5b) 1 2 3 4 5 6 7 8

two odd / two even = ${}^4C_2 \times {}^4C_2$

c) exactly one odd = ${}^4C_1 \times {}^4C_3$

d) all even = ${}^4C_4$

e) ≥ 1 odd = ${}^8C_4 - {}^4C_4$

7d) 6M, 8W

committees of 5

exactly 2M = ${}^{6}C_2 \times {}^{8}C_3$

f) majority women = ${}^{6}C_2 \times {}^{8}C_3 + {}^{6}C_1 \times {}^{8}C_4 + {}^{6}C_0 \times {}^{8}C_5$

g) man A is included = $1 \times {}^{13}C_4$

h) man A <u>NOT</u> included = ${}^{13}C_5$

7f) 6 m 8 w

majority w = 3w + 4w + 5w

$$= {}^{8}C_3 \times {}^{6}C_2 + {}^{8}C_4 \times {}^{6}C_1 + {}^{8}C_5$$

g) Committees = $1 \times {}^{13}C_4$

h) Committees = ${}^{14}C_4 - {}^{13}C_4$

9. $\cancel{A} \longrightarrow L$

c) Ways J/K excluded = ${}^{10}C_7$

d) A included, H is not = $1 \times {}^{10}C_6$

e) one of F/L included, other not = ${}^{2}C_1 \times {}^{10}C_6$

10 ?? :.

9 8 7 6 5 4 3 2 1 0

5 digits in ascending order = ${}^{9}C_5$

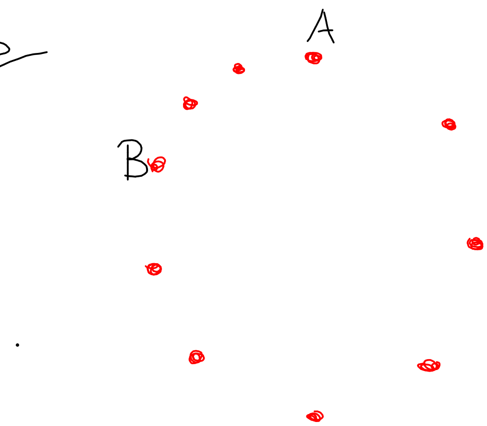
descending order = ${}^{10}C_5$

11)

6 4 2

12 people assigned = ${}^{12}C_6 \times {}^{6}C_4 \times {}^{2}C_2$

13



a) # lines = ${}^{10}C_2$

b) $\triangle$'s = ${}^{10}C_3$

c) $\triangle$'s use A = ${}^{9}C_2$

d) $\triangle$'s use A, B = ${}^{8}C_1$

14.

6 people $\Rightarrow$ 2 unequal groups.

5, 1

4, 2

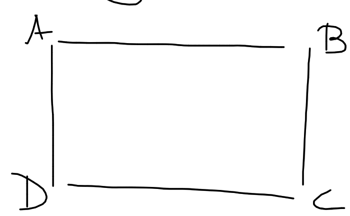
Ways $= {}^6C_5 + {}^6C_4$

15/ A B C D E F

2 unequal groups

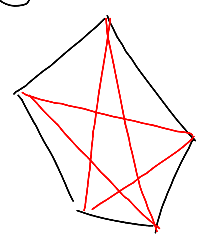
ways = ${}^6C_4 + {}^6C_5$

17a) # diagonals in a quadrilateral



$= 2 = {}^{4}C_2 - 4$

b) # diagonals in a pentagon

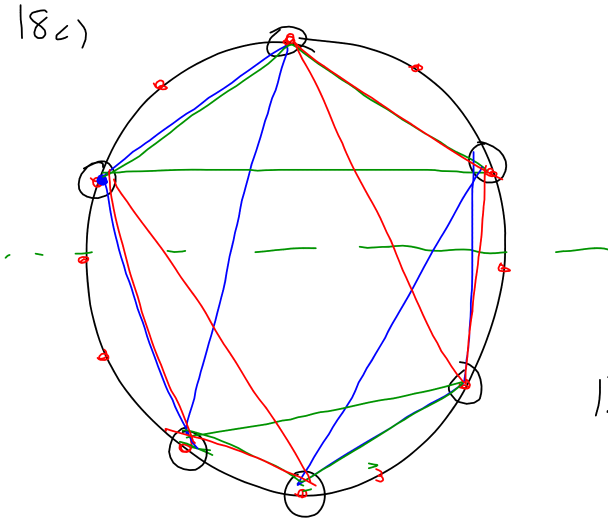


$= 5 = {}^{5}C_2 - 5$

c) # diagonals in a decagon $= {}^{10}C_2 - 10$

d) # diagonals in a n-gon $= {}^{n}C_2 - n$

18c)



not overlap.

