CANDIDATE NUMBER

SYDNEY GRAMMAR SCHOOL

2015 Trial Examination

FORM VI

MATHEMATICS EXTENSION 1

Wednesday 5th August 2015

General Instructions

- Writing time — 2 hours
- Write using black or blue pen.
- Board-approved calculators and templates may be used.
- A list of standard integrals is provided at the end of the examination paper.

Total — 70 Marks

- All questions may be attempted.

Section I – 10 Marks

- Questions 1–10 are of equal value.
- Record your answers to the multiple choice on the sheet provided.

Section II – 60 Marks

- Questions 11–14 are of equal value.
- All necessary working should be shown.
- Start each question in a new booklet.

Collection

- Write your candidate number on each answer booklet and on your multiple choice answer sheet.
- Hand in the booklets in a single well-ordered pile.
- Hand in a booklet for each question in Section II, even if it has not been attempted.
- If you use a second booklet for a question, place it inside the first.
- Write your candidate number on this question paper and hand it in with your answers.
- Place everything inside the answer booklet for Question Eleven.

Checklist

- SGS booklets — 4 per boy
- Multiple choice answer sheet
- Candidature — 112 boys

Examiner

PKH

SECTION I - Multiple Choice

Answers for this section should be recorded on the separate answer sheet handed out with this examination paper.

QUESTION ONE

Which of the following is an odd function?

(A) $f(x) = \tan^{-1} x$

(B) $f(x) = \cos x$

(C) $f(x) = \sin(x - \dfrac{\pi}{4})$

(D) $f(x) = \cos^{-1} x$

QUESTION TWO

Suppose θ is the acute angle between the lines $y - 2x = 3$ and $3y = -x + 2$. Which of the following is the value of $\tan\theta$?

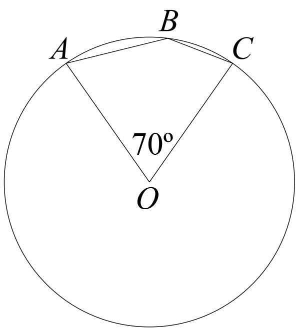
(A) 7

(B) -7

(C) 1

(D) -1

QUESTION THREE



What is the size of $\angle ABC$?

(A) 110°

(B) 145°

(C) 140°

(D) 130°

Exam continues next page ...

QUESTION FOUR

What is the inverse function of $f(x) = x^2 + 1$ for $x \leq 0$?

(A) $f^{-1}(x) = -\sqrt{x-1}$, for $x \leq 0$

(B) $f^{-1}(x) = \sqrt{x-1}$, for $x \leq 0$

(C) $f^{-1}(x) = -\sqrt{x-1}$, for $x \geq 1$

(D) $f^{-1}(x) = \sqrt{x-1}$, for $x \geq 1$

QUESTION FIVE

Find $\displaystyle\lim_{x \to 2} \frac{x^2 + x - 6}{x - 2}$.

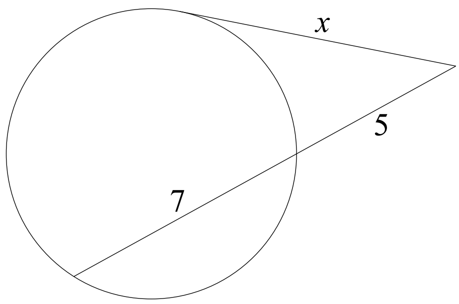
(A) ∞

(B) $-\infty$

(C) -5

(D) 5

QUESTION SIX



Find the length of x.

(A) $\sqrt{35}$

(B) $\sqrt{12}$

(C) $\sqrt{60}$

(D) $\sqrt{84}$

QUESTION SEVEN

If $f(x) = \tan^{-1}\dfrac{1}{x}$, find $f'(x)$.

(A) $\dfrac{x^2}{1+x^2}$

(B) $-\dfrac{1}{1+x^2}$

(C) $\dfrac{1}{1-x^2}$

(D) $-\dfrac{x^2}{1-x^2}$

QUESTION EIGHT

How many solutions does the equation $x^{\frac{1}{3}} = |x-2| - 3$ have?

(A) 0

(B) 1

(C) 2

(D) 3

QUESTION NINE

The parametric form of a parabola is $(6t, -3t^2)$. Its focal length is:

(A) $\dfrac{1}{4}$

(B) $-\dfrac{1}{4}$

(C) -3

(D) 3

QUESTION TEN

The polynomial $P(x)$ has degree 4 and the polynomial $Q(x)$ has degree 2. If you divide $P(x)$ by $Q(x)$, the remainder has degree:

(A) 1

(B) 2

(C) 0 or 1

(D) 0, 1 or 2

End of Section I

Exam continues next page ...

