(ii) $\cos^4 x + \cos^2 x \sin^2 x = \cos^2 x$

$\cos^4 x + \cos^2 x \sin^2 x$

$= \cos^2 x(\cos^2 x + \sin^2 x)$

$= \underline{\underline{\cos^2 x}}$

12b) $(\cos\phi + \cot\phi)\sec\phi = 1 + \operatorname{cosec}\phi$

$$(\cos\phi + \cot\phi)\sec\phi$$

$$= \left(\cos\phi + \frac{\cos\phi}{\sin\phi}\right)\frac{1}{\cos\phi}$$

$$= 1 + \frac{1}{\sin\phi}$$

$$= \underline{1 + \operatorname{cosec}\phi}$$

12j)

$$\frac{\cos\alpha}{1+\sin\alpha} = \sec\alpha(1-\sin\alpha)$$

$$\frac{\cos\alpha}{1+\sin\alpha} = \frac{\cos\alpha}{1+\sin\alpha} \times \frac{1-\sin\alpha}{1-\sin\alpha}$$

$$= \frac{\cos\alpha(1-\sin\alpha)}{1-\sin^2\alpha}$$

$$= \frac{\cos\alpha(1-\sin\alpha)}{\cos^2\alpha}$$

$$= \frac{1-\sin\alpha}{\cos\alpha}$$

$$= \underline{\sec\alpha(1-\sin\alpha)}$$

13c) $x = r\cos\theta\sin\phi$

$y = r\sin\theta\sin\phi$

$z = r\cos\phi$

$$x^2+y^2+z^2 = r^2\cos^2\theta\sin^2\phi + r^2\sin^2\theta\sin^2\phi + r^2\cos^2\phi$$

$$= r^2\left\{\sin^2\phi(\cos^2\theta+\sin^2\theta)+\cos^2\phi\right\}$$

$$= r^2(\sin^2\phi+\cos^2\phi)$$

$$= \underline{\underline{r^2}}$$

14

a) $x = a\cos\theta \Rightarrow \cos\theta = \frac{x}{a}$

$y = b\sin\theta \Rightarrow \sin\theta = \frac{y}{b}$

$\sin^2\theta + \cos^2\theta = 1$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

14b)

$x = a\tan\theta$ $\qquad$ $y = b\sec\theta$

$\tan\theta = \frac{x}{a}$ $\qquad$ $\sec\theta = \frac{y}{b}$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \frac{x^2}{a^2} = \frac{y^2}{b^2}$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

14d)

$$x = \sin\theta + \cos\theta$$

$$y = \sin\theta - \cos\theta$$

$$x^2 = \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta \qquad +$$

$$y^2 = \sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta$$

$$x^2 + y^2 = 2\sin^2\theta + 2\cos^2\theta$$

$$\underline{\underline{x^2 + y^2 = 2}}$$

15c)

$$\frac{\tan\theta + \cot\theta}{\sec\theta \operatorname{cosec}\theta} = \frac{\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}}{\dfrac{1}{\cos\theta} \times \dfrac{1}{\sin\theta}}$$

$$= \frac{\dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}}{\dfrac{1}{\cos\theta\sin\theta}}$$

$$= \frac{1}{1}$$

$$= \underline{\underline{1}}$$ which is independant of θ.

15d)

$$\frac{\tan\theta + 1}{\sec\theta} - \frac{\cot\theta + 1}{\operatorname{cosec}\theta}$$

$$= \frac{\frac{\sin\theta}{\cos\theta} + 1}{\frac{1}{\cos\theta}} - \frac{\frac{\cos\theta}{\sin\theta} + 1}{\frac{1}{\sin\theta}}$$

$$= \sin\theta + \cos\theta - (\cos\theta + \sin\theta)$$

$$= \underline{\underline{0}}$$

16a)

$$\frac{2\cos^3\theta - \cos\theta}{\sin\theta\cos^2\theta - \sin^3\theta} = \cot\theta$$

