1/

$$PS = \sqrt{(a\cos\theta - ae)^2 + (b\sin\theta)^2}$$

$$= \sqrt{a^2\cos^2\theta - 2a^2e\cos\theta + a^2e^2 + b^2\sin^2\theta}$$

$$= \sqrt{a^2\cos^2\theta - 2a^2e\cos\theta + a^2e^2 + a^2(1-e^2)\sin^2\theta}$$

$$= \sqrt{a^2\cos^2\theta - 2a^2e\cos\theta + a^2e^2 + a^2\sin^2\theta - a^2e^2\sin^2\theta}$$

$$= a\sqrt{1 - 2e\cos\theta + e^2 - e^2\sin^2\theta}$$

$$= a\sqrt{1 - 2e\cos\theta + e^2\cos^2\theta}$$

$$= a\sqrt{(1 - e\cos\theta)^2}$$

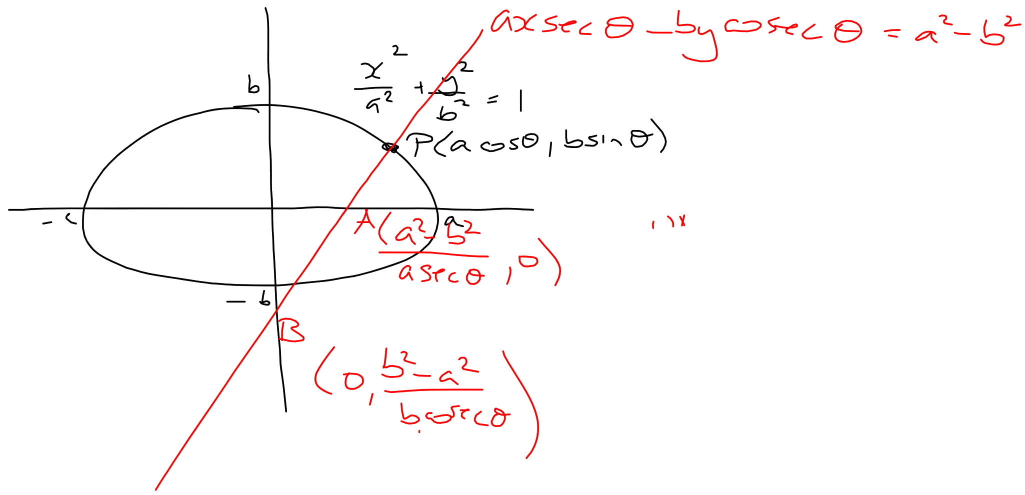
$$= \underline{\underline{a(1 - e\cos\theta)}}$$

$$PS' = a(1 + e\cos\theta)$$

$$PS + PS' = a(1 - e\cos\theta) + a(1 + e\cos\theta)$$

$$= \underline{\underline{2a}}$$

4c)


$ax\sec\theta - by\,\mathrm{cosec}\,\theta = a^2 - b^2$
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
$P(a\cos\theta, b\sin\theta)$
b
$-a$
a
$-b$
$A\left(\frac{a^2-b^2}{a\sec\theta}, 0\right)$
B
$\left(0, \frac{b^2-a^2}{b\,\mathrm{cosec}\,\theta}\right)$

$$PA = \sqrt{\left(a\cos\theta - \frac{a^2-b^2}{a}\cos\theta\right)^2 + b^2\sin^2\theta}$$

$$= \sqrt{\left(\frac{a^2-a^2+b^2}{a}\right)^2\cos^2\theta + b^2\sin^2\theta}$$

$$= b\sqrt{\frac{b^2\cos^2\theta}{a^2} + \sin^2\theta} = \frac{b}{a}\sqrt{b^2\cos^2\theta + a^2\sin^2\theta}$$

$$PB = \sqrt{a^2\cos^2\theta + \left(\frac{b^2-a^2}{b}\sin\theta - b\sin\theta\right)^2}$$

