Student's Name: ______________________

Student Number: | | | | | | | | | |

Teacher's Name: ______________________

ABBOTSLEIGH

2016
HIGHER SCHOOL CERTIFICATE
Assessment 4
Trial Examination

Mathematics Extension 2

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black pen.
- **Board-approved** calculators may be used.
- A reference sheet is provided.
- All necessary working should be shown in every question.
- Make sure your HSC candidate Number is on the front cover of each booklet.
- Start a new booklet for Each Question.
- Answer the Multiple Choice questions on the answer sheet provided.
- If you do not attempt a whole question, you must still hand in the Writing Booklet, with the words **'NOT ATTEMPTED'** written clearly on the front cover.

Total marks - 100

- Attempt Sections I and II.

Section I — Pages 3 - 6

10 marks
- Attempt Questions 1–10
- Allow about 15 minutes for this section.

Section II — Pages 7 - 13

90 marks
- Attempt Questions 11 – 16.
- Allow about 2 hr and 45 minutes for this section.
- All questions are of equal value.

Outcomes to be assessed:

Mathematics Extension 2

A student:

HSC :

E1 Appreciates the creativity, power and usefulness of Mathematics to solve a broad range of problems.

E2 Chooses appropriate strategies to construct arguments and proofs in both concrete and abstract settings

E3 Uses the relationship between algebraic and geometric representations of complex numbers and of conic sections

E4 Uses efficient techniques for the algebraic manipulation required in dealing with questions such as those involving conic sections and polynomials

E6 Combines the ideas of algebra and calculus to determine the important features of the graphs of a wide variety of functions

E7 Uses the techniques of slicing and cylindrical shells to determine volumes

E8 Applies further techniques of integration, including partial fractions, integration by parts and recurrence formulae, to problems

E9 Communicates abstract ideas and relationships using appropriate notation and logical argument

Mathematics Extension 1

A student:

Preliminary course:

PE1 Appreciates the role of mathematics in the solution of practical problems

PE2 Uses multi-step deductive reasoning in a variety of contexts

PE3 Solves problems involving inequalities, polynomials, circle geometry and parametric representations

PE4 Uses the parametric representation together with differentiation to identify geometric properties of parabolas

PE5 Determines derivatives that require the application of more than one rule of differentiation

PE6 Makes comprehensive use of mathematical language, diagrams and notation for communicating in a wide variety of situations

HSC course:

HE1 Appreciates interrelationships between ideas drawn from different areas of mathematics

HE2 Uses inductive reasoning in the construction of proofs

HE3 Uses a variety of strategies to investigate mathematical models of situations involving projectiles

HE4 Uses the relationship between functions, inverse functions and their derivatives

HE5 Applies the chain rule to problems including those involving velocity and acceleration as functions of displacement

HE6 Determines integrals by reduction to a standard form through a given substitution

HE7 Evaluates mathematical solutions to problems and communicates them in an appropriate form

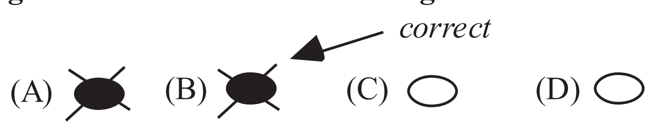
SECTION I

10 marks
Attempt Questions 1 – 10
Use the multiple-choice answer sheet
Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows.



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1. What is the eccentricity of the ellipse $\frac{x^2}{9}+\frac{y^2}{16}=1$?

(A) $\frac{7}{16}$ (B) $\frac{\sqrt{7}}{4}$ (C) $\frac{9}{16}$ (D) $\frac{7}{9}$

2. $\int_0^{\frac{\pi}{6}} \sin^3 x \; dx$ is equal in value to:

(A) $\int_0^{\frac{\pi}{6}} \sin x - \sin x \cos^2 x \, dx$

(B) $\int_0^{\frac{\pi}{6}} \frac{\pi}{6} - \sin^3 x \, dx$

(C) $\frac{1}{2}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \sin^3 x \, dx$

(D) $\int_0^{\frac{\pi}{6}} \cos^3 x \, dx$

3. Diagram A shows the complex number z represented in the Argand plane.

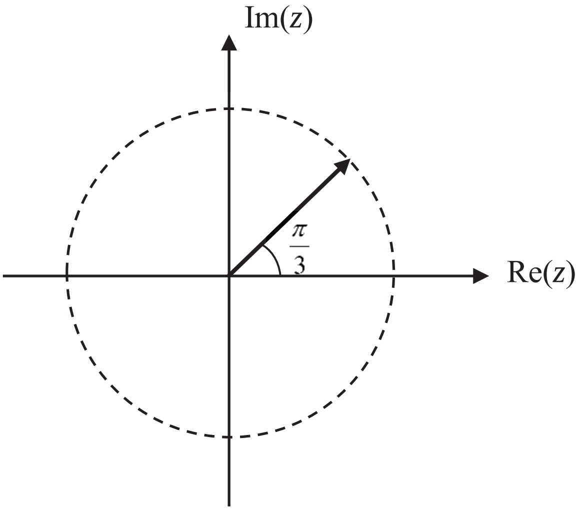


Diagram A

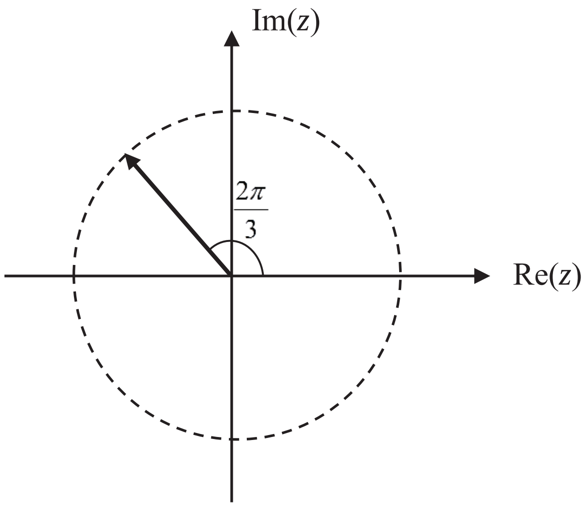
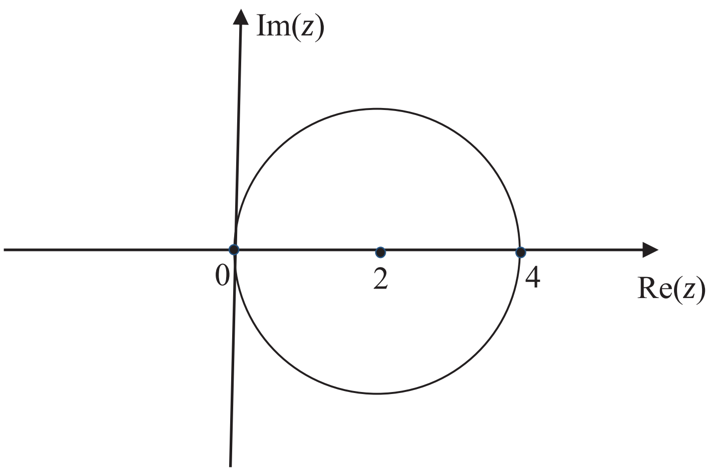


Diagram B

Diagram B shows:

(A) $\overline{z}$ (B) $2iz$ (C) $-2z$ (D) z^2

4. Which of the following is the equation of the circle below?



(A) $(z+2)(\overline{z}+2)=4$ (B) $(z-2)(\overline{z}-2)=4$

(C) $(z+2i)(\overline{z}-2i)=4$ (D) $(z+2)(\overline{z}-2)=4$

5. z and w are two complex numbers. Which of the the following statements is always TRUE?

(A) $|z|-|w| \geq |z+w|$

(B) $|z|+|w| \leq |z-w|$

(C) $|z|+|w| \leq |z+w|$

(D) $|z+w|+|z| \geq |w|$

6. If α, β and γ are the roots of the equation $x^3-3x+4=0$.

Then the cubic equation with roots α^2, β^2 and γ^2 is:

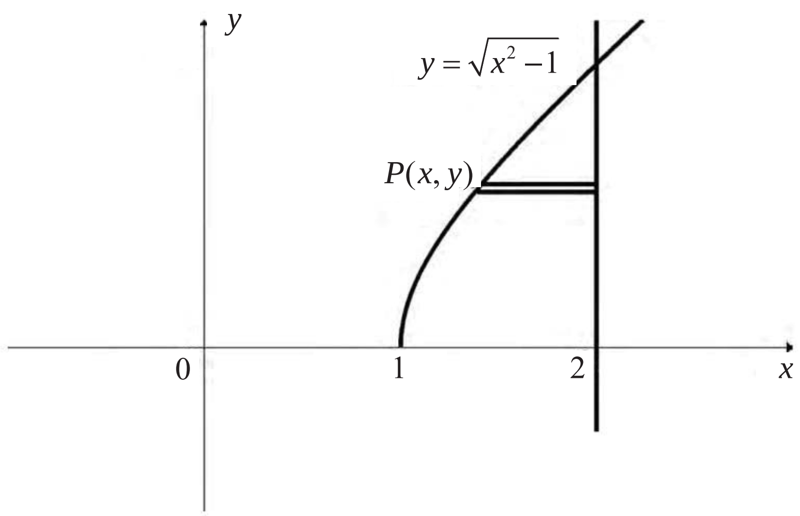
(A) $8x^3-9x+4=0$

(B) $x^3+9x^2-12x+4=0$

(C) $x^3-6x^2+9x-16=0$

(D) $8x^3+4x^2-9x+16=0$

7.