$$\frac{2\cos^3\theta - \cos\theta}{\sin\theta\cos^2\theta - \sin^3\theta} = \frac{\cos\theta(2\cos^2\theta - 1)}{\sin\theta(\cos^2\theta - \sin^2\theta)}$$

$$= \frac{\cos\theta(2\cos^2\theta - 1)}{\sin\theta(\cos^2\theta - (1 - \cos^2\theta))}$$

$$= \frac{\cos\theta(2\cos^2\theta - 1)}{\sin\theta(2\cos^2\theta - 1)}$$

$$= \frac{\cos\theta}{\sin\theta}$$

$$= \underline{\underline{\cot\theta}}$$

16b) $\sec y + \tan y + \cot y \equiv \dfrac{1+\sin y}{\sin y \cos y}$

$$\sec y + \tan y + \cot y = \frac{1}{\cos y} + \frac{\sin y}{\cos y} + \frac{\cos y}{\sin y}$$

$$= \frac{\sin y + \sin^2 y + \cos^2 y}{\sin y \cos y}$$

$$= \frac{\sin y + 1}{\sin y \cos y}$$

16c) $\dfrac{\cos A - \tan A \sin A}{\cos A + \tan A \sin A} = 1 - 2\sin^2 A$

$$\frac{\cos A - \tan A \sin A}{\cos A + \tan A \sin A} = \frac{\cos A - \frac{\sin A}{\cos A}\cdot \sin A}{\cos A + \frac{\sin A}{\cos A}\cdot \sin A}$$

$$= \frac{\cos^2 A - \sin^2 A}{\cos^2 A + \sin^2 A}$$

$$= \cos^2 A - \sin^2 A$$

$$= 1 - \sin^2 A - \sin^2 A$$

$$= 1 - 2\sin^2 A$$

16(d) $(\sin\phi + \cos\phi)(\sec\phi + \operatorname{cosec}\phi) \equiv 2 + \tan\phi + \cot\phi$

$(\sin\phi + \cos\phi)(\sec\phi + \operatorname{cosec}\phi)$

$= \dfrac{\sin\phi}{\cos\phi} + 1 + 1 + \dfrac{\cos\phi}{\sin\phi}$

$= \underline{\tan\phi + 2 + \cot\phi}$

16e) $$\frac{1}{1+\tan^2 x} - \frac{1}{1+\sec^2 x} = \frac{\cos^4 x}{1+\cos^2 x}$$

$$\frac{1}{1+\tan^2 x} - \frac{1}{1+\sec^2 x} = \frac{1}{\sec^2 x} - \frac{1}{1+\sec^2 x}$$

$$= \cos^2 x - \frac{1}{1+\frac{1}{\cos^2 x}}$$

$$= \cos^2 x - \frac{\cos^2 x}{\cos^2 x + 1}$$

$$= \frac{\cos^4 x + \cos^2 x - \cos^2 x}{\cos^2 x + 1}$$

$$= \frac{\cos^4 x}{\cos^2 x + 1}$$

16f) $\dfrac{\cos\theta}{1-\tan\theta}+\dfrac{\sin\theta}{1-\cot\theta}\equiv\sin\theta+\cos\theta$

$$\frac{\cos\theta}{1-\tan\theta}+\frac{\sin\theta}{1-\cot\theta}=\frac{\cos\theta}{1-\frac{\sin\theta}{\cos\theta}}+\frac{\sin\theta}{1-\frac{\cos\theta}{\sin\theta}}$$

$$=\frac{\cos^2\theta}{\cos\theta-\sin\theta}+\frac{\sin^2\theta}{\sin\theta-\cos\theta}$$

$$=\frac{\cos^2\theta-\sin^2\theta}{\cos\theta-\sin\theta}$$

$$=\frac{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)}{(\cos\theta-\sin\theta)}$$

$$=\underline{\cos\theta+\sin\theta}$$

1(g) $\left(\tan\alpha+\cot\alpha-1\right)\left(\sin\alpha+\cos\alpha\right)=\dfrac{\sec\alpha}{\operatorname{cosec}^2\alpha}+\dfrac{\operatorname{cosec}\alpha}{\sec^2\alpha}$