$$= \sqrt{a^2\cos^2\theta + \left(\frac{b^2-a^2-b^2}{b}\right)^2\sin^2\theta}$$

$$= \sqrt{a^2\cos^2\theta + \frac{a^4}{b^2}\sin^2\theta}$$

$$= \frac{a}{b}\sqrt{b^2\cos^2\theta + a^2\sin^2\theta}$$

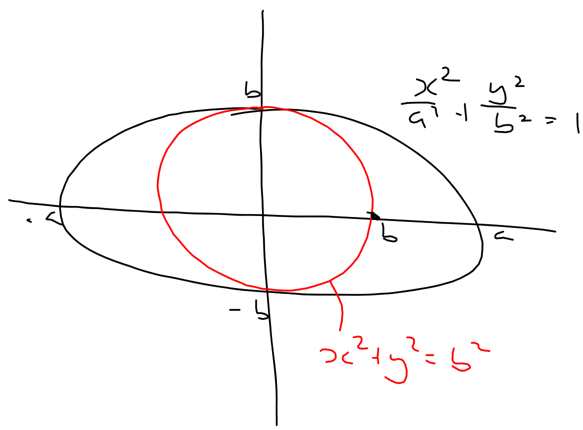
$$\frac{PA}{PB} = \frac{\frac{b}{a}\sqrt{b^2\cos^2\theta + a^2\sin^2\theta}}{\frac{a}{b}\sqrt{b^2\cos^2\theta + a^2\sin^2\theta}}$$

$$= \frac{b^2}{a^2}$$

$$= \frac{a^2(1-e^2)}{a^2}$$

$$= \underline{\underline{1-e^2}}$$

7b)



$$ae = b$$

$$e^2 = \frac{a^2-b^2}{a^2}$$

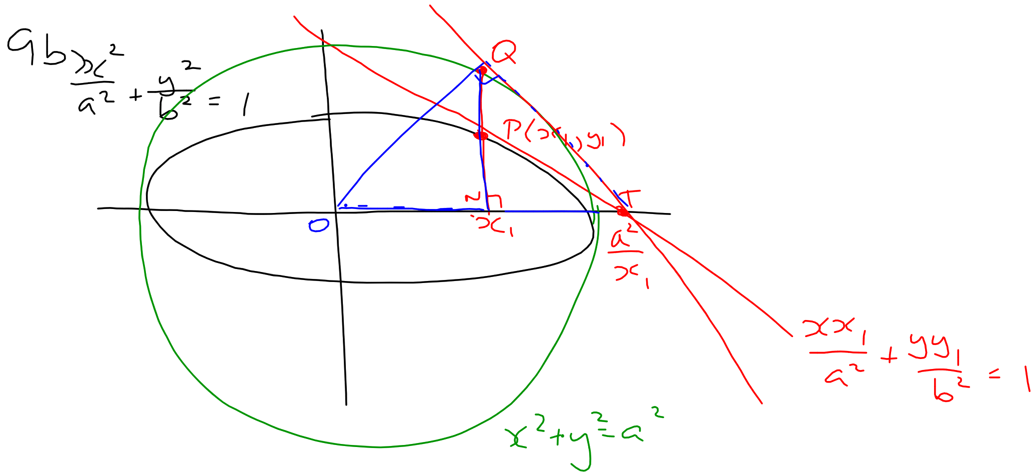
$$= \frac{a^2-a^2e^2}{a^2}$$

$$e^2 = 1-e^2$$

$$2e^2 = 1$$

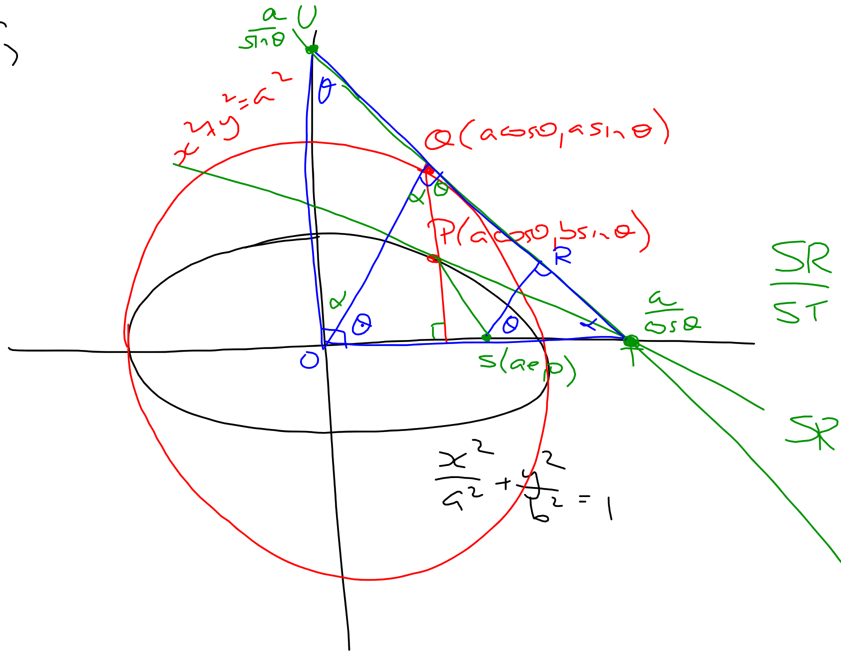
$$e^2 = \frac{1}{2}$$

$$e = \frac{1}{\sqrt{2}}$$



9b $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$
Q
$P(x_1, y_1)$
N
x_1
O
T
$\frac{a^2}{x_1}$
$\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$
$x^2+y^2=a^2$