The region bounded by the x-axis, the curve $y=\sqrt{x^2-1}$ and the line $x=2$ is rotated about the y-axis.

The slice at $P(x,y)$ on the curve is perpendicular to the axis of rotation.

What is the volume of δV of the annular slice formed?

(A) $\pi\left(5-y^2\right)\delta y$

(B) $\pi\left(4-\left(y^2+1\right)\right)\delta y$

(C) $\pi\left(4-x^2\right)\delta x$

(D) $\pi\left(2-x^2\right)\delta x$

8. What is the acute angle between the asymptotes of the hyperbola $\frac{x^2}{3} - y^2 = 1$?

(A) $\frac{\pi}{3}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{6}$

(D) $\frac{\pi}{2}$

9. Which of the following is the range of the function $f(x) = \sin^{-1} x + \tan^{-1} x$?

(A) $-\pi < y < \pi$

(B) $-\pi \leq y \leq \pi$

(C) $\frac{-3\pi}{4} \leq y \leq \frac{3\pi}{4}$

(D) $\frac{-\pi}{2} \leq y \leq \frac{\pi}{2}$

10. Using the substitution $x = \pi - y$, the definite integral $\int_0^{\pi} x \sin x \, dx$

will simplify to:

(A) 0

(B) $\int_0^{\pi} \sin x \, dx$

(C) $\frac{\pi}{2} \int_0^{\pi} \sin x \, dx$

(D) $\frac{\pi^2}{4}$

End of Section I

SECTION II

Total Marks – 90
Attempt Questions 11 - 16
All questions are of equal value

Answer each question in a **SEPARATE** writing booklet. Extra writing booklets are available.

In Questions 11-16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) Given that $z_1 = 5 + 2i$ and $z_2 = 3 - 4i$, find the value of $\text{Re}\left(\frac{z_1}{z_2}\right)$ in $x + iy$ form. **2**

(b) (i) Show that the square roots of $-35 + 12i$ are $\pm(1 + 6i)$. **2**

(ii) Hence solve $z^2 - (5 + 4i)z + 11 + 7i = 0$. **2**

(c) (i) Express $z_1 = -1 + \sqrt{3}i$ in modulus argument form. **2**

(ii) Given $z_2 = 3\left(\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)\right)$, find the value of $z_1 z_2$ in modulus argument form. **2**

(iii) Hence express $(z_1 z_2)^3$ in the form $x + iy$, where x and y are real numbers. **2**

(d) (i) On an Argand diagram sketch $\arg(z - 2) = \arg z + \frac{\pi}{2}$. **2**

(ii) Describe the locus. **1**

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet.

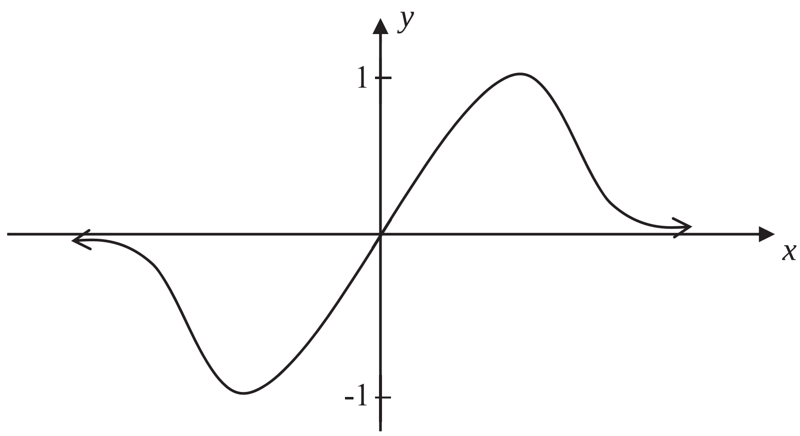
(a) (i) Express $\dfrac{3x+1}{(x+1)(x^2+1)}$ in the form $\dfrac{a}{x+1}+\dfrac{bx+c}{x^2+1}$. **2**

(ii) Hence find $\displaystyle\int \frac{3x+1}{(x+1)(x^2+1)}\,dx$. **2**

(b) (i) If $t=\tan\dfrac{\theta}{2}$, show that $d\theta=\dfrac{2}{1+t^2}\,dt$. **1**

(ii) By using the substitution $t=\tan\dfrac{\theta}{2}$, show that. $\displaystyle\int_0^{\frac{\pi}{3}}\sec\theta\,d\theta=\ln\left(2+\sqrt{3}\right)$. **3**

(c) The diagram shows $y=f(x)$ which is an **odd function**. There is a turning point at $(1,1)$.



Draw a separate sketch of each of the following graphs.

Use about one third of a page for each graph. Show all significant features.

(i) $y=\dfrac{1}{f(x)}$. **2**

(ii) $y=f\left(|x|\right)$. **1**

(iii) $y=e^{f(x)}$. **2**

(iv) $y=f(x)\times\sin^{-1}x$ (show the coordinates of the endpoints). **2**

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet.

(a) Given that the roots of the equation $4x^3 - 24x^2 + 45x - 26 = 0$ form an arithmetic sequence, solve the equation. **3**

(b) The polynomial $P(x) = 12x^3 + 44x^2 - 5x - 100$ has a double root. **2**

Factorise $P(x)$ over the real number system.

(c) The hyperbola H has equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and eccentricity e, while the ellipse E has equation $\frac{x^2}{a^2 + b^2} + \frac{y^2}{b^2} = 1$.

(i) Show that E has eccentricity $\frac{1}{e}$ and hence that E has equation $\frac{x^2}{a^2e^2} + \frac{y^2}{b^2} = 1$. **2**

(ii) Show that E passes through one focus of H, **and** H passes through one focus of E. **2**

(iii) Sketch H and E on the same diagram, labelling the foci S, S' of H and T, T' of E, and the directrices of H and E. Give the coordinates of the foci and the equations of the directrices in terms of a and e. **2**

(iv) If H and E intersect at P in the first quadrant. **2**

Show that the coordinates of P is $\left(ae\sqrt{\frac{2}{e^2+1}}, \frac{a\left(e^2-1\right)}{\sqrt{1+e^2}} \right)$.

(v) Show that the acute angle α between the tangents to the curves at P satisfies $\tan\alpha = \sqrt{2}\left(e + \frac{1}{e}\right)$. **2**

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet.

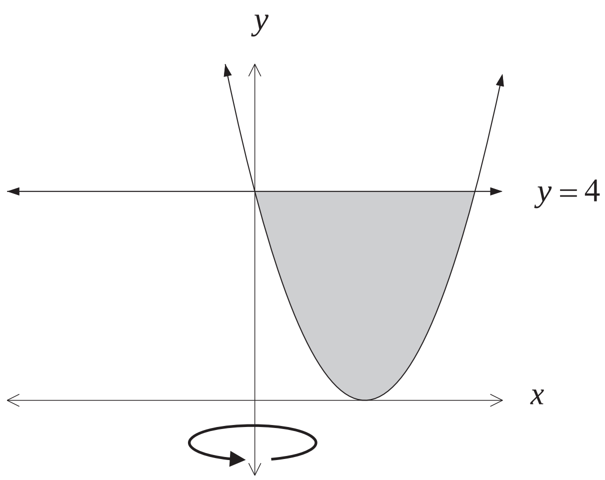
(a) The equation $x^3 + 2x - 1 = 0$ has roots α, β, γ. Evaluate $\alpha^3 + \beta^3 + \gamma^3$. **2**

(b) For $n = 0, 1, 2, ...$ let $I_n = \int_0^{\frac{\pi}{4}} \tan^n \theta \, d\theta$.

(i) Show that $I_1 = \frac{1}{2} \ln 2$. **2**

(ii) Use **integration by parts** to show that, for $n \geq 2$, $I_n + I_{n-2} = \frac{1}{n-1}$. **4**

(c) The area enclosed by the curve $y = (x-2)^2$ and the line $y = 4$ is rotated around the $y-$axis. Use the method of cylindrical shells to find the volume formed. **3**



(d) (i) On the same number plane diagram sketch the curves **2**

$$y = |x| - 2 \text{ and } y = 4 + 3x - x^2.$$

(ii) Hence or otherwise solve the inequality $\frac{|x| - 2}{4 + 3x - x^2} > 0$. **2**

End of Question 14

Question 15 (15 marks) Use a SEPARATE writing booklet.

(a) The points $P\left(cp, \dfrac{c}{p}\right)$ and $Q\left(cq, \dfrac{c}{q}\right)$ lie on the rectangular hyperbola $xy = c^2$.

The chord PQ meets the x-axis at C.

O is the centre of the hyperbola and R is the midpoint of PQ.