SECTION II - Written Response

Answers for this section should be recorded in the booklets provided.

Show all necessary working.

Start a new booklet for each question.

QUESTION ELEVEN (15 marks) Use a separate writing booklet. **Marks**

(a) Let $A = (-1, 4)$ and $B = (5, -5)$. Find the co-ordinates of the point P which divides interval AB in the ratio $1 : 2$. **2**

(b) Solve the inequation $\dfrac{x}{2x+1} < 2$. **2**

(c) Sketch the graph of $y = 2\cos^{-1}(x-1)$, clearly marking the domain and range. **2**

(d) Differentiate $e^{\tan x} \ln x$. **2**

(e) Find the coefficient of a^3 in the expansion of $(2a-1)^{20}$. **2**

(f) Taking $x = 1{\cdot}4$ as a first approximation, use one application of Newton's method to find a better approximation to $3\sin 2x - x = 0$.
Give your answer correct to 3 significant figures. **2**

(g) (i) Prove that $\dfrac{\sin 2A}{1 - \cos 2A} = \cot A$. **1**

(ii) Hence find the values of a and b if $\cot \dfrac{3\pi}{8} = a + \sqrt{b}$ for integers a and b. **2**

QUESTION TWELVE (15 marks) Use a separate writing booklet. **Marks**

(a) Use the substitution $u = \tan x$ to evaluate $\displaystyle\int \frac{\sec^2 x}{\tan^2 x + 3}\, dx$. **2**

(b) Prove by Mathematical Induction that, for $n \geq 1$, **3**

$$\frac{1 \times 2^0}{2 \times 3} + \frac{2 \times 2^1}{3 \times 4} + \frac{3 \times 2^2}{4 \times 5} + \ldots + \frac{n\, 2^{n-1}}{(n+1)(n+2)} = \frac{2^n}{n+2} - \frac{1}{2}\,.$$

(c) Find the area bounded by $y = \dfrac{1}{\sqrt{1 - 9x^2}}$, the line $x = 0$, the line $x = \dfrac{\sqrt{3}}{6}$ and the x-axis. **3**

(d) Consider the function $y = x^2 + \dfrac{16}{x}$.

(i) Find $\dfrac{dy}{dx}$. **1**

(ii) Find the co-ordinates of any stationary points and determine their nature. **2**

(iii) Show that there is a point of inflexion at the x-intercept. **2**

(iv) Sketch the graph $y = x^2 + \dfrac{16}{x}$, showing the above information. **2**

Exam continues next page ...

QUESTION THIRTEEN (15 marks) Use a separate writing booklet. **Marks**

(a) Find $\displaystyle\lim_{x \to 0} \frac{\sin ax}{x}$. **1**

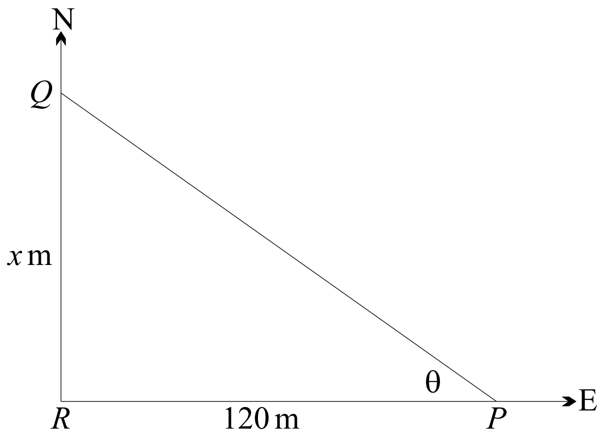
(b) (i) Show that $\ddot{x} = \dfrac{d}{dx}(\dfrac{1}{2}v^2)$. **1**

(ii) If $v^2 = 24 - 6x - 3x^2$, find the acceleration of the particle at the particle's greatest displacement from the origin. **3**

(c) Let α, β and γ be the roots of the equation $x^3 - px + q = 0$. In terms of p and q find an expression for $\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma}$. **2**

(d) Show that $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = \pi$. **2**