13f)



$SR = SP$

$SP = a(1 - e\cos\theta)$

$$\frac{SR}{ST} = \frac{OU}{UT}$$

$$SR = \frac{ST \times OU}{UT}$$

$=$

$$SR = \frac{ST \times OU}{UT}$$

$$= \frac{\left(\frac{a}{\cos\theta} - ae\right) \times \frac{a}{\sin\theta}}{a\sqrt{\frac{1}{\sin^2\theta} + \frac{1}{\cos^2\theta}}}$$

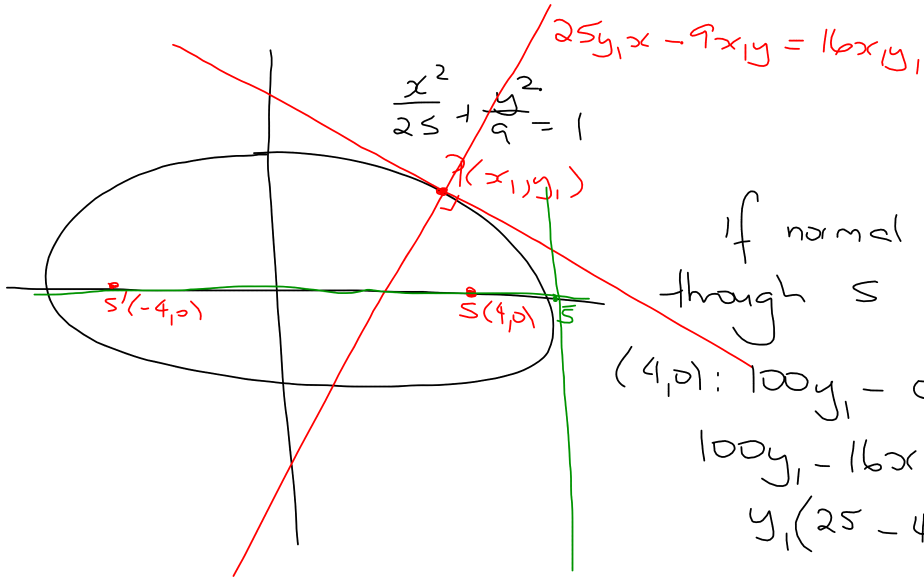
$$= \frac{1 - e\cos\theta}{\cos\theta} \times \frac{a}{\sin\theta} \times \sqrt{\frac{\sin^2\theta \cos^2\theta}{\sin^2\theta + \cos^2\theta}}$$

$$= \frac{(1 - e\cos\theta)}{\cos\theta} \times \frac{a}{\sin\theta} \times \frac{\sin\theta\cos\theta}{1}$$

$$= a(1 - e\cos\theta)$$

$$= \underline{\underline{SP}}$$

11d)



