3 i)

$$y = \frac{1}{1+\sin x}$$

$$y' = \frac{-\cos x}{(1+\sin x)^2}$$

k)

$$y = \frac{1-\sin x}{\cos x}$$

$$y' = \frac{(\cos x)(-\cos x) - (1-\sin x)(-\sin x)}{\cos^2 x}$$

$$= \frac{-\cos^2 x + \overset{\sin x}{\cancel{y}} - \sin^2 x}{\cos^2 x}$$

$$= \frac{\sin x - 1}{\cos^2 x}$$

$$= \frac{\sin x - 1}{1-\sin^2 x}$$

$$= \frac{-1}{1+\sin x}$$

6/

$$y = e^{-x}(\cos 2x + \sin 2x)$$

$$y' = (e^{-x})(-2\sin 2x + 2\cos 2x) + (\cos 2x + \sin 2x)(-e^{-x})$$

$$= e^{-x}(\cos 2x - 3\sin 2x)$$

$$y'' = (e^{-x})(-2\sin 2x - 6\cos 2x) + (\cos 2x - 3\sin 2x)(-e^{-x})$$

$$= e^{-x}(-7\cos 2x + \sin 2x)$$

$$y'' + 2y' + 5y$$

$$= e^{-x}\left(-7\cos 2x + \sin 2x + 2\cos 2x - 6\sin 2x + 5\cos 2x + 5\sin 2x\right)$$

$$= \underline{0}$$

12)

$y = \sin x$

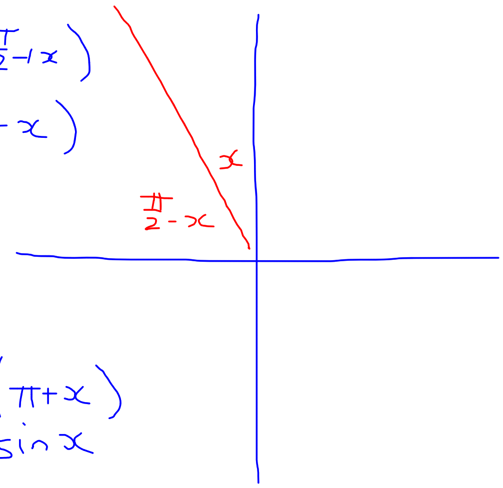
$\frac{dy}{dx} = \cos x$

$= \sin\left(\frac{\pi}{2}+x\right)$

$\sin\left(\frac{\pi}{2}+x\right)$

$= \sin\left(\frac{\pi}{2}-x\right)$

$= \cos x$



(ii) $\frac{d^2y}{dx^2} = -\sin x$

$= \sin(\pi+x)$

$\sin(\pi+x)$

$= -\sin x$

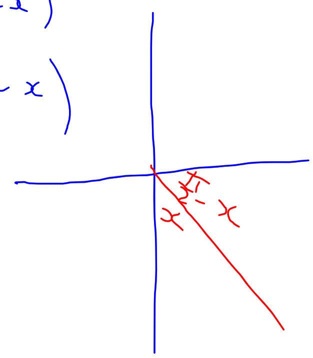
(iii) $\frac{d^3y}{dx^3} = -\cos x$

$= \sin\left(\frac{3\pi}{2}+x\right)$

$\sin\left(\frac{3\pi}{2}+x\right)$

$= -\sin\left(\frac{\pi}{2}-x\right)$

$= -\cos x$



(iv) $\frac{d^ny}{dx^n} = \sin\left(\frac{n\pi}{2}+x\right)$

13c) $\frac{d}{dx}\left(\frac{\cos x+\sin x}{\cos x-\sin x}\right)$

$$= \frac{(\cos x-\sin x)(-\sin x+\cos x)-(\cos x+\sin x)(-\sin x-\cos x)}{(\cos x-\sin x)^2}$$

$$= 1 + \frac{\cos^2 x+2\sin x\cos x+\sin^2 x}{\cos^2 x-2\sin x\cos x+\sin^2 x}$$

$$= 1+\frac{1+\sin 2x}{1-\sin 2x}$$ let $t=\tan x$

$$= 1+\frac{1+\frac{2t}{1+t^2}}{1-\frac{2t}{1+t^2}}$$

$$= 1+\frac{1+2t+t^2}{1-2t+t^2}$$

$$= 1+\left(\frac{1+t}{1-t}\right)^2$$

$$= 1+\left(\frac{\tan\frac{\pi}{4}+\tan x}{1-\tan\frac{\pi}{4}\tan x}\right)^2$$

$$= 1+\tan^2\left(\frac{\pi}{4}+x\right)$$

$$= \sec^2\left(\frac{\pi}{4}+x\right)$$

<u>OR</u>

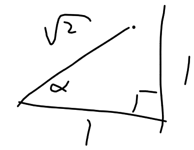
$$\frac{d}{dx}\left(\frac{\cos x+\sin x}{\cos x-\sin x}\right)$$

$$= \frac{d}{dx}\left(\frac{\cancel{\sqrt{2}}\,\sin\left(x+\frac{\pi}{4}\right)}{\sqrt{2}\,\cos\left(x+\frac{\pi}{4}\right)}\right)$$

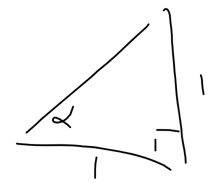
$$= \frac{d}{dx}\left(\tan\left(x+\frac{\pi}{4}\right)\right)$$

$$= \sec^2\left(x+\frac{\pi}{4}\right)$$

$$\cos x+\sin x$$
$$=\sqrt{2}\,\sin\left(x+\frac{\pi}{4}\right)$$



$$\cos x-\sin x$$
$$=\sqrt{2}\,\cos\left(x+\frac{\pi}{4}\right)$$



16a)

$$\sin x + \cos y = 1$$

$$\cos x - \sin y \cdot \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{\cos x}{\sin y}$$

16c)

$$\sin(x+y) = \cos(x-y)$$

$$\cos(x+y) \times \frac{d(x+y)}{dx} = -\sin(x-y) \times \frac{d(x-y)}{dx}$$

$$\cos(x+y)\left(1+\frac{dy}{dx}\right) = -\sin(x-y)\left[1-\frac{dy}{dx}\right]$$

$$\cos(x+y) + \cos(x+y)\cdot\frac{dy}{dx} = -\sin(x-y) + \sin(x-y)\frac{dy}{dx}$$

$$\left[\sin(x-y) - \cos(x+y)\right]\frac{dy}{dx} = \sin(x-y) + \cos(x+y)$$

$$\frac{dy}{dx} = \frac{\sin(x-y) + \cos(x+y)}{\sin(x-y) - \cos(x+y)}$$