 12\text{ cms}^{-1}$

max acceleration when $t = \frac{\pi}{2}$
(positive direction

$\ddot{x} = -36\sin 3t$

$= 36\text{cm}^{-2}$

Marker's Comments

- errors were made differentiating $x = 4\sin 3t$
- common error $t = \frac{\pi}{18} = 10\text{sec}$
- errors differentiat $\dot{x} = 12\cos 3t$
- using degrees instead of radians
- -36cms^{-2} was accepted

Exam Trial Maths Ext I MATHEMATICS : Question...... 14 b

Suggested Solutions	Marker's Comments
	• errors were common when evaluating constants

bi. $\ddot{x} = 0$

$\dot{x} = c_1$

when $t=0$ $\dot{x} = v\cos\theta$

$\therefore c_1 = v\cos\theta$

$\dot{x} = v\cos\theta$

$x = vt\cos\theta + c_2$

when $t=0$ $x=0$ $\therefore c_2 = 0$

$x = vt\cos\theta$

$\ddot{y} = -g$

$\dot{y} = -gt + c_3$

when $t=0$ $\dot{y} = v\sin\theta$

$\therefore c_3 = v\sin\theta$

$\dot{y} = -gt + v\sin\theta$

$y = -\frac{gt^2}{2} + vt\sin\theta + c_4$

when $t=0$ $y=0$ $\therefore c_4 = 0$

$y = -\frac{gt^2}{2} + vt\sin\theta$

bii. $P \rightarrow y=0$

$0 = -\frac{1}{2}gt^2 + vt\sin\theta$

$0 = t\left(v\sin\theta - \frac{gt}{2}\right)$

$t=0$ or $v\sin\theta - \frac{gt}{2} = 0$

$\frac{gt}{2} = v\sin\theta$

$t = \frac{2v\sin\theta}{g}$

iii. $\tan\theta = \frac{-y}{x}$

$\tan\theta = \frac{-(vt\sin\theta - \frac{1}{2}gt^2)}{vt\cos\theta}$

$\tan\theta = -\tan\theta + \frac{gt}{2v\cos\theta}$

$2\tan\theta = \frac{gt}{2v\cos\theta}$

$\frac{4v\sin\theta}{g} = t$ which is twice $\frac{2v\sin\theta}{g}$

Marker's Comments: • be careful quoting the cartesian eq'n for the trajectory (a method some used). You may need to derive it.