(i) Draw a sketch showing all information. **1**

(ii) Find the equation of the chord PQ. **2**

(iii) Find the coordinates of C. **1**

(iv) Find the coordinates of R. **1**

(v) Show that $OR = RC$. **2**

(b) (i) Show that $\sin(A+B) - \sin(A-B) = 2\cos A \sin B$. **1**

(ii) Use the method of Mathematical Induction to show that for all positive integers n: **4**

$$\cos x + \cos 2x + \cos 3x + \dots + \cos nx = \frac{\sin\left(n+\frac{1}{2}\right)x - \sin\frac{1}{2}x}{2\sin\frac{1}{2}x}$$

(iii) Hence show that: **3**

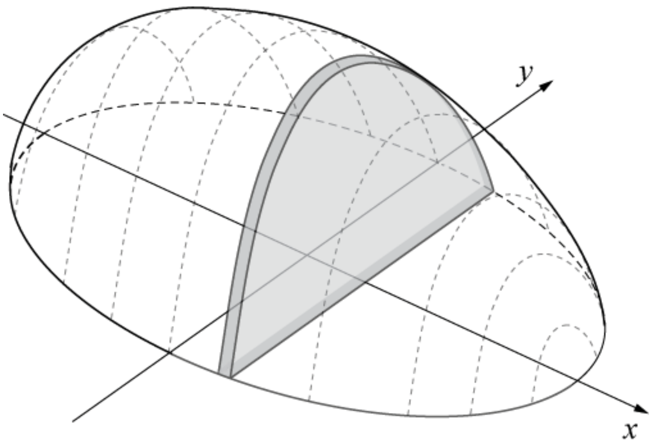
$$\cos 2x + \cos 4x + \cos 6x + \dots + \cos 16x = 8\cos 9x \cos 4x \cos 2x \cos x.$$

End of Question 15

Question 16 (15 marks) Use a SEPARATE writing booklet.

(a) (i) Show that the area enclosed between the parabola $x^2 = 4ay$ and its latus rectum is $\dfrac{8a^2}{3}$ units^2. **2**

A solid figure (as shown below) has the ellipse $\dfrac{x^2}{16} + \dfrac{y^2}{4} = 1$ as its base in the xy plane. Cross-sections perpendicular to the x-axis are parabolas with latus rectums in the xy plane.



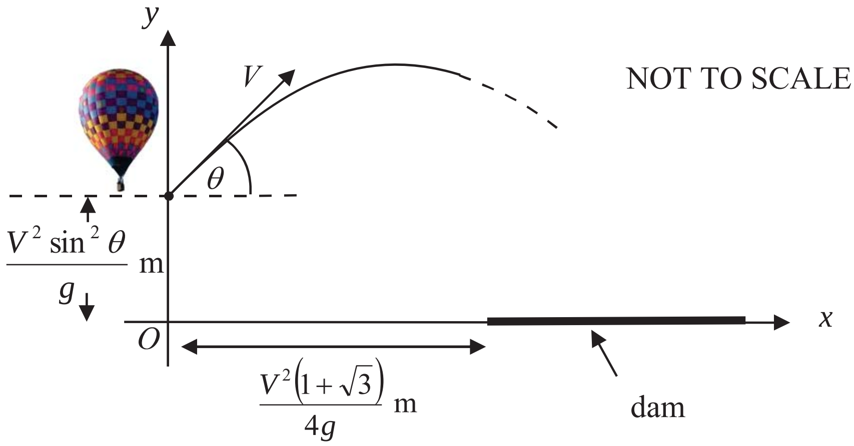
(ii) Show that the area of the cross-section at $x = h$ is $\dfrac{16 - h^2}{6}$ units^2. **3**

(iii) Hence, find the volume of this solid. **2**

Question 16 continues on the next page.

Question 16 continued.

(b)



A man ascending in a hot air balloon throws a set of car keys to his wife who is on the ground. The keys are projected at a constant velocity of $V\ \mathrm{m\,s^{-1}}$ at an angle of θ to the horizontal, $0° < \theta < 90°$, and from a point $\frac{V^2 \sin^2 \theta}{g}$ m vertically above the ground.

The edge of a dam closest to where the balloon took off, lies $\frac{V^2(1+\sqrt{3})}{4g}$ m horizontally from the point of projection. The dam is $\frac{V^2}{2g}$ m wide.

The position of the keys at time *t* seconds after they are projected is given by:

$$x = Vt\cos\theta$$

and

$$y = \frac{-gt^2}{2} + Vt\sin\theta + \frac{V^2 \sin^2 \theta}{g}$$

(i) Show that the Cartesian equation of the path of the keys is given by: **1**

$$y = \frac{-gx^2 \sec^2 \theta}{2V^2} + x\tan\theta + \frac{V^2 \sin^2 \theta}{g}$$

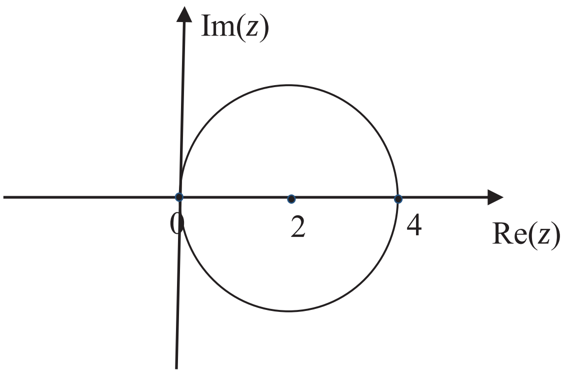
(ii) Show that the horizontal range of the keys on the ground is given by: **3**

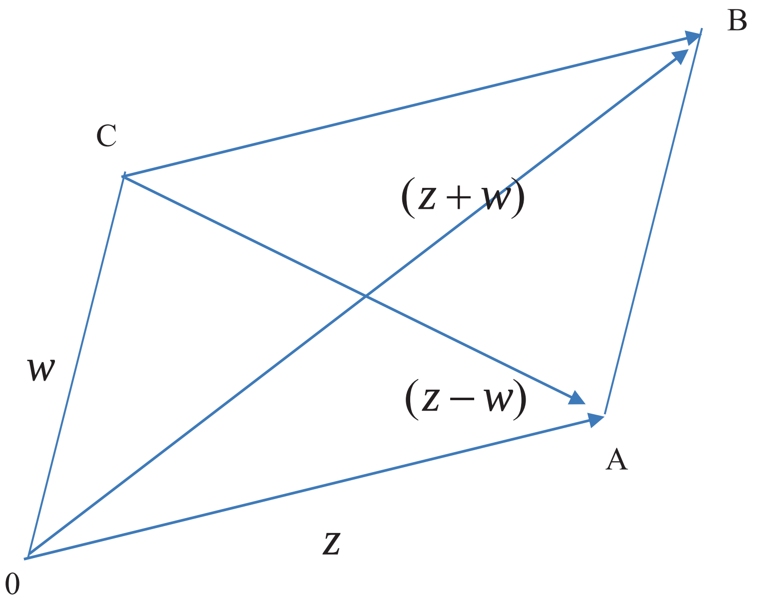
$$x = \frac{V^2\left(1+\sqrt{3}\right)\sin 2\theta}{2g}$$

(iii) Find the values of θ for which the keys will **NOT** land in the dam. **4**

END OF PAPER

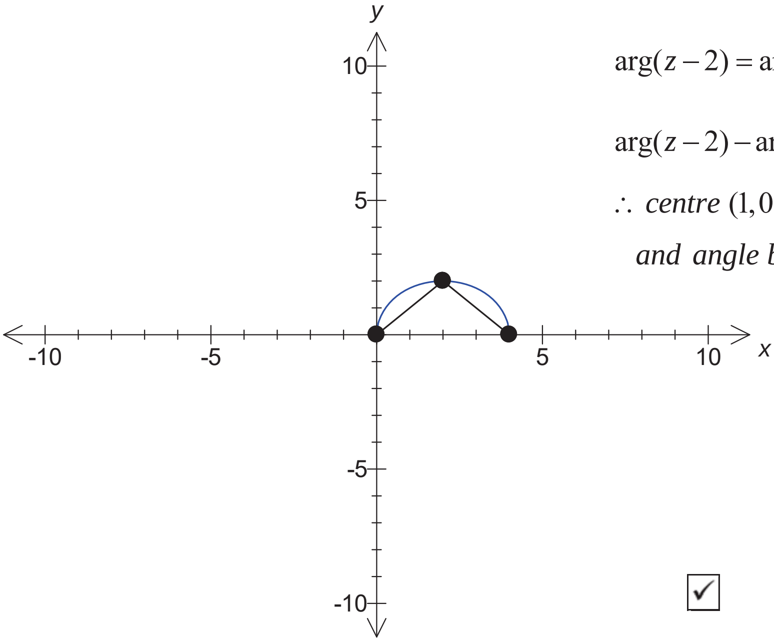
Extension 2 Mathematics Task 4 Trial Examination 2016 Solutions:

Question	Working	Solution
1	$\frac{x^2}{9}+\frac{y^2}{16}=1$ $\sin ce\ \ b>a$ $\therefore a^2=b^2(1-e^2)$ $9=16(1-e^2)$ $e^2=\frac{7}{16}$ $e=\frac{\sqrt{7}}{4}$ $\quad\therefore B$	B
2	$\int_0^{\frac{\pi}{6}} \sin^3 x\ \ dx$ $=\int_0^{\frac{\pi}{6}} \sin x.\sin^2 x\,dx$ $=\int_0^{\frac{\pi}{6}} \sin x\left(1-\cos^2 x\right)dx$ $=\int_0^{\frac{\pi}{6}} \sin x-\sin x\cos^2 x\,dx$ $\quad\therefore A$	A
3	z^2 double the argument $\quad\therefore D$	D
4	 Centre $(0,2)$ and radius of 2 $\therefore(z-2)\left(\bar{z}-2\right)=4$ $(x+iy-2)(x-iy-2)=4$ $(x+iy)(x-iy)-2(x+iy)-2(x-iy)+4=4$ $x^2+y^2-4x+4=4$ $(x-2)^2+y^2=2^2$ $\quad\therefore B$	B