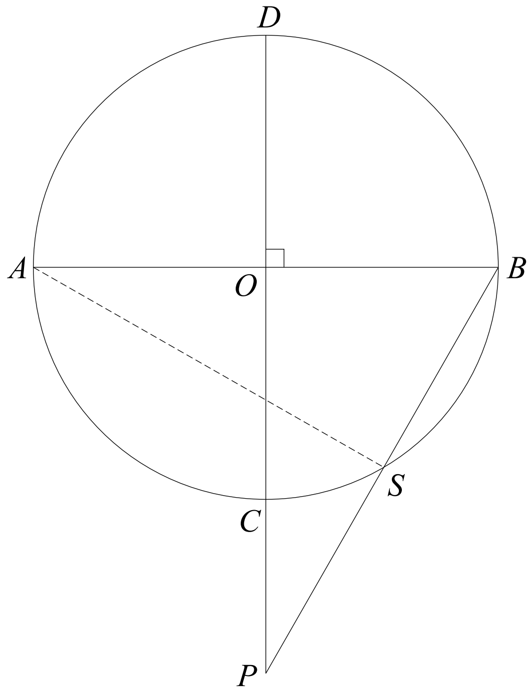
(e) **3**



An observer stands at P, 120 metres East of R. A second person is at Q, x metres due North of R and continues to move North. Let angle $RPQ = \theta$. Suppose θ is changing at 0·2 radians/minute.

Find the rate at which x is changing when $x = 90$ metres.

(f)



Two diameters AB and CD of a circle, with centre O, are at right angles. Diameter DC is produced to P and PB cuts the circle again at S.

(i) Prove that $AOSP$ is a cyclic quadrilateral. **1**

(ii) Prove that $\angle BCS = \angle SPO$. **2**

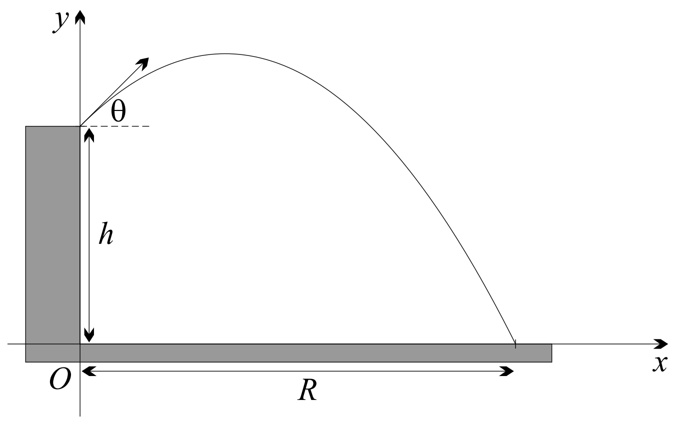
QUESTION FOURTEEN (15 marks) Use a separate writing booklet. **Marks**

(a) Consider the function $f(x) = \ln(x^{\frac{1}{x}})$, for $x > 0$.

(i) Show that $f'(x) = \dfrac{1}{x^2}(1 - \ln x)$. **2**

(ii) Find the range of $f(x)$, giving full reasons. **3**

(b)



A projectile is fired from the top of a cliff of height h above a horizontal plane with initial speed V at an angle of elevation θ. The horizontal range of the projectile is R. The magnitude of the gravitational acceleration of the projectile is g. Take the origin at the base of the cliff directly below the launch point of the projectile.
It is known that the vertical and horizontal displacements satisfy

$$x = V\cos\theta\, t \quad \text{and} \quad y = h + V\sin\theta\, t - \tfrac{1}{2}gt^2.$$

(i) Show that the Cartesian equation of motion is **2**

$$y = h + x\tan\theta - \frac{gx^2}{2V^2}\sec^2\theta.$$

(ii) Show that $R^2\sec^2\theta - 2R\dfrac{V^2}{g}\tan\theta - 2h\dfrac{V^2}{g} = 0$. **2**

(iii) Show that $R^2 = \left(\dfrac{V^4}{g^2} + 2h\dfrac{V^2}{g}\right) - \left(R\tan\theta - \dfrac{V^2}{g}\right)^2$. **2**

(iv) Deduce that the maximum range is $\dfrac{1}{g}\sqrt{V^4 + 2hV^2g}$. **1**

(v) Show that the angle of elevation satisfies $\tan\theta = \dfrac{V^2}{gR_1}$ where R_1 is the maximum range. **1**

(vi) Show that $\tan 2\theta = \dfrac{R_1}{h}$. **2**

End of Section II

END OF EXAMINATION

SOLUTION TO FORM VI SGS
TRIAL AUGUST 2015

1 A

2 $y - 2x = 3$ $\quad\quad$ $3y = -x + 2$

$y = 2x + 3$ $\quad\quad$ $y = -\frac{1}{3}x + \frac{2}{3}$

So $m_1 = 2$ and $m_2 = -\frac{1}{3}$

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\tan\theta = \left|\frac{2 + \frac{1}{3}}{1 - \frac{2}{3}}\right| = \left|\frac{\frac{7}{3}}{\frac{1}{3}}\right| = 7$$

