Insertion Method

Useful when some objects are NOT allowed to be together

e.g. The letters of the word **BETWEEN** are arranged in a line. In how many ways can they be arranged if;

(*i*) All the **E**'s are separated?

Traditional method: *work out the number of ways the E's can be placed*

___	___	___	___	___	___	___	
E		**E**		**E**	**E**	**E**	3
E			**E**		**E**	**E**	2
E				**E**		**E**	1
	E		**E**		**E**	**E**	2
	E			**E**		**E**	1
		E		**E**		**E**	1
							10

$$\text{Ways} = 10 \times 4!$$
$$= 240$$

Insertion method*: place the unrestricted objects first, creating spots for the E's to be placed.*

___ **B** ___ **T** ___ **W** ___ **N** ___

Once **B, T, W** and **N** have ben arranged there are now 5 spaces that the **E**'s can go (note: order not important as objects are the same)

$$\text{Ways } = 4! \times {}^5C_3$$
$$= \underline{240}$$

(ii) Exactly two of the **E**'s are together?

(note: this time order is important as objects are not the same **EE** and **E**)

$$\text{Ways } = 4! \times {}^5P_2$$
$$= \underline{480}$$

Using Separators

Useful when dividing large groups into smaller groups

e.g. Three pirates are sharing out the contents of a treasure chest containing forty-eight gold coins and two lead coins.

The first pirate takes out coins one at a time until a lead coin is taken.

The second pirate then takes out coins one at a time until the second lead coin is taken.

The third pirate then takes all of the remaining coins.

In how many ways can the coins be distributed?

The question is equivalent to how many ways can 2 L's and 48 G's be arranged. (The 2 L's act as separators of the 3 pirates)

$$\text{Ways} = \frac{50!}{48!2!}$$

$$= \underline{1225}$$

2012 Extension 2 HSC Question 16 a) (ii)

In how many ways can 10 identical coins be allocated to 4 different boxes?

The question is equivalent to how many ways can 3 S's and 10C's be arranged. (The 3 S's act as separators of the 4 boxes)

$$\text{Ways} = \frac{13!}{3!10!}$$

$$= \underline{286}$$

Note: 1% of the state got this correct!!!

2013 Extension 2 HSC Question 10

A hostel has four vacant rooms. Each room can accommodate a maximum of four people.

In how many ways can six people be accommodated in the four rooms?

$$\text{Total Ways no restrictions} = 4^6$$

$$= 4096$$

Each person has a choice of 4 rooms

$$\text{Less ways with 6 in a room} = {}^4C_1$$

$$= 4$$

Choose which of the 4 rooms will have the six people

$$\text{Less ways with 5 in a room} = {}^4C_1 \times {}^6C_5 \times {}^3C_1$$

$$= 72$$

Choose which of the 4 rooms will have the five people, then choose the five people to go in that room, then which of the remaining rooms will have one

$$\text{Ways} = 4096 - 4 - 72$$

$$= \underline{4020}$$

Note: 15% of the state got this correct!!!
It was multiple choice

Expanding Perfect Parentheses

$$(a+b)^2 = a^2 + 2ab + b^2$$

A different way of thinking about it

$$(a+b)^2$$

$$= \underline{\ 1\ }\left(a^2+b^2\right)+\underline{\ 2!\ }\,ab$$

$$=\left(a^2+b^2\right)+2ab$$

1. What are all the different ways of writing two pronumerals using ***a*** and ***b***?
2. How many ways can you arrange **two *a*'s** or **two *b*'s**
3. How many ways can you arrange **one *a* and one *b***

$$(a+b+c+...+n)^2$$

$$=\left(a^2+b^2+c^2+...+n^2\right)+2\left(ab+ac+an+bc+bn+...+cn\right)$$

$$(a+b)^3$$

1. What are all the different ways of writing three pronumerals using ***a*** and ***b***?

$$=\underline{\;1\;}\left(a^3+b^3\right)+\frac{3!}{\underline{\;2!\;}}\left(ab^2+a^2b\right)$$

2. How many ways can you arrange **three *a*'s** or **three *b*'s**
3. How many ways can you arrange **two *a*'s and one *b*** or **two *b*'s and one *a***

$$=\left(a^3+b^3\right)+3\left(a^2b+ab^2\right)$$

$$(a+b+c)^3$$

$$=\underline{\;1\;}\left(a^3+b^3+c^3\right)+\frac{3!}{\underline{\;2!\;}}\left(a^2b+a^2c+ab^2+ac^2+b^2c+bc^2\right)+\underline{\;3!\;}\,abc$$

$$=\left(a^3+b^3+c^3\right)+3\left(a^2b+a^2c+ab^2+ac^2+b^2c+bc^2\right)+6abc$$

$$(a+b)^4$$

$$=\underline{\;1\;}\left(a^4+b^4\right)+\frac{4!}{\underline{\;3!\;}}\left(ab^3+a^3b\right)+\frac{4!}{\underline{2!2!}}a^2b^2$$

$$=\left(a^4+b^4\right)+4\left(ab^3+a^3b\right)+6a^2b^2$$

$$(a+b+c+d)^6$$

$$=1\left(a^6+b^6+c^6+d^6\right)+\frac{6!}{5!}\left(a^5b+a^5c+a^5d+ab^5+\ldots+cd^5\right)$$

$$+\frac{6!}{4!2!}\left(a^4b^2+a^4c^2+a^4d^2+a^2b^4+\ldots+c^2d^4\right)$$

$$+\frac{6!}{3!3!}\left(a^3b^3+a^3c^3+a^3d^3+b^3c^3+b^3d^3+c^3d^3\right)$$

$$+\frac{6!}{4!}\left(a^4bc+a^4bd+a^4cd+ab^4c+\ldots+bcd^4\right)$$

$$+\frac{6!}{3!2!}\left(a^3b^2c+a^3b^2d+a^3c^2d+a^2b^3c+\ldots+bc^2d^3\right)$$

$$+\frac{6!}{2!2!2!}\left(a^2b^2c^2+a^2b^2d^2+a^2c^2d^2+b^2c^2d^2\right)$$

$$+\frac{6!}{3!}\left(a^3bcd+ab^3cd+abc^3d+abcd^3\right)$$

$$+\frac{6!}{2!2!}\left(a^2b^2cd+a^2bc^2d+a^2bcd^2+ab^2c^2d+ab^2cd^2+abc^2d^2\right)$$