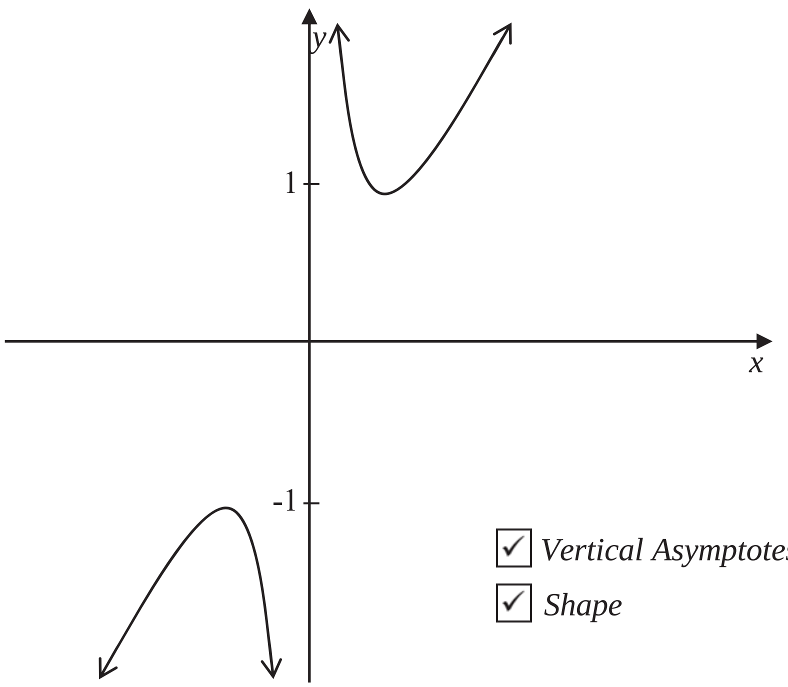
5	 The length of any two sides of a triangle must exceed the length of the third side. Therefore $\Delta\ OAB \quad \lvert z\rvert-\lvert w\rvert \geq \lvert z+w\rvert$ ☒ $\Delta\ OAC \quad \lvert z\rvert+\lvert w\rvert \leq \lvert z-w\rvert$ ☒ $\Delta\ AOB \quad \lvert z\rvert+\lvert w\rvert \leq \lvert z+w\rvert$ ☒ $\Delta\ AOB \quad \lvert z+w\rvert+\lvert z\rvert \geq \lvert w\rvert$ ☑ $\therefore D$	D
6	If replace x with $\sqrt{x}$ $\left(\sqrt{x}\right)^3-3\left(\sqrt{x}\right)+4=0$ $x\sqrt{x}-3\sqrt{x}=-4$ $\sqrt{x}\left(x-3\right)=-4$ $x\left(x-3\right)^2=16$ $x\left(x^2-6x+9\right)=16$ $x^3-6x^2+9x-16=0 \qquad \therefore C$	C
7	$y=\sqrt{x^2-1} \qquad \therefore x^2=y^2+1$ $V=\pi\left(2^2-x^2\right)\delta y$ $\therefore V=\pi\left(4-(y^2+1)\right)\delta y \qquad \therefore B$	B

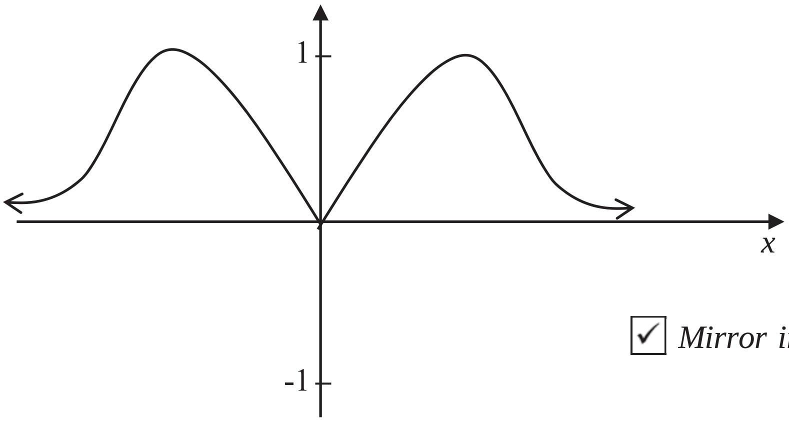
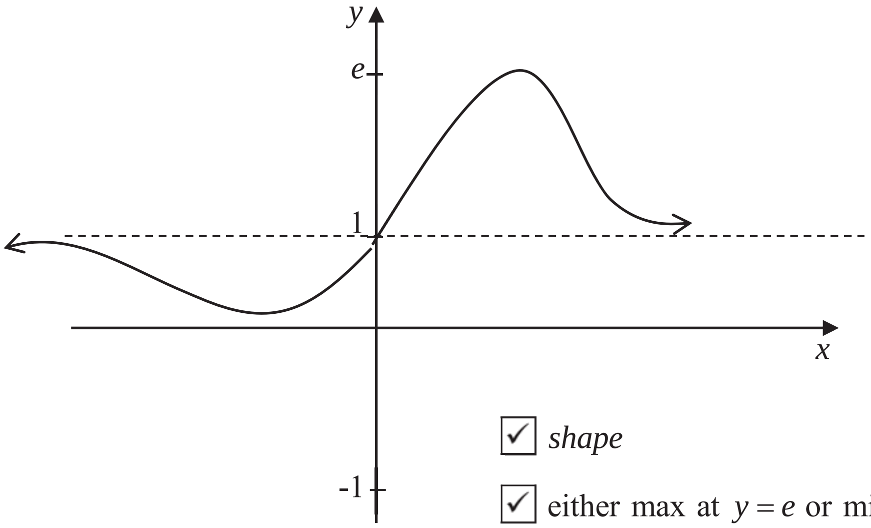
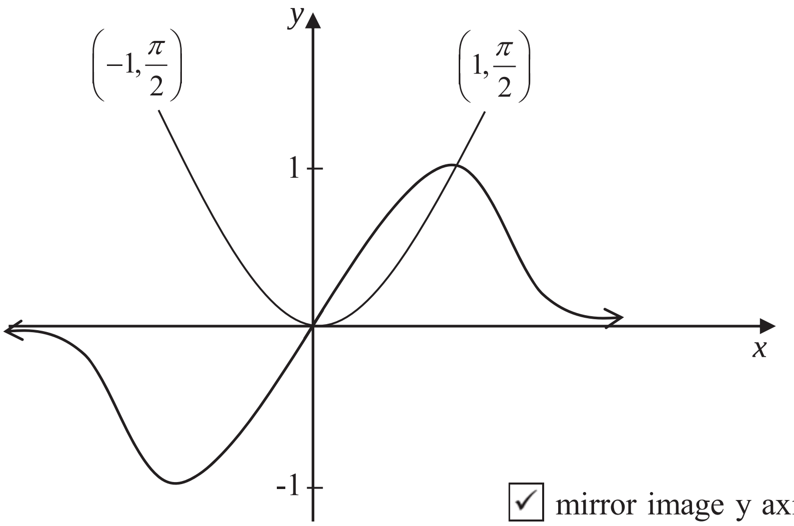
8	*Asymptotes* $y = \pm\frac{1}{\sqrt{3}}x$ $\therefore \tan\theta = \left\|\frac{m_1 - m_2}{1 + m_1 m_2}\right\|$ $\tan\theta = \left\|\frac{\left(\frac{1}{\sqrt{3}}\right) - \left(-\frac{1}{\sqrt{3}}\right)}{1 + \left(\frac{1}{\sqrt{3}}\right)\left(-\frac{1}{\sqrt{3}}\right)}\right\|$ $\tan\theta = \frac{3}{\sqrt{3}}$ $\therefore \theta = 60° = \frac{\pi}{3}$ $\therefore A$	A
9	*Domain*: $-1 \le x \le 1$ $\sin^{-1}(-1) + \tan^{-1}(-1) \le y \le \sin^{-1}(1) + \tan^{-1}(1)$ $\frac{-\pi}{2} - \frac{-\pi}{4} \le y \le \frac{\pi}{2} + \frac{\pi}{4}$ $\frac{-3\pi}{4} \le y \le \frac{3\pi}{4}$ $\therefore C$	C
10	$\int_0^{\pi} x\sin x\ dx = \int_{\pi}^{0} (\pi - y)\sin(\pi - y).-dy$ $= \int_0^{\pi} \pi\sin(\pi - y) - y\sin(\pi - y)\ dy$ $= \int_0^{\pi} \pi\sin y - y\sin y\ dy$ $= \int_0^{\pi} \pi\sin x - x\sin x\ dx$ $\therefore\ 2\int_0^{\pi} x\sin x\ dx = \pi\int_0^{\pi} \sin x\ dx$ $\therefore \int_0^{\pi} x\sin x\ dx = \frac{\pi}{2}\int_0^{\pi} \sin x\ dx$ $\therefore C$	C