△1 3 adj points ${}^{9}C_3$

△2

$${}^{12}C_6 \times 3$$

19 $S = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$

$$\text{Subsets} = {}^{10}C_{10} + {}^{10}C_9 + {}^{10}C_8 + {}^{10}C_7 + \dots + {}^{10}C_1 + {}^{10}C_0$$

$$= 2^{10}$$

$$= \underline{\underline{1024}}$$

$$\text{Subsets} \geq 3 \text{ numbers} = 1024 - {}^{10}C_2 - {}^{10}C_1 - {}^{10}C_0$$

$$= \underline{968}$$

Subsets $\geq$ 3 numbers, not '7' $= 2^9 - {}^9C_2 - {}^9C_1 - {}^9C_0$

Subsets $\geq$ 3 numbers, not '7', must have '13'

$$= 2^8 - {}^8C_1 - {}^8C_0$$

21) b) 10 basketballers, divide into 2 teams of 5

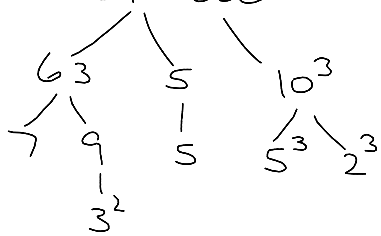
$$\text{Ways} = \frac{{}^{10}C_5}{2!}$$

when groups are = in size divide by # of groups!

ie #ways groups can be arranged.

23/

$315000 = 2^3 \times 5^4 \times 7 \times 3^2$



$\# \text{divisors} = {}^4C_1 \times {}^5C_1 \times {}^2C_1 \times {}^3C_1$

$= \underline{\underline{120}}$

24 $\quad {}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$

$$\frac{n!}{r!(n-r)!} + \frac{n!}{(r+1)!(n-r-1)!}$$

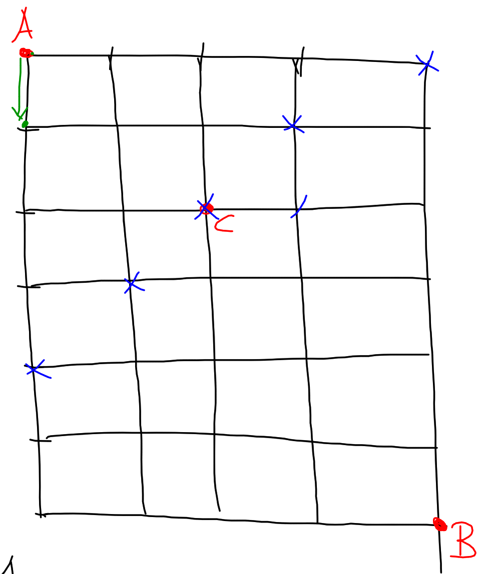
$$= \frac{n!(r+1) + n!(n-r)}{(r+1)!(n-r)!}$$

$$= \frac{n!(r+1+n-r)}{(r+1)!(n-r)!}$$

$$= \frac{n!(n+1)}{(r+1)!(n-r)!}$$

$$= \frac{(n+1)!}{(r+1)!(n-r)!} = {}^{n+1}C_{r+1}$$

25/



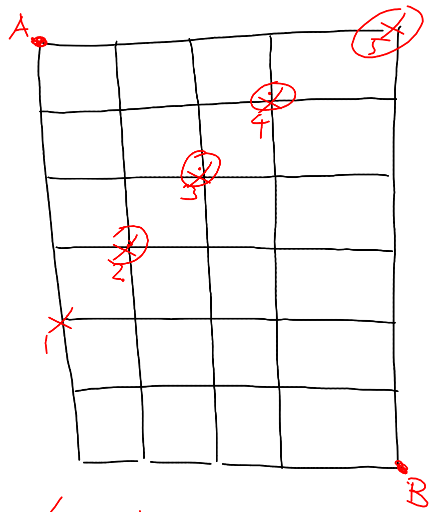
a) A to B = ${}^{10}C_6$

b) pass through C = ${}^{4}C_2 \times {}^{6}C_4$

c) can't use top line = $1 \times {}^{9}C_5$

d) can't use 2nd row = ${}^{9}C_5$

$${}^{10}C_4 = {}^{4}C_4 \times {}^{6}C_6 + {}^{4}C_3 \times {}^{6}C_5$$



Prove:

$$ {}^{10}C_4 = {}^4C_0 \times {}^6C_4 + {}^4C_1 \times {}^6C_3 + {}^4C_2 \times {}^6C_2 + {}^4C_3 \times {}^6C_1 + {}^6C_0 $$

$X_1 = {}^6C_4 \cdot {}^4C_0$

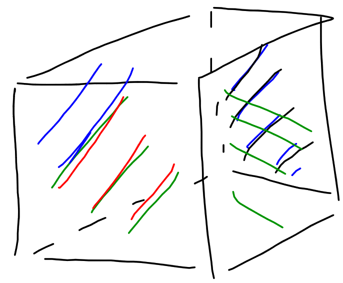
$X_2 = {}^4C_1 \times {}^6C_3$

$X_3 = {}^4C_2 \times {}^6C_2$

$X_4 = {}^4C_3 \times {}^6C_1$

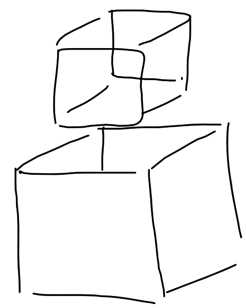
$X_5 = {}^4C_4 \times {}^6C_0$

26)



