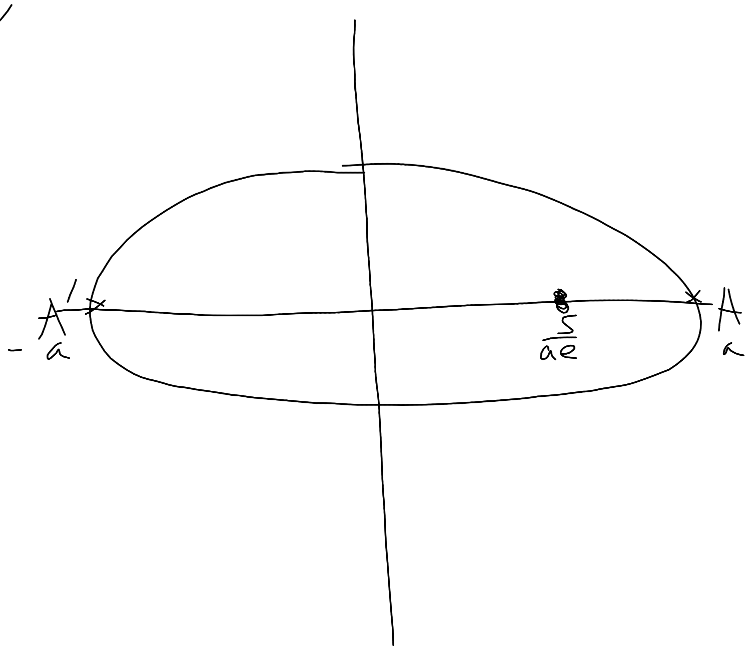
8/



$$\frac{AS'}{AS} = \frac{30}{29}$$

$$\frac{a+ae}{a-ae} = \frac{30}{29}$$

$$\frac{1+e}{1-e} = \frac{30}{29}$$

$$29+29e = 30-30e$$

$$59e = 1$$

$$e = \frac{1}{59}$$

11(c) $x = 3\cos\theta$ $y = 2\sin\theta$

$\frac{dx}{d\theta} = -3\sin\theta$ $\frac{dy}{d\theta} = 2\cos\theta$

$\frac{dy}{dx} = -\frac{2\cos\theta}{3\sin\theta}$

When $\theta = \frac{\pi}{6}$, $\frac{dy}{dx} = \frac{-4\sqrt{3}}{3}$

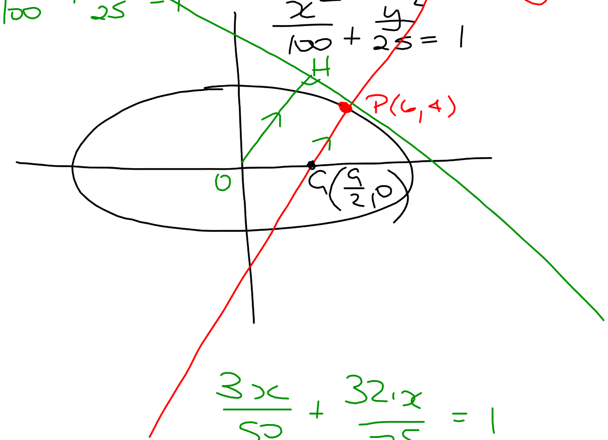
$\therefore$ slope of normal is $\frac{3}{4\sqrt{3}}$

12b)

$\frac{6x}{100} + \frac{4y}{25} = 1$

$\frac{x^2}{100} + \frac{y^2}{25} = 1$

$8x - 3y = 36$



$PG \times OH = 25$

$m_{PG} = \frac{4}{6 - \frac{9}{2}}$

$= \frac{8}{3}$

OH is $y = \frac{8}{3}x$

$\frac{3x}{50} + \frac{32x}{75} = 1$

$x = \frac{150}{73}$ H is $\left(\frac{150}{73}, \frac{400}{73}\right)$

$PG \times OH = \sqrt{\left(6 - \frac{9}{2}\right)^2 + 4^2} \times \sqrt{\left(\frac{150}{73}\right)^2 + \left(\frac{400}{73}\right)^2}$