Question	Working	Solution
11(a)	$\left(\frac{z_1}{z_1}\right)=\left(\frac{5+2i}{3-4i}\right)$ $=\left(\frac{5+2i}{3-4i}\times\frac{3+4i}{3+4i}\right)$ $=\left(\frac{15+20i+6i+8i^2}{9-16i^2}\right)$ $=\left(\frac{7+26i}{25}\right)$ ☑ $\therefore \text{Re}\left(\frac{7+26i}{25}\right)=\frac{7}{25}$ ☑	**2**
11(b)(i)	$\sqrt{-35+12i}=a+ib$ $-35+12i=a^2-b^2+2abi$ $\therefore a^2-b^2=-35$ *and* $2ab=12$ $(ie: b=\frac{6}{a})$ *So* $a^2-\left(\frac{6}{a}\right)^2=-35$ $a^2-\frac{36}{a^2}=-35$ $\therefore a^4+35a^2-36=0$ $(a^2+36)(a^2-1)=0$ $\therefore a=\pm 6i$ $a=\pm 1$ ☑ *So* $a=1$ $b=6$ $a=-1$ $b=-6$ *Therefore* $\sqrt{-35+12i}=1+6i$ *and* $-1-6i$ ☑	**2**
11(b)(ii)	$z=\frac{5+4i\pm\sqrt{(5+4i)^2-4(11+7i)}}{2}$ $z=\frac{5+4i\pm\sqrt{25+40i-16-44-28i}}{2}$ $=\frac{5+4i\pm\sqrt{-35+12i}}{2}$ ☑ $=\frac{5+4i\pm(1+6i)}{2}$ *from* (i) $=3+5i$ *or* $2-i$ ☑	**2**
11(c)(i)	$z_1=-1+\sqrt{3}i \Rightarrow z_1=r(\cos\theta+i\sin\theta)$	

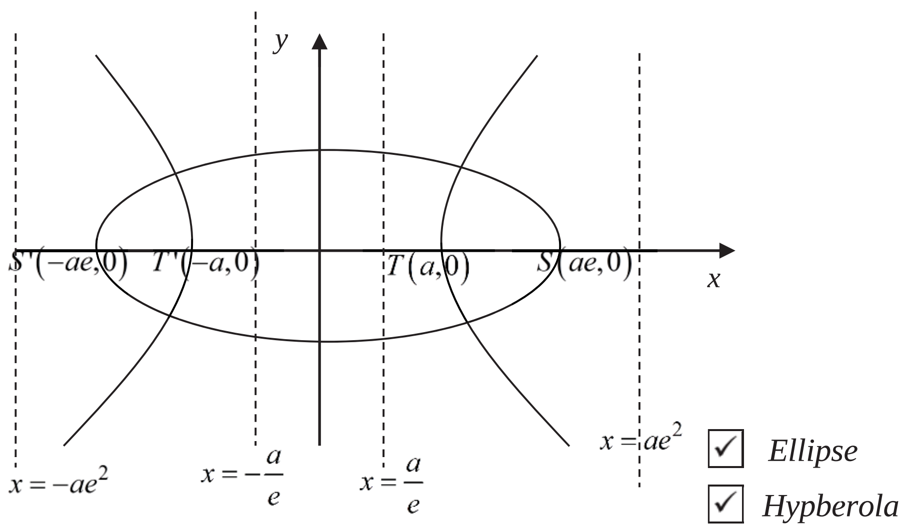
	$\text{mod}\, z = \lvert z \rvert \quad \Rightarrow \quad r = \sqrt{(-1)^2 + (\sqrt{3})^2} = 2$ ✓ $\arg(z) \quad \Rightarrow \quad \tan\theta = \frac{\sqrt{3}}{1} \quad \therefore \theta = \left(\pi - \frac{\pi}{3}\right) = \frac{2\pi}{3}$ ✓ $\therefore z_1 = 2\left(\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right)$	**2**
11(c)(ii)	$z_1 = 2\left(\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right)$ and $z_2 = 3\left((\cos(\frac{\pi}{6}) + i\sin(\frac{\pi}{6})\right)$ $\therefore z_1 z_2 = \left(2\left(\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right)\right)\left(3\left((\cos(\frac{\pi}{6}) + i\sin(\frac{\pi}{6})\right)\right)$ $= 6(\cos(\frac{2\pi}{3} + \frac{\pi}{6}) + i\sin(\frac{2\pi}{3} + \frac{\pi}{6}))$ ✓ $= 6(\cos(\frac{5\pi}{6}) + i\sin(\frac{5\pi}{6}))$ ✓	**2**
11(c)(iii)	$\left(z_1 z_2\right)^3 = \left(6(\cos(\frac{5\pi}{6}) + i\sin(\frac{5\pi}{6}))\right)^3$ $= 6^3(\cos(3 \times \frac{5\pi}{6}) + i\sin(3 \times \frac{5\pi}{6}))$ ✓ $= 216(\cos(\frac{5\pi}{2}) + i\sin(\frac{5\pi}{2}))$ $= 216(0 + 1i)$ $= 216i$ ✓	**2**
11(d)(i)	 $\arg(z-2) = \arg z + \frac{\pi}{2}$ $\arg(z-2) - \arg z = \frac{\pi}{2}$ $\therefore$ *centre* $(1,0)$ *and* $(0,0)$ *and angle between lines is* $90°$ ✓ ✓	**2**
11(d)(ii)	Locus: semi circle with radius 1 and centre (1,0) ✓ $\left(ie:\ y = \sqrt{1-(x-1)^2}\right)$	**1**

Question	Working	Solution
12(a)(i)	$\frac{3x+1}{(x+1)(x^2+1)}=\frac{a}{x+1}+\frac{bx+c}{x^2+1}$ $a(x^2+1)+(bx+c)(x+1)=3x+1$ $\therefore$ Let $x=-1$ $a((-1)^2+1)+(b(-1)+c)((-1)+1)=3(-1)+1$ $\boxed{a=-1}$ $\therefore ax^2+bx^2=0$ $-1+b=0$ $\boxed{b=1}$ ☑ $\therefore x=0$ $a((0)^2+1)+(b(0)+c)((0)+1)=3(0)+1$ $a+c=1$ $\boxed{c=2}$ $\therefore \frac{3x+1}{(x+1)(x^2+1)}=\frac{-1}{x+1}+\frac{x+2}{x^2+1}$ ☑	**2**
12(a)(ii)	$\int \frac{3x+1}{(x+1)(x^2+1)}\,dx=\int \frac{-1}{x+1}+\frac{x+2}{x^2+1}\,dx$ $= -\ln(x+1)+\int \frac{x}{x^2+1}+\frac{2}{x^2+1}\,dx$ ☑ $=-\ln\left\|(x+1)\right\|+\frac{1}{2}\ln\left\|(x^2+1)\right\|+2\tan^{-1}x+C$ ☑	**2**
12(b)(i)	$t=\tan\frac{\theta}{2}$ $\frac{dt}{d\theta}=\frac{1}{2}\sec^2\frac{\theta}{2}$ $=\frac{1}{2}(1+\tan^2\frac{\theta}{2})$ $=\frac{1}{2}(1+t^2)$ $\therefore \frac{dt}{d\theta}(\tan\frac{\theta}{2})=\frac{1+t^2}{2}$ *or* $d\theta=\frac{2\,dt}{1+t^2}$ ☑ OR $t=\tan\frac{\theta}{2}$ $\tan^{-1}t=\frac{\theta}{2}$ $\theta=2\tan^{-1}t$ $\Rightarrow \frac{d\theta}{dt}=\frac{2}{1+t^2}$ $d\theta=\frac{2}{1+t^2}\,dt$	**1**

12(b)(ii)	$t_1 = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$ $t_2 = \tan 0 = 0$ $\therefore \sec\theta = \frac{1+t^2}{1-t^2}$ ☑ $= \int_0^{\frac{\pi}{3}} \sec\theta \, d\theta$ $= \int_0^{\frac{1}{\sqrt{3}}} \frac{1+t^2}{1-t^2} \times \frac{2dt}{1+t^2}$ $= \int_0^{\frac{1}{\sqrt{3}}} \frac{2}{1-t^2} dt$ $= \int_0^{\frac{1}{\sqrt{3}}} \left(\frac{1}{1+t} + \frac{1}{1-t} \right) dt$ $= \left[\ln\left(\frac{1+t}{1-t} \right) \right]_0^{\frac{1}{\sqrt{3}}}$ ☑ $= \ln\left(\frac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}} \right)$ $= \ln\left(\frac{\sqrt{3}+1}{\sqrt{3}-1} \right)$ ☑ $= \ln\left(\frac{4+2\sqrt{3}}{3-1} \right)$ $= \ln\left(2+\sqrt{3}\right)$	**3**
12(c)(i)	 ☑ *Vertical Asymptotes and Odd* ☑ *Shape*	2

12(c)(ii)	 ☑ *Mirror image through y axis*	**1**
12(c)(iii)	 ☑ *shape* ☑ either max at $y = e$ or min at $y = \frac{1}{e}$	**2**
12(c)(iv)	 ☑ mirror image y axis and shape ☑ end points	**2**