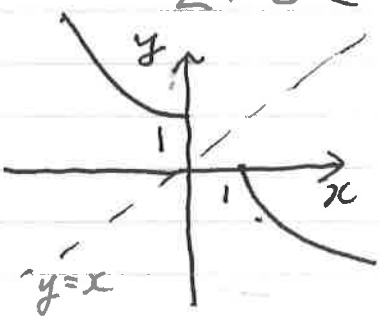
A

3 $\angle AOC = 70^\circ$

$\angle AOC(\text{reflex}) = 290^\circ$

$\angle ABC = 145^\circ$

B

4 

Let $y = x^2 + 1$ $\quad x \leq 0$, $y \geq 1$

Swap x and y

$x = y^2 + 1$

$y = \pm\sqrt{x-1}$

$f(x) = -\sqrt{x-1}$, $x \geq 1$

C

5 $$\lim_{x \to 2} \frac{(x+3)(x-2)}{x-2}$$

$= 5$

D

(2)

6. $x^2 = 12 \times 5$

$x = \sqrt{60}$

C

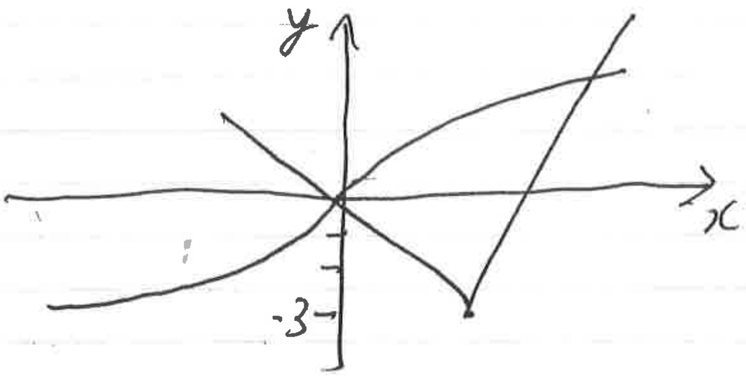
7. $y = \tan^{-1}\frac{1}{x}$

$\frac{dy}{dx} = \frac{1}{1+\frac{1}{x^2}} \times -\frac{1}{x^2}$ (using the chain rule)

$= -\frac{1}{x^2+1}$

B

8



Two solutions

C

9 $P(6t, -3t^2)$

$x = 6t \qquad y = -3t^2$

$t = \frac{x}{6} \qquad y = -3\frac{x^2}{36}$

$x^2 = -12y$

$x^2 = -4(3)y$

Focal length is 3

D

10 C

11

(a) $\quad \underset{(x_1, y_1)}{A(-1, 4)} \quad \underset{(x_2, y_2)}{B(5, -5)} \quad m:n = 1:2$

$$P = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right)$$

$$P = \left(\frac{1 \times 5 + 2 \times -1}{3}, \frac{1 \times -5 + 2 \times 4}{3} \right)$$

$$= (1, 1)$$

one mark for each co-ordinate.

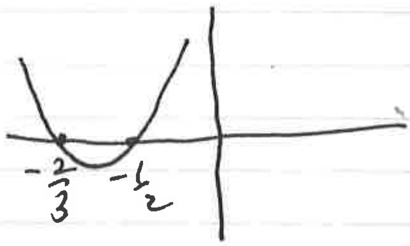
(b)
$$\frac{x}{2x+1} < 2$$

$$(2x+1)x < 2(2x+1)^2$$

$$2(2x+1)^2 - (2x+1)x > 0$$

$$(2x+1)(4x+2-x) > 0$$

$$(2x+1)(3x+2) > 0$$



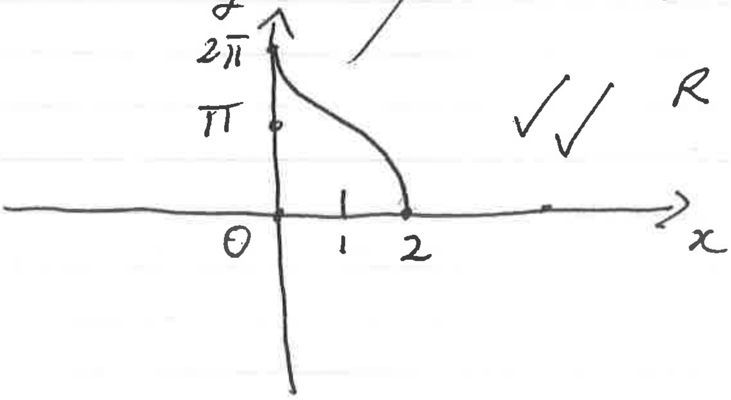
$$x < -\frac{2}{3} \text{ or } x > -\frac{1}{2}$$