$= 25$

16/ $y = mx + b$ is tangent to $\frac{x^2}{9} + y^2 = 1$

a) $$\frac{x^2}{9} + (mx+b)^2 = 1$$

$$(9m^2+1)x^2 + 18bmx + (9b^2-9) = 0$$

$$\Delta = 0$$

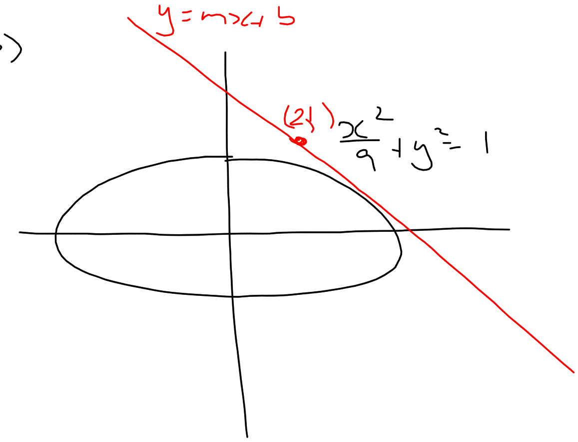
$$324b^2m^2 - 4(9m^2+1)(9b^2-9) = 0$$

$$9b^2m^2 - (9m^2+1)(b^2-1) = 0$$

$$9b^2m^2 - 9m^2b^2 + 9m^2 - b^2 + 1 = 0$$

$$\underline{\underline{b^2 = 9m^2+1}}$$

b)



$b^2 = 9m^2 + 1$

$(2,1): \; 1 = 2m + b$

$b = 1 - 2m$

$\therefore (1-2m)^2 = 9m^2 + 1$

$1 - 4m + 4m^2 = 9m^2 + 1$

$5m^2 + 4m = 0$

$m(5m+4) = 0$

$m = 0$ or $m = -\frac{4}{5}$

$y - 1 = -\frac{4}{5}(x-2)$

$5y - 5 = -4x + 8$

$4x + 5y - 13 = 0$

$\therefore$ tangents are

$4x + 5y - 13 = 0$ and $y = 1$

17/ ellipse $0 \leq e \leq 1$

$\frac{b}{a} = ?$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $\quad e^2 = \frac{a^2 - b^2}{a^2}$