Question	Working	Solution
13(a)	$4x^3-24x^2+45x-26=0$ $\boxed{roots\ \ \alpha-d,\ \alpha,\ \alpha+d}$ $\alpha+\beta+\gamma=-\frac{b}{a}$ $(\alpha-d)+(\alpha)+(\alpha+d)=-\frac{-24}{4}$ $\therefore 3\alpha=6$ $\boxed{\alpha=2}$ ☑ $\alpha\beta+\alpha\gamma+\beta\gamma=\frac{c}{a}$ $\alpha(\alpha-d)+\alpha(\alpha+d)+(\alpha-d)(\alpha+d)=\frac{45}{4}$ $\alpha^2-\alpha d+\alpha^2+\alpha d+\alpha^2-d^2=\frac{45}{4}$ $3\alpha^2-d^2=\frac{45}{4}$ $3(2)^2-d^2=\frac{45}{4}$ $\therefore d=\pm\frac{\sqrt{3}}{2}$ ☑ $\therefore$ Roots $2-\frac{\sqrt{3}}{2},\ 2,\ 2+\frac{\sqrt{3}}{2}$ ☑	**3**
13(b)	$P(x)=12x^3+44x^2-5x-100$ $P'(x)=36x^2+88x-5$ $=(18x-1)(2x+5)$ ☑ $P(x)=(2x+5)^2(3x-4)$ ☑	**2**
13(c)(i)	For hyperbola $b^2=a^2(e^2-1)\Rightarrow e^2=\frac{b^2}{a^2}+1$ or $\frac{b^2+a^2}{a^2}$ For ellipse let eccentricity be ε. $b^2=a^2(1-\varepsilon^2)$ but $a^2=b^2+a^2$ $\therefore \frac{b^2}{b^2+a^2}=1-\varepsilon^2$ $\varepsilon^2=1-\frac{b^2}{b^2+a^2}=\frac{b^2+a^2-b^2}{b^2+a^2}=\frac{a^2}{b^2+a^2}=\frac{1}{e^2}$ $\therefore \varepsilon=\frac{1}{e}$ ☑ $\frac{a^2}{a^2+b^2}=\frac{1}{e^2}$ $a^2+b^2=e^2a^2$ $\therefore$ ellipse has equation: $\frac{x^2}{a^2e^2}+\frac{y^2}{b^2}=1$ ☑	**2**
13(c)(ii)	Focus of hyperbola: $(\pm ae,\ 0)$	

	Focus of ellipse: $(\pm ae,\ 0)$ but $a = ae$ and $\varepsilon = \frac{1}{e}$ $\therefore \left(\pm ae\left(\frac{1}{e}\right), 0\right) = (\pm a,\ 0)$ ☑ Sub $(\pm a, 0)$ into $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, $LHS = 1 - 0$ $RHS = 1$ $\therefore$ the hyperbola passes through $(\pm a, 0)$ ☑ Sub $(\pm ae,\ 0)$ into $\frac{x^2}{a^2e^2} + \frac{y^2}{b^2} = 1$ $LHS = 1 + 0$ $RHS = 1$ $\therefore$ the ellipse passes through $(\pm ae, 0)$	**2**
13(c)(iii)		**2**

13(c)(iv)	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \quad [1] \qquad \frac{x^2}{a^2e^2}+\frac{y^2}{b^2}=1 \quad [2]$ $\frac{x^2}{a^2}+\frac{x^2}{a^2e^2}=2$ $x^2\left(e^2+1\right)=2a^2e^2$ $x=\pm\sqrt{\frac{2a^2e^2}{\left(e^2+1\right)}} \text{ or } \pm ae\sqrt{\frac{2}{\left(e^2+1\right)}}$ ☑ $\frac{y^2}{b^2}=1-\frac{x^2}{a^2e^2}$ $=1-\frac{2a^2e^2}{1+e^2}\times\frac{1}{a^2e^2}$ $y^2=b^2\left\{\frac{e^2-1}{1+e^2}\right\} \qquad b^2=a^2\left(e^2-1\right)$ $\therefore y^2=\frac{a^2\left(e^2-1\right)^2}{1+e^2}$ $\therefore y=\pm\frac{a\left(e^2-1\right)}{\sqrt{1+e^2}}$ ☑ $\therefore P\left(ae\sqrt{\frac{2}{e^2+1}},\frac{a\left(e^2-1\right)}{\sqrt{1+e^2}}\right)$	**2**
13(c)(v)	hyperbola: $\frac{dy}{dx}=\frac{b^2}{a^2}\left(\frac{x}{y}\right)$ ellipse: $\frac{dy}{dx}=\frac{-b^2}{a^2e^2}\left(\frac{x}{y}\right)$ at P $\quad \frac{dy}{dx}=\sqrt{2}e \qquad \frac{dy}{dx}=-\frac{\sqrt{2}}{e}$ ☑ $\tan\alpha=\left\|\frac{\sqrt{2}e+\frac{\sqrt{2}}{e}}{1+\sqrt{2}e\left(\frac{-\sqrt{2}}{e}\right)}\right\|$ $\tan\alpha=\left\|\frac{\sqrt{2}e+\frac{\sqrt{2}}{e}}{1-2}\right\|$ $\tan\alpha=\sqrt{2}\left\|\frac{e+\frac{1}{e}}{-1}\right\|$ ☑ $\tan\alpha=\sqrt{2}\left(e+\frac{1}{e}\right)$	**2**

Question	Working	Solution
14(a)	$\alpha^3+2\alpha-1=0 \quad or \quad \alpha^3=-2\alpha+1 \quad ...(1)$ $\beta^3+2\beta-1=0 \quad or \quad \beta^3=-2\beta+1 \quad ....(2)$ $\gamma^3+2\gamma-1=0 \quad or \quad \gamma^3=-2\gamma+1....(3)$ $\therefore (1)+(2)+(3)=\alpha^3+\beta^3+\gamma^3$ $=-2\alpha-2\beta-2\gamma+3$ $=2(\alpha+\beta+\gamma)+3$ ☑ since $(\alpha+\beta+\gamma)=\frac{-b}{a}=0$ then $2(0)+3=3$ ☑	**2**
14(b)(i)	$I_1=\int_0^{\frac{\pi}{4}}\tan\theta d\theta=\int_0^{\frac{\pi}{4}}\frac{\sin\theta}{\cos\theta}d\theta=-\left[\ln(\cos\theta)\right]_0^{\frac{\pi}{4}}$ ☑ $=-\left[\ln\frac{1}{\sqrt{2}}-\ln 1\right]$ $=\ln\sqrt{2} \quad or \quad \frac{1}{2}\ln 2$ ☑	**2**
14(b)(ii)	$I_n=\int_0^{\frac{\pi}{4}}\tan^n\theta d\theta=\int_0^{\frac{\pi}{4}}\tan^{n-2}\theta\times\tan^2\theta d\theta$ $=\int_0^{\frac{\pi}{4}}\tan^{n-2}\theta\times(\sec^2\theta-1)d\theta$ $=\int_0^{\frac{\pi}{4}}\tan^{n-2}\theta\times\sec^2\theta d\theta-\int_0^{\frac{\pi}{4}}\tan^{n-2}\theta d\theta$ $I_n=\int_0^{\frac{\pi}{4}}\tan^{n-2}\theta\times\sec^2\theta d\theta-I_{n-2} \quad (1)$ ☑	**4**

$$Now\int_0^{\frac{\pi}{4}}\tan^{n-2}\theta\times\sec^2\theta d\theta \qquad **$$

$u=\tan^{n-2}\theta,\ u'=(n-2)\tan^{n-3}\theta\times\sec^2\theta$

$v'=\sec^2\theta,\qquad v=\tan\theta$

$$\int_0^{\frac{\pi}{4}}\tan^{n-2}\theta\sec^2\theta d\theta=\left[\tan^{n-2}\theta\tan\theta\right]_0^{\frac{\pi}{4}}-\int_0^{\frac{\pi}{4}}(n-2)\tan^{n-2}\theta\sec^2\theta d\theta \quad ✓$$

$$\therefore\int_0^{\frac{\pi}{4}}\tan^{n-2}\theta\times\sec^2\theta d\theta=1-(n-2)\int_0^{\frac{\pi}{4}}\tan^{n-2}\theta\times\sec^2\theta d\theta$$

$$(1+(n-2))\int_0^{\frac{\pi}{4}}\tan^{n-2}\theta\times\sec^2\theta d\theta=1 \quad ✓$$

$$\int_0^{\frac{\pi}{4}}\tan^{n-2}\theta\times\sec^2\theta d\theta=\frac{1}{n-1}$$

$$I_n=\frac{1}{n-1}-I_{n-2} \quad ✓$$

$$\therefore I_n+I_{n-2}=\frac{1}{n-1}$$

OR from **

$$Now\int_0^{\frac{\pi}{4}}\tan^{n-2}\theta\times\sec^2\theta d\theta$$

$u=\tan^{n-2}\theta,\ u'=(n-2)\tan^{n-3}\theta\times\sec^2\theta$

$v'=\sec^2\theta,\qquad v=\tan\theta$

$$\int_0^{\frac{\pi}{4}}\tan^{n-2}\theta\sec^2\theta d\theta=\left[\tan^{n-2}\theta\tan\theta\right]_0^{\frac{\pi}{4}}-\int_0^{\frac{\pi}{4}}(n-2)\tan^{n-2}\theta\sec^2\theta d\theta \quad ✓$$

$$\therefore\int_0^{\frac{\pi}{4}}\tan^{n-2}\theta\times\sec^2\theta d\theta=1-(n-2)\int_0^{\frac{\pi}{4}}\tan^{n-2}\theta\times\sec^2\theta d\theta$$