(c)
$$y = 2\cos^{-1}(x-1)$$

$D: -1 \leqslant x-1 \leqslant 1$

$\quad\; 0 \leqslant x \leqslant 2$

$R: \; 0 \leqslant y \leqslant 2\pi$



(d) Let $y = e^{\tan x} \ln x$

$$\frac{dy}{dx} = e^{\tan x} \times \frac{1}{x} + e^{\tan x} \sec^2 x \ln x$$

(e) $(2a-1)^{20}$

General term is ${}^{20}C_r\,(2a)^{20-r}(-1)^r$

$= {}^{20}C_r\;2^{20-r}(-1)^r\;a^{20-r}$ ✓

Term in a^3 is ${}^{20}C_{17}\;2^3(-1)^{17}\;a^3$ ✓

$= {}^{20}C_3\;2^3 \times (-1)^{17}\;a^3$

Coeffuent is $-1 \times {}^{20}C_3 \times 8$

$= -9120$

(f) Let $f(x) = 3\sin 2x - x$

$f'(x) = 6\cos 2x - 1$

$x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}$ ✓

$x_1 = 1.4 - \dfrac{3\sin 2.8 - 1.4}{6\cos 2.8 - 1}$

$= 1.34$ ✓

(g) $LHS = \dfrac{2\sin A\cos A}{1-(1-2\sin^2 A)}$ — *

$= \dfrac{\cos A}{\sin A} = \cot A$ ✓

Using $\dfrac{\sin 2A}{1-\cos 2A} = A$ with $A = \dfrac{3\pi}{8}$

$\cot\dfrac{3\pi}{8} = \dfrac{\sin\frac{3\pi}{4}}{1-\cos\frac{3\pi}{4}}$ ✓

$= \dfrac{\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}} = \dfrac{1}{\sqrt{2}+1}$

$= \sqrt{2}-1$ So $\left.\begin{array}{l} a=-1 \\ b=2 \end{array}\right\}$ ✓

12

(a) $I = \int \frac{\sec^2 x}{\tan^2 x + 3}\, dx$

Let $u = \tan x$

$du = \sec^2 x\, dx$ ✓

$I = \int \frac{1}{u^2+3}\, du$

$= \frac{1}{\sqrt{3}} \tan^{-1} \frac{u}{\sqrt{3}} + c$ ✓

$= \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{\tan x}{\sqrt{3}}\right)$

(b) $\frac{1\times 2^0}{2\times 3} + \frac{2\times 2^1}{3\times 4} + \dots \frac{n\,2^{n-1}}{(n+1)(n+2)} = \frac{2^n}{n+2} - \frac{1}{2}$

When $n=1$, $LHS = \frac{1\times 2^0}{2\times 3} = \frac{1}{6}$

$RHS = \frac{2^1}{3} - \frac{1}{2} = \frac{1}{6}$

So true when $n = 1$.

Assume true when $n = K$

ie. $\frac{1\times 2^0}{2\times 3} + \frac{2\times 2^1}{3\times 4} + \dots \frac{K\,2^{K-1}}{(K+1)(K+2)} = \frac{2^K}{K+2} - \frac{1}{2}$

RTP true when $n = K+1$

ie $\frac{1\times 2^0}{2\times 3} + \frac{2\times 2^1}{3\times 4} + \dots \frac{K\,2^{K-1}}{(K+1)(K+2)} + \frac{(K+1)\,2^K}{(K+2)(K+3)} = \frac{2^{K+1}}{K+3} - \frac{1}{2}$

$LHS = \frac{2^K}{K+2} - \frac{1}{2} + \frac{(K+1)\,2^K}{(K+2)(K+3)}$ (by assumption)

$= \frac{(K+3)2^K + (K+1)\,2^K}{(K+2)(K+3)} - \frac{1}{2}$

$= \frac{2^K(2K+4)}{(K+2)(K+3)} = \frac{2^{K+1}}{K+3} - \frac{1}{2} = RHS$

Hence true by Mathematical Induction

(c) $A = \int_0^{\frac{\sqrt{3}}{6}} \frac{1}{\sqrt{1-(3x)^2}}\,dx$

$= \frac{1}{3}\sin^{-1}(3x)\Big]_0^{\frac{\sqrt{3}}{6}}$ ✓

$= \frac{1}{3}\sin^{-1}\frac{\sqrt{3}}{2}$ ✓

$= \frac{1}{3}\cdot\frac{\pi}{3}$ ✓ $= \frac{\pi}{9}$

Maybe only worth 2.