$$\left(\tan\alpha+\cot\alpha-1\right)\left(\sin\alpha+\cos\alpha\right)=\left(\frac{\sin\alpha}{\cos\alpha}+\frac{\cos\alpha}{\sin\alpha}-1\right)\left(\sin\alpha+\cos\alpha\right)$$

$$=\frac{\sin^2\alpha}{\cos\alpha}+\cos\alpha-\sin\alpha+\sin\alpha+\frac{\cos^2\alpha}{\sin\alpha}-\cos\alpha$$

$$=\frac{\sin^2\alpha}{\cos\alpha}+\frac{\cos^2\alpha}{\sin\alpha}$$

$$=\frac{\frac{1}{\operatorname{cosec}^2\alpha}}{\frac{1}{\sec\alpha}}+\frac{\frac{1}{\sec^2\alpha}}{\frac{1}{\operatorname{cosec}\alpha}}$$

$$=\frac{\sec\alpha}{\operatorname{cosec}^2\alpha}+\frac{\operatorname{cosec}\alpha}{\sec^2\alpha}$$

16h) $\dfrac{1}{\sec\theta+\tan\theta} = \sec\theta - \tan\theta = \dfrac{\cos\theta}{1+\sin\theta}$

$$\frac{1}{\sec\theta+\tan\theta} = \frac{1}{\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}}$$

$$= \frac{\cos\theta}{1+\sin\theta}$$

$$= \frac{\cos\theta}{1+\sin\theta} \times \frac{1-\sin\theta}{1-\sin\theta}$$

$$= \frac{\cos\theta(1-\sin\theta)}{1-\sin^2\theta}$$

$$= \frac{\cos\theta(1-\sin\theta)}{\cos^2\theta}$$

$$= \frac{1-\sin\theta}{\cos\theta}$$

$$= \sec\theta - \tan\theta$$

1b i) $\dfrac{1}{\cot\theta - \cos\theta} \equiv \dfrac{\tan\theta}{1-\sin\theta} \equiv \dfrac{\sin\theta + \sin^2\theta}{\cos^3\theta}$

$$\frac{1}{\cot\theta - \cos\theta} = \frac{1}{\frac{\cos\theta}{\sin\theta} - \cos\theta}$$

$$= \frac{\sin\theta}{\cos\theta - \sin\theta\cos\theta}$$

$$= \frac{\sin\theta}{\cos\theta(1-\sin\theta)}$$

$$= \frac{\tan\theta}{1-\sin\theta}$$

$$\frac{\sin\theta}{\cos\theta(1-\sin\theta)}$$

$$= \frac{\sin\theta}{\cos\theta(1-\sin\theta)} \times \frac{1+\sin\theta}{1+\sin\theta}$$

$$= \frac{\sin\theta + \sin^2\theta}{\cos\theta(1-\sin^2\theta)}$$

$$= \frac{\sin\theta + \sin^2\theta}{\cos\theta\cos^2\theta}$$

$$= \frac{\sin\theta + \sin^2\theta}{\cos^3\theta}$$

16j) $\sin^2 x\left(1+n\cot^2 x\right)+\cos^2 x\left(1+n\tan^2 x\right) \equiv n+1$

$\equiv \sin^2 x\left(n+\cot^2 x\right)+\cos^2 x\left(n+\tan^2 x\right)$

$\sin^2 x\left(1+n\cot^2 x\right)+\cos^2 x\left(1+n\tan^2 x\right)$

$= \sin^2 x\left(1+\frac{n\cos^2 x}{\sin^2 x}\right)+\cos^2 x\left(1+\frac{n\sin^2 x}{\cos^2 x}\right)$