If normal passes through S

$(4,0)$: $100y_1 - 0 = 16x_1y_1$

$100y_1 - 16x_1y_1 = 0$

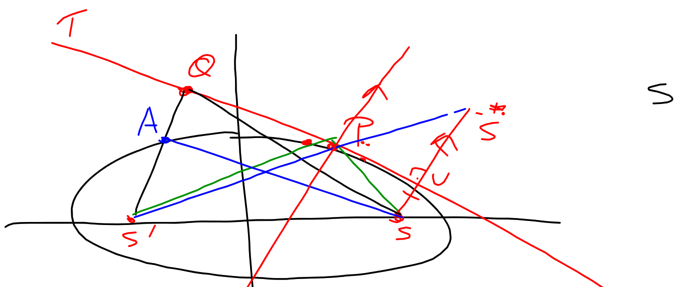
$y_1(25 - 4x_1) = 0$

$y_1 = 0$ or $x_1 = \frac{100}{16}$

$= \frac{25}{4}$

not ~~possible~~ as $-5 \leq x_1 \leq 5$

15



$S'Q + QS \geq S'P + PS.$

$S'Q + QS$

$= S'A + AQ + QS$

$> S'A + AQ + AS \quad (QS > AS)$

$> S'A + AS$

$= 2a$

$= S'P + PS$

$\therefore \underline{S'Q + QS \geq S'P + PS}$

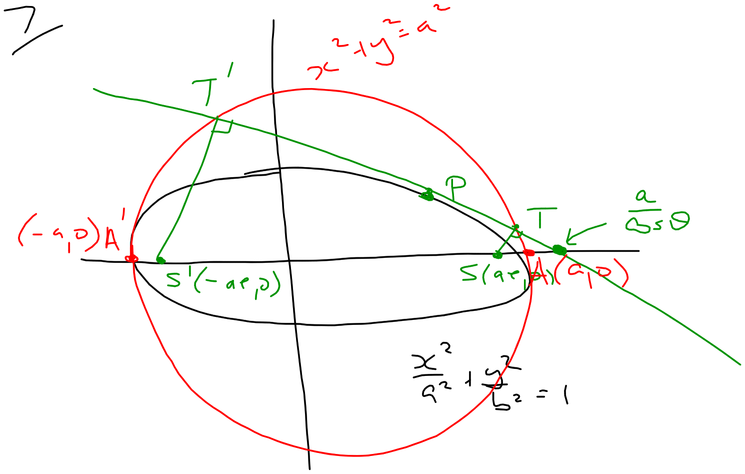
b) $\min(S'Q + QS) = S'P + PS$

$= S'P + PS^* \quad (PS = PS^*)$

$\therefore$ occurs when $Q = P$

$\therefore P$ lies on SS^*

17/



$\boxed{ST \times S'T' = b^2}$

$$\frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}=1$$

$$bx\cos\theta + ay\sin\theta = ab$$

$ST \times S'T'$

$$= \frac{\left| abe\cos\theta - ab \right|}{\sqrt{b^2\cos^2\theta + a^2\sin^2\theta}} \times \frac{\left| -abe\cos\theta - ab \right|}{\sqrt{b^2\cos^2\theta + a^2\sin^2\theta}}$$

$$= \frac{a^2b^2 \left| (e\cos\theta - 1)(e\cos\theta + 1) \right|}{b^2\cos^2\theta + a^2\sin^2\theta}$$

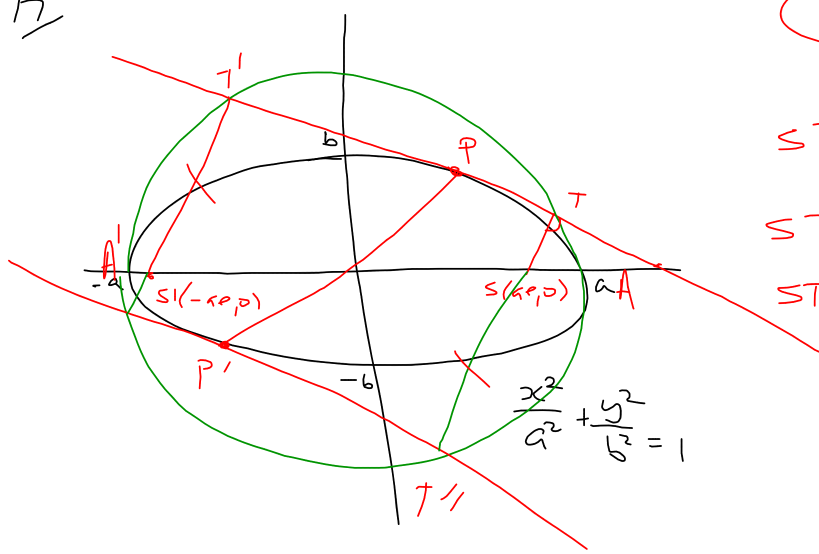
$$= b^2 \left[\frac{\frac{b^2}{(1-e^2)} \left| e^2\cos^2\theta - 1 \right|}{b^2\cos^2\theta + \frac{b^2}{(1-e^2)}\sin^2\theta} \right]$$

$$= b^2 \left[\frac{\left| e^2\cos^2\theta - 1 \right|}{\cos^2\theta - e^2\cos^2\theta + \sin^2\theta} \right]$$

$$= b^2 \left(\frac{1 - e^2\cos^2\theta}{1 - e^2\cos^2\theta} \right)$$

$$= \underline{\underline{b^2}}$$

17/



$$ST \times S'T' = b^2$$

$$ST'' = S'T'$$

$$ST \times ST'' = AS \times A'S$$

$$ST \times S'T' = (a-ae)(a+ae)$$

$$= a^2 - a^2e^2$$

$$= a^2(1-e^2)$$

$$= b^2$$