(d) (i) $y = \frac{x^3+16}{x}$

$y = x^2 + 16x^{-1}$

$y' = 2x - \frac{16}{x^2} = 2x - 16x^{-2}$ ✓

(ii) Stat pts where $y' = 0$

$2x - \frac{16}{x^2} = 0$

$x^3 = 8$

$x = 2, \quad y = 12$ ✓

Table of values for y'

x	1	2	3
y'	-14	0	$\frac{38}{9}$

✓ Min point at $(2, 12)$ ✓

(iii) $y'' = 2 + \frac{32}{x^3}$

Possible point of infexion where

$y'' = 0$

$2 + \frac{32}{x^3} = 0$ ✓

$x = \sqrt[3]{-16}$ which is an x-intercept

(See over)

Table of values for y''

x	-3	$\sqrt[3]{-16}$	-2
y''	$\frac{22}{7}$	0	-2

When $x = -3$

$$y'' = 2 + \frac{32}{-27} = \frac{22}{27}$$

When $x = -2$

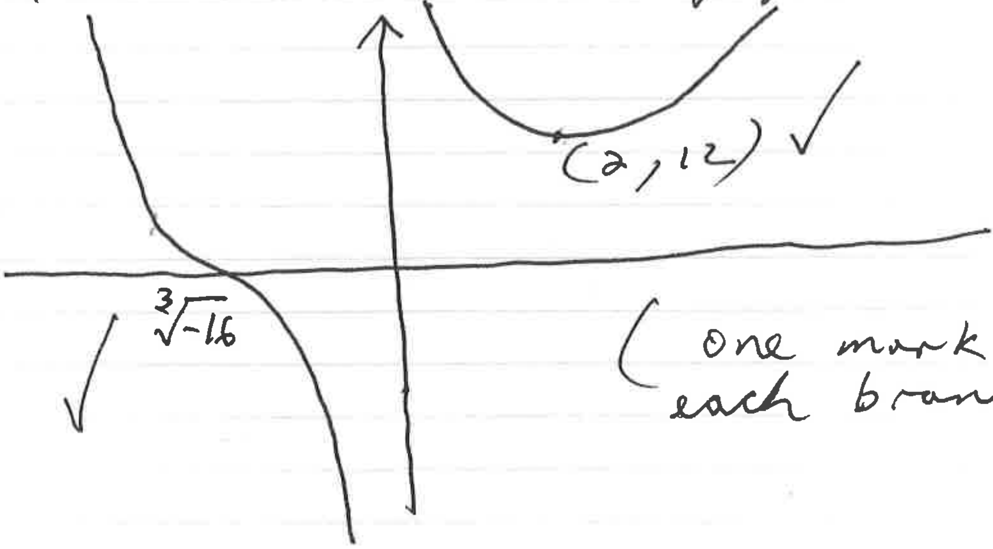
$$y'' = 2 + \frac{32}{-8} = -2$$

There is a change in concavity at $x = \sqrt[3]{-16}$

There is a point of inflexion at $x = \sqrt[3]{-16}$ (which is an x-intercept)

(iv) Now $x + \frac{16}{x} = \frac{x^3+16}{x}$

So there is a vertical asymptote at $x = 0$



(one mark for each branch)

8

13

(a) $\lim_{x \to 0} \frac{\sin ax}{x} = a \lim_{x \to 0} \frac{\sin ax}{ax}$

$= a \times 1$

$= a$

(b) (i) $\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

$RHS = 1v^1 \times \frac{dv}{dx}$

$= \frac{dx}{dt} \times \frac{dv}{dx}$

$= \frac{dv}{dt} = LHS.$

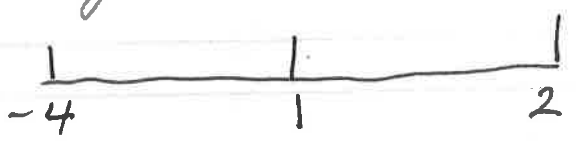
(ii) Particle is stationary where $v^2 = 0$

$24 - 6x - 3x^2 = 0$

$x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

Stationary at $x = -4$ and $x = 2$



Now $\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

$= \frac{d}{dx}\left(12 - 3x - \frac{3}{2}x^2\right)$

$= -3 - 3x$

Max displacement when $x = 2$

Acceleration when $x = 2$ is

$\ddot{x} = -3 - 3 \times 2$

$= -9\ m/s^2$

(9)