SO $I_n=1-(n-2)\int_0^{\frac{\pi}{4}}\tan^{n-2}\theta\times\sec^2\theta d\theta-I_{n-2}$ BUT $\int_0^{\frac{\pi}{4}}\tan^{n-2}\theta\times\sec^2\theta d\theta=I_n+I_{n-2}$ from (1)

$$I_n=1-(n-2)(I_n+I_{n-2})-I_{n-2}$$

$$I_n+I_{n-2}=1-(n-2)(I_n+I_{n-2})$$

$$(I_n+I_{n-2})+(n-2)(I_n+I_{n-2})=1$$

$$(I_n+I_{n-2})(1+n-2)=1$$

$$(I_n+I_{n-2})(n-1)=1$$

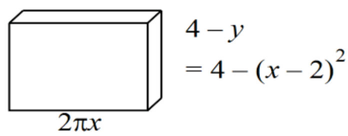
$$I_n+I_{n-2}=\frac{1}{n-1}$$

14(c)

$V_{Shell}=2\pi rh.\ dx$

$=2\pi x\left(4-(x-2)^2\right)\ dx$

$=2\pi x\left(4x-x^2\right)\ dx \quad ✓$



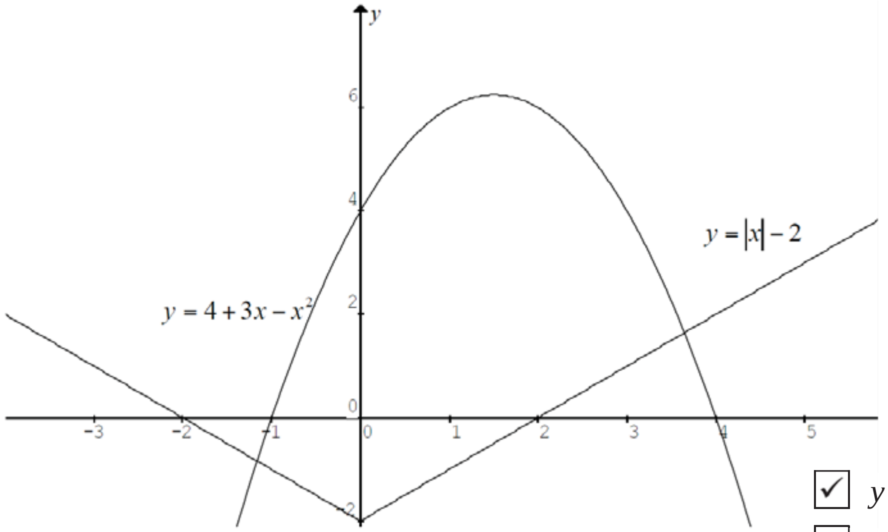
$$V=2\pi\int_0^4 x\left(4x-x^2\right)\ dx$$

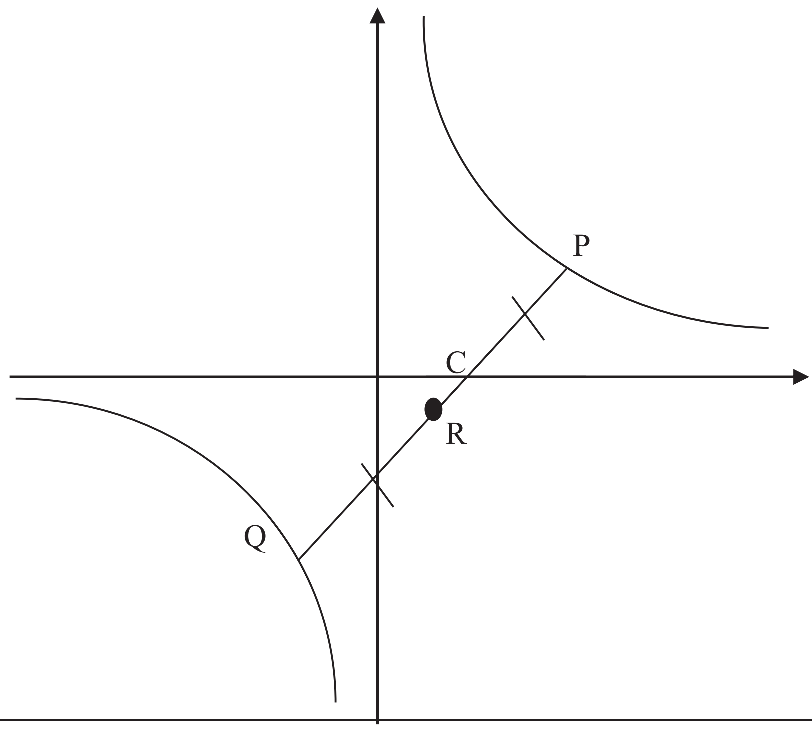
$$V=2\pi\int_0^4\left(4x^2-x^3\right)\ dx$$

$$V=2\pi\left[\frac{4x^3}{3}-\frac{x^4}{4}\right]_0^4 \quad ✓$$

$$V=\frac{128\pi}{3} \quad ✓$$

3

14(d)(i)	 ☑ $y = \|x\| - 2$ ☑ $y = 4 + 3x - x^2$	**2**
14(c)(ii)	$\frac{\|x\|-2}{4+3x-x^2} > 0$ when $\|x\|-2>0$ and $4+3x-x^2$ are **both** positive or **both** negative By inspection of the graphs above 2 < x < 4 ☑ and -2 < x < -1 ☑	**2**

Question	Working	Solution
15(a)(i)		**1**
15(a)(ii)	$m_{pq} = \dfrac{\frac{c}{p} - \frac{c}{q}}{cp - cq} = \dfrac{-1}{pq}$ ☑ *Equation PQ*: $y - \dfrac{c}{p} = \dfrac{-1}{pq}(1 - cp)$ $\Rightarrow\ x + pqy = c(p+q)$ ☑	**2**

15(a)(ii)	$Let\ \ y=0$ $x=c(p+q)$ $\therefore C\left(c(p+q),0\right)$ ☑	**1**
15(a)(iii)	$Midpoint: \left(\frac{cp+cq}{2},\frac{\frac{c}{p}+\frac{c}{q}}{2}\right)=R\left(\frac{c}{2}(p+q),\frac{c(p+q)}{2pq}\right)$ ☑	**1**
15(a)(iv)	$OR^2=\frac{c^2(p+q)^2}{4p^2q^2}+\frac{c^2}{4}(p+q)^2$ ☑ $RC^2=\frac{c^2(p+q)^2}{4p^2q^2}+\left[\frac{c}{2}(p+q)-c(p-q)\right]^2$ $=\frac{c^2(p+q)^2}{4p^2q^2}+\left[-\frac{cp}{2}-\frac{cq}{2}\right]^2$ $=\frac{c^2(p+q)^2}{4p^2q^2}+\left[-\frac{c}{2}(p+q)\right]^2$ $=\frac{c^2(p+q)^2}{4p^2q^2}+\frac{c^2}{4}\left[(p+q)\right]^2$ $=OR^2$ $\therefore\ \ RC=OR$ ☑	**2**
15(b)(i)	$\sin(A+B)-\sin(A-B)$ $=\sin A\cos B+\cos A\sin B-\sin A\cos B+\cos A\sin B$ $=2\sin A\cos B$ ☑	**1**
15(b)(ii)	$n=1 \qquad LHS=\cos x$ $RHS=\frac{\sin\left(\frac{3}{2}x\right)-\sin\left(\frac{1}{2}x\right)}{2\sin\left(\frac{1}{2}x\right)}=\frac{2\cos x\sin\left(\frac{1}{2}x\right)}{2\sin\left(\frac{1}{2}x\right)}=\cos x$ $\therefore$ true for $n=1$ ☑	**4**

	Assume true for $n=k$ $$\cos x+\cos 2x+\cos 3x+...+\cos kx=\frac{\sin\left(k+\frac{1}{2}\right)x-\sin\left(\frac{1}{2}\right)x}{2\sin\left(\frac{1}{2}\right)x}$$ Prove true for $n=k+1$ $$RTP:\quad S_{k+1}=\frac{\sin\left(k+\frac{3}{2}\right)x-\sin\left(\frac{1}{2}\right)x}{2\sin\left(\frac{1}{2}\right)x}$$ $Now\ S_{k+1}=S_k+\cos(k+1)x$ $=\frac{\sin\left(k+\frac{1}{2}\right)x-\sin\left(\frac{1}{2}\right)x}{2\sin\left(\frac{1}{2}\right)x}+\cos(k+1)x$ ✓ $=\frac{\sin\left(k+\frac{1}{2}\right)x-\sin\left(\frac{1}{2}\right)x+2\sin\left(\frac{1}{2}x\right)\cos(k+1)x}{2\sin\left(\frac{1}{2}\right)x}$ $=\frac{\sin\left(k+\frac{1}{2}\right)x-\sin\left(\frac{1}{2}\right)x+\sin\left(\frac{1}{2}x+(k+1)x\right)-\sin\left((k+1)x-\frac{1}{2}x\right)}{2\sin\left(\frac{1}{2}\right)x}$ ✓ $=\frac{\sin\left(k+\frac{1}{2}\right)x-\sin\left(\frac{1}{2}\right)x+\sin\left(k+\frac{3}{2}\right)x-\sin\left(k+\frac{1}{2}\right)x}{2\sin\left(\frac{1}{2}\right)x}$ $=\frac{\sin\left(k+\frac{3}{2}\right)x-\sin\left(\frac{1}{2}\right)x}{2\sin\left(\frac{1}{2}\right)x}$ ✓ $\therefore$ true for $n=k+1$ Now the statement is true for $n=1$ and we have just proved it true for $n=k+1$. $\therefore$ It is true for $n=1+1, i,.e\ n=2,3,....$ Hence it is true for all positive integers n	