6 x 5

b)



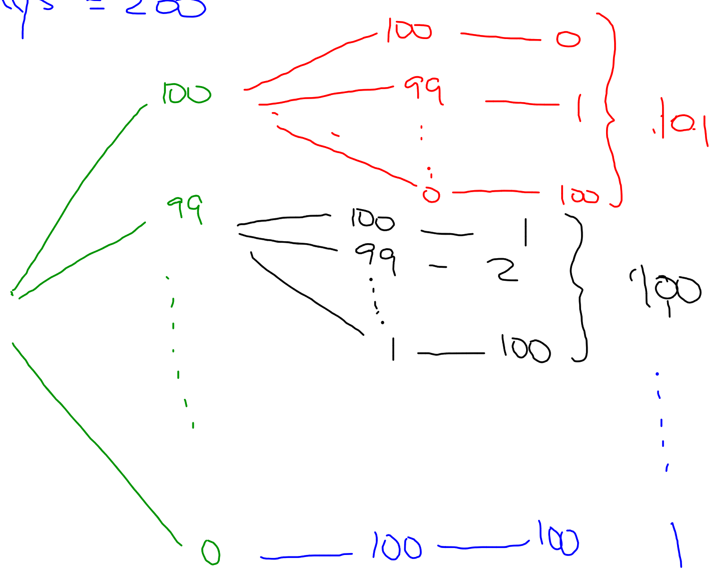
Ways = 6 x 4

= 24

27

$$\frac{D}{100} \quad \frac{T}{100} \quad \frac{O}{100} = \frac{\text{Mark}}{300}$$

Ways "= 200"



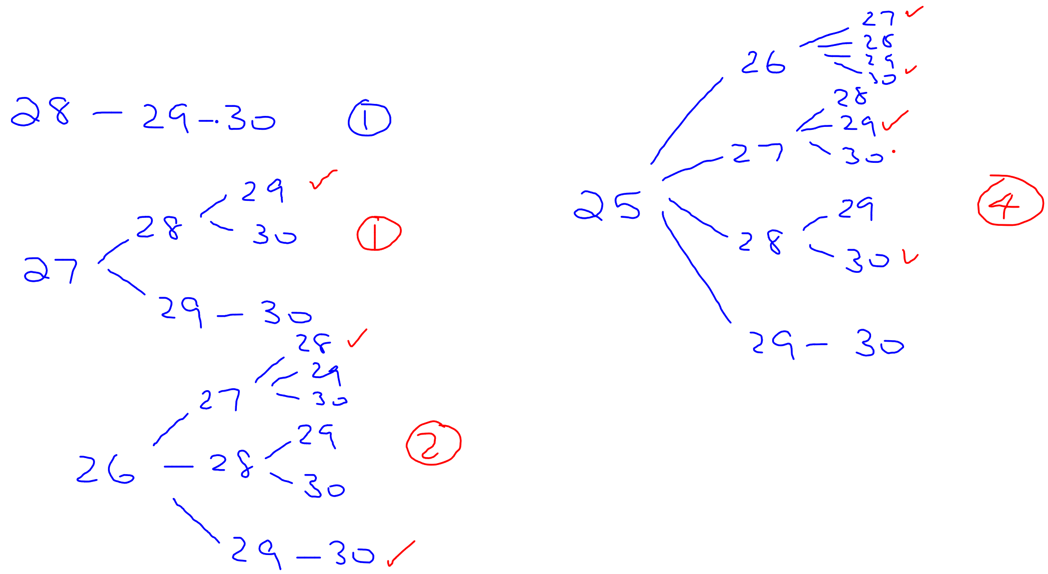
Ways

$= 1 + 2 + \dots + 101$

$= \dfrac{101 \times 102}{2}$

$= \underline{\underline{5151}}$

23



28 — 29 — 30
1
27
28
29
30
1
29 — 30
26
27
28
29
30
28
29
30
2
29 — 30
25
26
27
28
29
30
27
28
29
30
28
29
30
29 — 30
4

29/ doubles tennis

a) 4 players $= \frac{{}^{4}C_{2}}{2!}$

b) 8 players / 2 games $= \frac{{}^{8}C_{4}}{2!} \times \frac{{}^{4}C_{2}}{2!} \times \frac{{}^{4}C_{2}}{2!}$

c) 6 married couples (12) $= \frac{{}^{12}C_{4} \times {}^{8}C_{4}}{3!} \times \frac{{}^{4}C_{2}}{2!} \times \frac{{}^{4}C_{2}}{2!} \times \frac{{}^{4}C_{2}}{2!}$

d) mixed doubles

$$= \frac{{}^{6}C_{2} \times {}^{6}C_{2} \times {}^{4}C_{2} \times {}^{4}C_{2}}{3!} \times \left(\frac{{}^{2}C_{1} \times {}^{2}C_{1}}{2!} \right)^{3}$$