12 a) $a > b$

$\underline{\underline{-a < -b}}$ (multiplying by -1, changes inequality sign)

b) $a > b$

$a \times b^2 > b \times b^2$ $(b^2 > 0)$

$\underline{ab^2 > b^3}$

18f) If $0 < a < b$

then $\sqrt{a^2 + b^2} = a + b$

FALSE

$a = 1 \quad b = 2$

$\sqrt{1^2 + 2^2} \neq 1 + 2$

22

$x^2 + y^2 \geq 2xy$

a) $\boxed{a + \frac{1}{a} \geq 2}$

$x^2 + y^2 \geq 2xy$

let $x = \sqrt{a}$ $\quad y = \frac{1}{\sqrt{a}}$

$(\sqrt{a})^2 + \left(\frac{1}{\sqrt{a}}\right)^2 \geq 2(\sqrt{a})\left(\frac{1}{\sqrt{a}}\right)$

$a + \frac{1}{a} \geq 2$

b) $\boxed{\frac{a+b}{2} \geq \sqrt{ab}}$

$$x^2 + y^2 \geq 2xy$$

let $x = \sqrt{a}$ $\quad y = \sqrt{b}$

$$(\sqrt{a})^2 + (\sqrt{b})^2 \geq 2(\sqrt{a})(\sqrt{b})$$

$$a + b \geq 2\sqrt{ab}$$

$$\frac{a+b}{2} \geq \sqrt{ab}$$

23/ Prove $x^2 + xy + y^2 > 0$

$$x^2 + xy + y^2$$
$$= \frac{1}{4}x^2 + xy + y^2 + \frac{3}{4}x^2$$
$$= \left(\frac{1}{2}x + y\right)^2 + \frac{3}{4}x^2 > 0$$

24 a) $(x+y)^2 \geq 4xy$

$(x+y)^2 - 4xy$

$= x^2 + 2xy + y^2 - 4xy$

$= x^2 - 2xy + y^2$

$= (x-y)^2$

≥ 0

$\therefore (x+y)^2 \geq 4xy$

24 a) $(x+y)^2 \geq 4xy$

b) Prove $\dfrac{1}{x^2} + \dfrac{1}{y^2} \geq \dfrac{4}{x^2+y^2}$

$$\left(\frac{1}{x} + \frac{1}{y}\right)^2 \geq \frac{4}{xy}$$

$$\frac{1}{x^2} + \frac{2}{xy} + \frac{1}{y^2} \geq \frac{4}{xy}$$

$$\frac{1}{x^2} + \frac{1}{y^2} \geq \frac{2}{xy} \quad \ldots\ldots \quad 1$$

Prove $\frac{2}{xy} \geq \frac{4}{x^2+y^2}$

$$a^2+b^2 \geq 2ab$$

$$\frac{a^2+b^2}{2} \geq ab$$

$$\frac{2}{a^2+b^2} \leq \frac{1}{ab}$$

$$\frac{4}{a^2+b^2} \leq \frac{2}{ab} \quad \ldots \text{(2)}$$

Using (2)

$$\frac{2}{xy} \geq \frac{4}{x^2+y^2}$$

But from (1)

$$\frac{1}{x^2} + \frac{1}{y^2} \geq \frac{2}{xy}$$

$$\therefore \frac{1}{x^2} + \frac{1}{y^2} \geq \frac{4}{x^2+y^2}$$

b) OR Hence prove $\frac{1}{x^2} + \frac{1}{y^2} \geq \frac{4}{x^2+y^2}$

$$\boxed{(x+y)^2 \geq 4xy}$$

let $x = x^2$, $y = y^2$

$$(x^2+y^2)^2 \geq 4x^2y^2$$

$$x^4 + 2x^2y^2 + y^4 \geq 4x^2y^2$$

$$x^4 + y^4 \geq 2x^2y^2$$

$$\frac{x^4+y^4}{x^2y^2} \geq 2$$

$$\frac{x^4+y^4}{x^2y^2} + 2 \geq 4$$

$$\frac{x^4+y^4}{x^2y^2} + 1 + 1 \geq 4$$

$$\frac{x^4+y^4}{x^2y^2} + \frac{x^2}{x^2} + \frac{y^2}{y^2} \geq 4$$

$$\frac{x^2}{x^2} + \frac{x^4+y^4}{x^2y^2} + \frac{y^2}{y^2} \geq 4$$

$$\frac{x^2}{x^2} + \frac{x^2}{y^2} + \frac{y^2}{x^2} + \frac{y^2}{y^2} \geq 4$$

$$\left(\frac{1}{x^2} + \frac{1}{y^2}\right)\left(x^2+y^2\right) \geq 4$$

$$\frac{1}{x^2} + \frac{1}{y^2} \geq \frac{4}{x^2+y^2}$$

OR let $x = x^2, \quad y = y^2$

$$(x^2+y^2)^2 \geqslant 4x^2y^2$$

$$\frac{x^2+y^2}{x^2y^2} \geqslant \frac{4}{x^2+y^2}$$

$$\frac{1}{y^2} + \frac{1}{x^2} \geqslant \frac{4}{x^2+y^2}$$

OR

$$\frac{1}{x^2} + \frac{1}{y^2} - \frac{4}{x^2+y^2} = \frac{y^2(x^2+y^2) + x^2(x^2+y^2) - 4x^2y^2}{x^2y^2(x^2+y^2)}$$

$$= \frac{(x^2+y^2)^2 - 4x^2y^2}{x^2y^2(x^2+y^2)}$$

$$\geq \frac{4x^2y^2 - 4x^2y^2}{x^2y^2(x^2+y^2)} \quad \because (x^2+y^2)^2 \geq 4x^2y^2$$

$$= 0$$