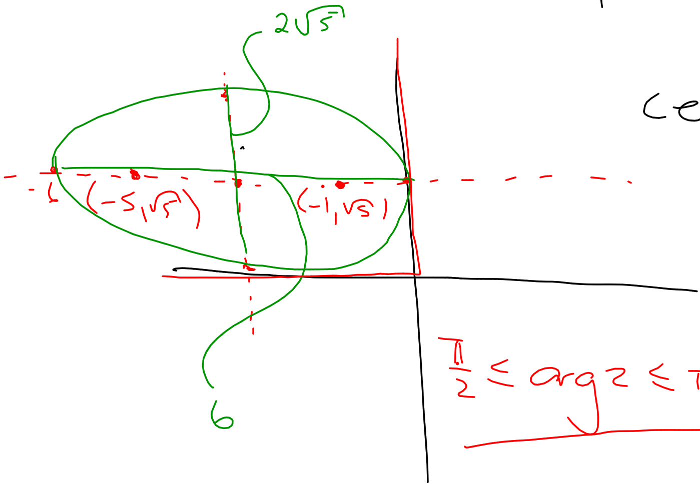
$e \to 0, \quad a^2 - b^2 \to 0$

$\frac{b}{a} \to 1$

$e \to 1 \quad a^2 - b^2 \to a^2$

$\frac{b}{a} \to 0$

18/ $$|z+1-i\sqrt{5}| + |z+5-i\sqrt{5}| = 6$$



$$\frac{\pi}{2} \le \arg z \le \pi$$

centre $= -3 + i\sqrt{5}$

$2a = 6$

$a = 3$

$ae = 2$

$e = \frac{2}{3}$ $\quad b^2 = 9\left(1 - \frac{4}{9}\right)$

$= 5$

19a) $$\frac{x^2}{4-\lambda} + \frac{y^2}{2-\lambda} = 1$$

$4-\lambda > 0$ $\quad$ $2-\lambda > 0$ $\quad$ $e^2 = \frac{a^2-b^2}{a^2}$

$\lambda < 4$ $\quad$ $\lambda < 2$

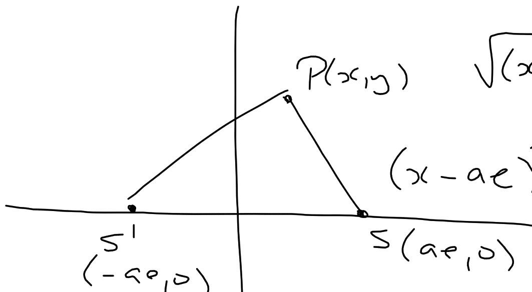
b) as $\lambda \mapsto 2$

$\underline{\underline{\lambda < 2}}$

$b^2 \to 0$

$e^2 \to 1$

20/



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$\sqrt{(x-ae)^2+y^2}+\sqrt{(x+ae)^2+y^2}=2a$$

$$(x-ae)^2+y^2+(x+ae)^2+y^2$$

$$=4a^2-2\sqrt{\left[(x-ae)^2+y^2\right]\left[(x+ae)^2+y^2\right]}$$

$$-4a^2+2y^2+2x^2+2a^2e^2=-2\sqrt{\left[(x-ae)^2+y^2\right]\left[(x+ae)^2+y^2\right]}$$

$$-2a^2+y^2+x^2+a^2e^2=-\sqrt{\left[(x-ae)^2+y^2\right]\left[(x+ae)^2+y^2\right]}$$

$$\left(x^2+y^2+a^2e^2-2a^2\right)^2=\left(x^2+a^2e^2+y^2-2aex\right)\left(x^2+a^2e^2+y^2+2aex\right)$$

$$=\left(x^2+a^2e^2+y^2\right)^2-4a^2e^2x^2$$

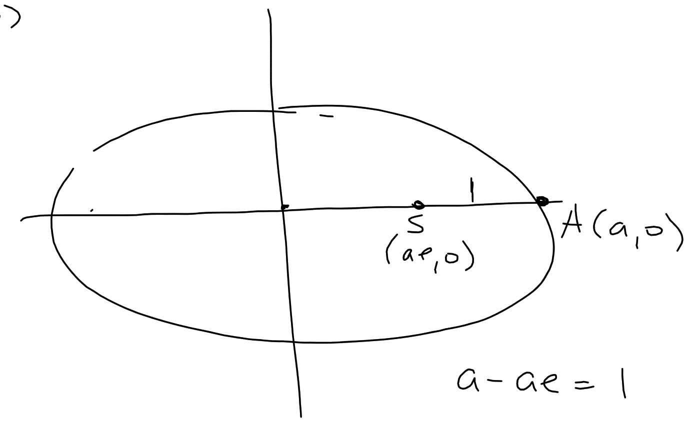
$$4a^4-4a^2x^2-4a^2y^2-4a^4e^2=-4a^2e^2x^2$$

$$a^2-x^2-y^2-a^2e^2=-e^2x^2$$

$$(1-e^2)x^2+y^2=a^2-a^2e^2$$

$$\frac{x^2}{a^2}+\frac{y^2}{a^2(1-e^2)}=1$$

22 b)



$a - ae = 1$

$b^2 = \frac{1+e}{1-e}$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\frac{(x+a)^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\frac{(x+a)^2}{a^2} + \frac{y^2(1-e)}{(1+e)} = 1$$

$$\boxed{a(1-e)=1}$$

$$\frac{(x+a)^2(1+e)}{a^2(1-e)} + y^2 = \frac{1+e}{1-e}$$

$$\frac{x^2(1+e)}{a^2(1-e)} + \frac{2ax(1+e)}{a^2(1-e)} + \frac{a^2(1+e)}{a^2(1-e)} - \frac{1+e}{1-e} + y^2 = 0$$

$$\frac{x^2(1+e)}{a} + \frac{2x(1+e)}{} + y^2 = 0$$

$$\frac{x^2(1+e)}{a} \times \frac{(1-e)}{(1-e)} + 2x(1+e) + y^2 = 0$$

$$x^2(1-e^2) + 2x(1+e) + y^2 = 0$$