(c) $x^3 - px + q = 0$ sum of roots two at a time

$$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\alpha\beta + \beta\gamma + \gamma\alpha}{\alpha\beta\gamma}$$

$$= \frac{\frac{c}{a}}{-\frac{d}{a}}$$

$$= \frac{\frac{-p}{1}}{\frac{-q}{1}} = \frac{p}{q}$$

(d) $\tan^{-1} + \tan^{-1} 2 + \tan^{-1} 3 = \pi$

$$\tan^{-1} 2 + \tan^{-1} 3 = \frac{3\pi}{4}$$

Let $\alpha = \tan^{-1} 2$ and $\beta = \tan^{-1} 3$

So $\tan\alpha = 2$ and $\tan\beta = 3$

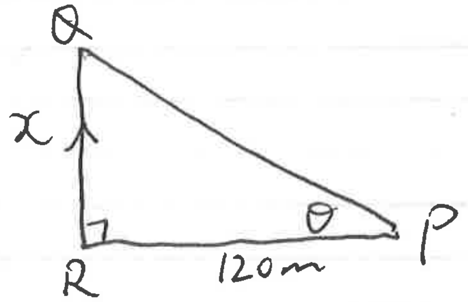
$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$= \frac{2+3}{1-6} = -1$$

$$\alpha + \beta = \tan^{-1}(-1)$$

ie. $\alpha + \beta = \frac{\pi}{4}$

(b)



$$\frac{x}{120} = \tan\theta$$

$$x = 120\tan\theta$$

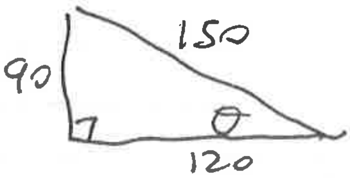
$$\frac{dx}{d\theta} = 120\sec^2\theta$$

$$\frac{dx}{dt} = \frac{dx}{d\theta} \times \frac{d\theta}{dt}$$

$$\frac{dx}{dt} = 120\sec^2\theta \times 0.2$$

$$= 120 \times \frac{25}{16} \times 0.2$$

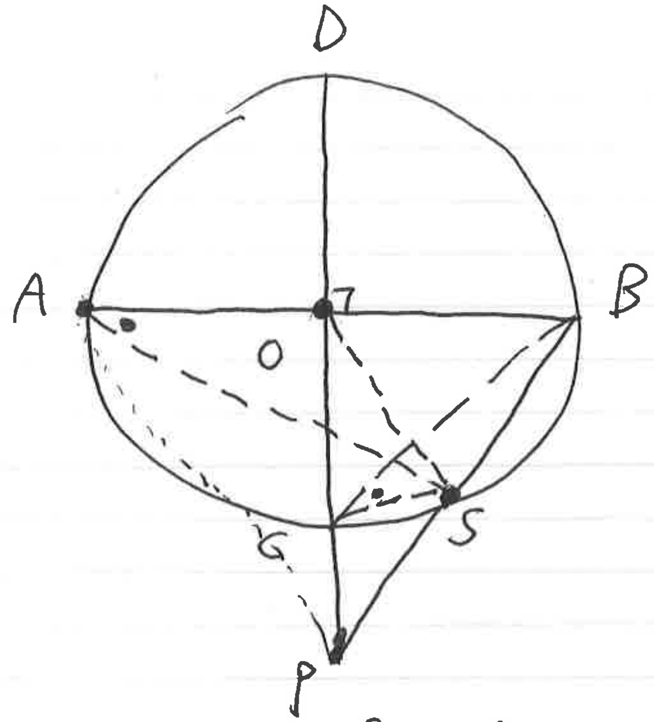
$$= 37.5 \text{ m/ min}$$



$$\sec\theta = \frac{150}{120} = \frac{5}{4}$$

(10)

(f)



(i)

$\angle AOP = 90^\circ$ (vertically opposite)

$\angle ASB = 90^\circ$ (angle in a semi-circle)

$\angle ASP = 90^\circ$ (st line)

✓ So $\angle AOP = \angle ASP$
But these stand on the same chord AP
$\therefore$ AOSP is a cyclic quad'l.

(ii)

$\angle BCS = \angle SAB$ (angles standing on the same chord in circle ADB) ✓

$\angle SPO = \angle SAO$ $(= \angle SAB)$ (angles standing on the same arc in circle AOS) ✓

$\therefore \angle BCS = \angle SPO$

15 (i) $f(x) = \ln\left(x^{\frac{1}{x}}\right) \quad x > 0$

$$f(x) = \frac{1}{x}\ln x$$

$$f'(x) = \frac{1}{x}\cdot\frac{1}{x} + \ln x \times -\frac{1}{x^2}$$

$$= \frac{1}{x^2}\left(1 - \ln x\right) \checkmark$$

(ii) Stationary pts where $f'(x) = 0$

$$\frac{1}{x^2}(1-\ln x) = 0$$

$$1 - \ln x = 0$$

$$\ln x = 1$$

$$x = e, \quad y = \frac{1}{e} \checkmark$$

Table of values for y'