15(a)(iii)	$\cos 2x+\cos 4x+\cos 6x+...+\cos 16x=\dfrac{\sin\left(8+\frac{1}{2}\right)2x-\sin\frac{1}{2}(2x)}{2\sin\frac{1}{2}(2x)}$ $=\dfrac{\sin 17x-\sin x}{2\sin x}$ $=\dfrac{\sin(9x+8x)-\sin(9x-8x)}{2\sin x}$ ☑ $=\dfrac{2\cos 9x\sin 8x}{2\sin x}$ $\sin 8x=2\sin 4x\cos 4x$ $=4\sin 2x\cos 2x\cos 4x$ $=8\sin x\cos x\cos 2x\cos 4x$ ☑ $\therefore \dfrac{2\cos 9x\sin 8x}{2\sin x}=\dfrac{8\cos 9x\sin x\cos x\cos 2x\cos 4x}{\sin x}$ $=8\cos 9x\cos x\cos 2x\cos 4x$ ☑	**3**

Question	Working	Solution
16(a)	End points of latus rectum are $(2a,a)$ and $(-2a,a)$ Area $=4a^2-\displaystyle\int_{-2a}^{2a}\frac{x^2}{4a}\,dx$ ☑ $=4a^2-\dfrac{1}{12a}\left[x^3\right]_{-2a}^{2a}$ $=4a^2-\dfrac{8a^2}{6}$ $=\dfrac{8a^2}{3}\,unit^2$ ☑	**2**
16(b)(i)	$at\quad x=h$ $\dfrac{y^2}{4}=1-\dfrac{h^2}{16}$ $y^2=4-\dfrac{h^2}{4}$ $y=\pm 2\sqrt{1-\dfrac{h^2}{16}}$ ☑ For parabola $2a=2\sqrt{1-\dfrac{h^2}{16}}$ $a=\sqrt{1-\dfrac{h^2}{16}}$ $\therefore Area=\dfrac{8}{3}\left(1-\dfrac{h^2}{16}\right)$ $=\dfrac{16-h^2}{6}\,units^2$ ☑ OR use Simpson's rule OR $A=2\int_0^{2a} a-y\;dx$	**3**

16(b)(ii)	$\partial V = \frac{16-h^2}{6}\partial h$ $V = \lim_{x\to\infty}\sum_{h=-4}^{4}\frac{16-h^2}{6}\ \partial h$ $= 2\int_0^4 \frac{16-h^2}{6}\ dh$ ☑ $= \frac{1}{3}\left[16h - \frac{h^3}{3}\right]_0^4$ $= \frac{1}{3}\left[\left(64 - \frac{64}{3}\right) - 0\right]$ $\text{Volume} = \frac{128}{9}$ units^3 $\left(or\ \ 14\frac{2}{9}\right)$ ☑	**2**
16(c)(i)	*Given* $x = Vt\cos\theta$ so $t = \frac{x}{V\cos\theta}$ in $y = \frac{-gt^2}{2} + Vt\sin\theta + \frac{V^2\sin^2\theta}{g}$ becomes $y = \frac{-gx^2}{2V^2\cos^2\theta} + \frac{Vx\sin\theta}{V\cos\theta} + \frac{V^2\sin^2\theta}{g}$ ☑ $\therefore\ y = \frac{-gx^2\sec^2\theta}{2V^2} + x\tan\theta + \frac{V^2\sin^2\theta}{g}$	**1**
16(c)(ii)	When, $y = 0$ $y = \frac{-gx^2\sec^2\theta}{2V^2} + x\tan\theta + \frac{V^2\sin^2\theta}{g}$ $0 = \frac{-gx^2\sec^2\theta}{2V^2} + x\tan\theta + \frac{V^2\sin^2\theta}{g}$ $x = \frac{-\tan\theta \pm \sqrt{\tan^2\theta + 4\times\frac{g\sec^2\theta}{2V^2}\times\frac{V^2\sin^2\theta}{g}}}{\frac{-g\sec^2\theta}{V^2}}$ ☑ $= \frac{-\tan\theta \pm \sqrt{\tan^2\theta + 2\tan^2\theta}}{\frac{-g\sec^2\theta}{V^2}}$ $= \frac{-\tan\theta \pm \sqrt{3\tan^2\theta}}{\frac{-g\sec^2\theta}{V^2}}$ $= \frac{\left(\tan\theta + \sqrt{3}\tan\theta\right)V^2}{g\sec^2\theta}$ or $\frac{\left(\tan\theta - \sqrt{3}\tan\theta\right)V^2}{g\sec^2\theta}$ $= \frac{V^2\tan\theta\left(1+\sqrt{3}\right)}{g\sec^2\theta}$ or $\frac{V^2\tan\theta\left(1-\sqrt{3}\right)}{g\sec^2\theta}$ $= \frac{V^2\left(1+\sqrt{3}\right)\sin\theta\cos\theta}{g}$ ☑ since $1-\sqrt{3} < 0$ this value of x occurs to the left of the $y-$axis ☑ So $x = \frac{V^2\left(1+\sqrt{3}\right)\sin 2\theta}{2g}$ as required. OR	**3**

16(c)(ii)	When, $y = 0$ $0 = -\frac{gt^2}{2} + Vt\sin\theta + \frac{V^2\sin^2\theta}{g}$ $t = \frac{V\sin\theta \pm \sqrt{V^2\sin^2\theta + 4\times\frac{g}{2}\times\frac{V^2\sin^2\theta}{g}}}{2\times\frac{g}{2}}$ $= \frac{V\sin\theta \pm \sqrt{V^2\sin^2\theta + 2V^2\sin^2\theta}}{g}$ $= \frac{V\sin\theta \pm \sqrt{3V^2\sin^2\theta}}{g}$ $= \frac{V\sin\theta \pm V\sin\theta\sqrt{3}}{g}$ $= \frac{V\sin\theta\left(1\pm\sqrt{3}\right)}{g}$ But $t > 0$ $= \frac{V\sin\theta\left(1+\sqrt{3}\right)}{g}$ sub into $x = Vt\cos\theta$ $x = V\cos\theta\times\frac{V\sin\theta\left(1+\sqrt{3}\right)}{g}$ $= \frac{V^2\sin\theta\cos\theta\left(1+\sqrt{3}\right)}{g}$ $= \frac{V^2\frac{\sin 2\theta}{2}\left(1+\sqrt{3}\right)}{g}$ $= \frac{V^2\sin 2\theta\left(1+\sqrt{3}\right)}{g}$	

16(c)(iii)	From part (ii) the range of the projectile is given by $x=\dfrac{V^2\left(1+\sqrt{3}\right)\sin 2\theta}{2g}$ The near edge of the dam is located at $\left(\dfrac{V^2\left(1+\sqrt{3}\right)}{4g},\ 0\right)$ At this point, $\dfrac{V^2\left(1+\sqrt{3}\right)}{4g}=\dfrac{V^2\left(1+\sqrt{3}\right)\sin 2\theta}{2g}$ ☑ $\therefore\quad \sin 2\theta=\dfrac{1}{2}$ $2\theta=30^\circ,150^\circ,\ldots$ $\theta=15^\circ,75^\circ$ since $0^\circ<\theta<90^\circ$ ☑ The far edge of the dam is located at $\left(\dfrac{V^2\left(1+\sqrt{3}\right)}{4g}+\dfrac{V^2}{2g},\ 0\right)$ that is, at $\left(\dfrac{V^2\left(3+\sqrt{3}\right)}{4g},\ 0\right)$ At this point, $\dfrac{V^2\left(3+\sqrt{3}\right)}{4g}=\dfrac{V^2\left(1+\sqrt{3}\right)\sin 2\theta}{2g}$ ☑ $3+\sqrt{3}=2\left(1+\sqrt{3}\right)\sin 2\theta$ $\sin 2\theta=\dfrac{3+\sqrt{3}}{2+2\sqrt{3}}$ $=\dfrac{3+\sqrt{3}}{2+2\sqrt{3}}\times\dfrac{2-2\sqrt{3}}{2-2\sqrt{3}}$ $=\dfrac{6-6\sqrt{3}+2\sqrt{3}-6}{4-12}$ $=\dfrac{-4\sqrt{3}}{-8}$ $\sin 2\theta=\dfrac{\sqrt{3}}{2}$ $2\theta=60^\circ,120^\circ,\ldots$ $\theta=30^\circ,60^\circ\quad 0^\circ<\theta<90^\circ$ ☑	**4**