$= \sin^2 x+n\cos^2 x+\cos^2 x+n\sin^2 x$

$= \left(\sin^2 x+\cos^2 x\right)\left(1+n\right)$

$= \underline{1+n}$

$$\sin^2 x\left(n+\cot^2 x\right)+\cos^2 x\left(n+\tan^2 x\right)$$

$$=\sin^2 x\left(n+\frac{\cos^2 x}{\sin^2 x}\right)+\cos^2 x\left(n+\frac{\sin^2 x}{\cos^2 x}\right)$$

$$=n\sin^2 x+\cos^2 x+n\cos^2 x+\sin^2 x$$

$$=\left(\sin^2 x+\cos^2 x\right)(n+1)$$

$$=\underline{\underline{n+1}}$$

sinα

16k)

$$\frac{(\sin^2\alpha - \cos^2\alpha)(1 - \sin\alpha\cos\alpha)}{\cos\alpha(\sec\alpha - \operatorname{cosec}\alpha)(\sin^3\alpha + \cos^3\alpha)}$$

$$= \frac{\cancel{(\sin\alpha + \cos\alpha)}(\sin\alpha - \cos\alpha)(1 - \sin\alpha\cos\alpha)}{\cos\alpha\left(\frac{1}{\cos\alpha} - \frac{1}{\sin\alpha}\right)\cancel{(\sin\alpha + \cos\alpha)}(\sin^2\alpha - \sin\alpha\cos\alpha + \cos^2\alpha)}$$

$$= \frac{\cancel{(\sin\alpha - \cos\alpha)}\cancel{(1 - \sin\alpha\cos\alpha)}}{\cos\alpha\left(\frac{\cancel{\sin\alpha - \cos\alpha}}{\cos\alpha\sin\alpha}\right)\cancel{(1 - \sin\alpha\cos\alpha)}}$$

$$= \frac{1}{\frac{\cos\alpha}{\cos\alpha\sin\alpha}}$$

$$= \sin\alpha$$

16 l)

$$\frac{1+\operatorname{cosec}^2 A \tan^2 C}{1+\operatorname{cosec}^2 B \tan^2 C} = \frac{1+\cot^2 A \sin^2 C}{1+\cot^2 B \sin^2 C}$$

$$\frac{1+\operatorname{cosec}^2 A \tan^2 C}{1+\operatorname{cosec}^2 B \tan^2 C} = \frac{1+\frac{1}{\sin^2 A}\cdot\frac{\sin^2 C}{\cos^2 C}}{1+\frac{1}{\sin^2 B}\cdot\frac{\sin^2 C}{\cos^2 C}}$$

$$= \frac{\cos^2 C + \operatorname{cosec}^2 A \sin^2 C}{\cos^2 C + \operatorname{cosec}^2 B \sin^2 C}$$

$$= \frac{\cos^2 C + (1+\cot^2 A)\sin^2 C}{\cos^2 C + (1+\cot^2 B)\sin^2 C}$$

$$= \frac{\cos^2 C + \sin^2 C + \cot^2 A \sin^2 C}{\cos^2 C + \sin^2 C + \cot^2 B \sin^2 C}$$

$$= \frac{1+\cot^2 A \sin^2 C}{1+\cot^2 B \sin^2 C}$$

17a)

$x = \operatorname{cosec}^2\theta + 2\cot^2\theta$ $\quad$ $y = 2\operatorname{cosec}^2\theta + \cot^2\theta$

$1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$1 + y - 2\operatorname{cosec}^2\theta = x - 2\cot^2\theta$

$x - y - 1 = 2\cot^2\theta - 2\operatorname{cosec}^2\theta$

$x - y - 1 = -2$

$\underline{x - y + 1 = 0}$

17b)

$x = \sin\theta - 3\cos\theta$

$y = \sin\theta + 2\cos\theta$

$(x+y)^2 = (2\sin\theta - \cos\theta)^2$

$= 4\sin^2\theta - 4\sin\theta\cos\theta + \cos^2\theta$

$(x+y)^2 = 1 + 3\sin^2\theta - 4\sin\theta\cos\theta$ $+$

$y^2 = \sin^2\theta + 4\sin\theta\cos\theta + 4\cos^2\theta$

$(x+y)^2 + y^2 = 1 + 4\sin^2\theta + 4\cos^2\theta$

$x^2 + 2xy + 2y^2 = 5$

17 c)

$$x = \sin\theta + \cos\theta$$

$$y = \tan\theta + \cot\theta$$

$$x^2 y = (\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta)(\tan\theta + \cot\theta)$$