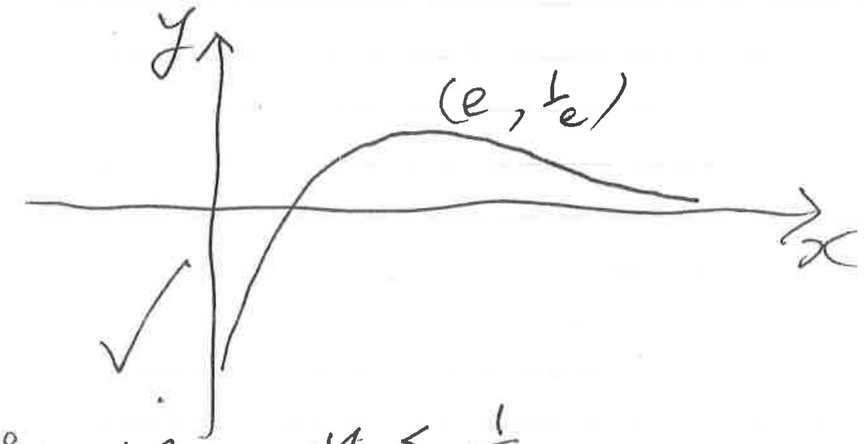
x	2	e	3
y'	0.08	0	−0.01

✓ Max point at $\left(e, \frac{1}{e}\right)$

Now as $x \to 0^+$, $y \to -\infty$

$x \to \infty$, $y \to 0^+$

OR Sketch



(and correct range) ✓

Range is $y \le \frac{1}{e}$

QUESTION 15

(b) $x = V\cos\theta\, t \qquad y = h + V\sin\theta\, t - \frac{1}{2}gt^2$

(i) $t = \dfrac{x}{V\cos\theta}$ ✓

$$y = h + V\sin\theta \cdot \frac{x}{V\cos\theta} - \frac{1}{2} g \frac{x^2}{V^2\cos^2\theta}$$ ✓

$$y = h + x\tan\theta - \frac{1}{2}\frac{gx^2}{V^2}\sec^2\theta.$$

(ii) Passes through $(R, 0)$

$$0 = h + R\tan\theta - \frac{1}{2}\frac{gR^2}{V^2}\sec^2\theta$$ ✓ ✓

$\times \frac{2V^2}{-g}$: $\quad R^2\sec^2\theta - \dfrac{2RV^2}{g}\tan\theta - \dfrac{2hV^2}{g} = 0$

(iii) But $\sec^2\theta = \tan^2\theta + 1$

$$R^2\tan^2\theta + R^2 - \frac{2RV^2}{g}\tan\theta + \frac{V^4}{g^2} - \frac{V^4}{g^2} - \frac{2hV^2}{g} = 0$$

$$R^2 + \left(\tan\theta - \frac{V^2}{g}\right)^2 - \left(\frac{V^4}{g^2} + \frac{2hV^2}{g}\right) = 0$$ ✓

$$R^2 = \left(\frac{V^4}{g^2} - \frac{2hV^2}{g}\right) - \left(R\tan\theta - \frac{V^2}{g}\right)^2$$ ✓

(iv) $$R^2 \leq \frac{V^4}{g^2} - \frac{2hV^2}{g}$$

$$R \leq \frac{1}{g}\sqrt{V^4 - 2hV^2g}.$$ ✓

Max R_1 occurs where

$$R_1 \tan\theta - \frac{v^2}{g} = 0$$

$$\tan\theta = \frac{v^2}{R_1 g}$$

(vi)

$$R_1 = \sqrt{\frac{v^2}{g^2}\left(\frac{v^2}{g} + 2h\right)}$$

$$= \frac{v}{g}\sqrt{\frac{v^2}{g} + 2h} \quad \text{— } *$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

$$= 2\frac{v^2}{R_1 g} \div 1 - \frac{v^4}{R_1^2 g^2}$$

$$= \frac{2v^2}{R_1 g} \times \frac{R_1^2 g^2}{R_1^2 g_1^2 - v^4} \quad **$$

See over

$$R_1 = \sqrt{\frac{v^4}{g^2} + \frac{2hv^2}{g}}$$

$$R_1 = \frac{v}{g}\sqrt{v^2 + 2hg}$$

$$R_1^2 = \frac{v^2}{g^2}\left(v^2 + 2hg\right)$$

$$g^2R_1^2 = v^4 + 2v^2hg$$

$$g^2R_1^2 - v^4 = 2v^2hg$$

Substitute into

$$\tan 2\theta = \frac{\cancel{2v^2}}{\cancel{R_1 g}} \times \frac{R_1^{\cancel{2}} g^2}{\cancel{2v^2}h\cancel{g}}$$

$$= \frac{R_1}{h}.$$