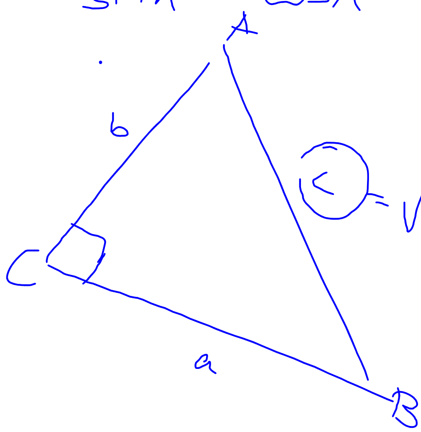
$$= (1 + 2\sin\theta\cos\theta)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$$

$$= (1 + 2\sin\theta\cos\theta)\left(\frac{1}{\sin\theta\cos\theta}\right)$$

$$= \frac{1}{\sin\theta\cos\theta} + 2$$

$$= \underline{y + 2}$$

18a) $\dfrac{a}{\sin A} = \dfrac{b}{\cos A}$



$c = \sqrt{a^2+b^2}$

$\sin A \cos A$

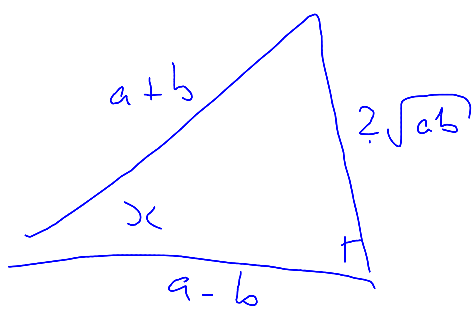
$$= \frac{a}{\sqrt{a^2+b^2}} \times \frac{b}{\sqrt{a^2+b^2}}$$

$$= \frac{ab}{a^2+b^2}$$

b) $\dfrac{a+b}{\operatorname{cosec} x} = \dfrac{a-b}{\cot x}$

$$(a+b)\sin x = \frac{(a-b)\sin x}{\cos x}$$

$$\cos x = \frac{a-b}{a+b}$$



$$y^2 = (a+b)^2 - (a-b)^2$$
$$= 4ab$$

$\operatorname{cosec} x \cot x$

$$= \frac{a+b}{2\sqrt{ab}} \times \frac{a-b}{2\sqrt{ab}}$$

$$= \frac{a^2-b^2}{4ab}$$

18 c) $\tan\theta + \sin\theta = x$

$\tan\theta - \sin\theta = y$

$x^4 + y^4 = 2xy(8 + xy)$

$x^4 + y^4$

$= (\tan\theta + \sin\theta)^4 + (\tan\theta - \sin\theta)^4$

$= 2\tan^4\theta + \underline{12\tan^2\theta\sin^2\theta} + 2\sin^4\theta$

$=$

$2xy(8+xy)$

$= 2\left[(\tan^2\theta - \sin^2\theta)(8 + \tan^2\theta - \sin^2\theta)\right]$

$= 2\left[8\tan^2\theta + \tan^4\theta - \tan^2\theta\sin^2\theta - 8\sin^2\theta - \tan^2\theta\sin^2\theta + \sin^4\theta\right)$

$= 2\left(\tan^4\theta + \sin^4\theta - 2\tan^2\theta\sin^2\theta + 8\left[\frac{\sin^2\theta - \sin^2\theta\cos^2\theta}{\cos^2\theta}\right]\right)$

$= 2\left(\tan^4\theta + \sin^4\theta - 2\tan^2\theta\sin^2\theta + 8\left(\frac{\sin^2\theta(1-\cos^2\theta}{\cos^2\theta}\right)\right.$